K_{e3} DECAY AS A TEST OF UNIVERSAL V-A LEPTON COUPLING*

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In this note, we show that the experimental spectrum of electrons from K_{e3} decay provides a direct test of universal vector-axial vector lepton coupling. The test is independent of the K-π form factor. We also discuss briefly the best way of treating the data so as to obtain information about the form factor itself.

K_{e3} decay has been treated extensively in the literature.[1] With the usual V-A lepton current, the matrix element for the decay is

$$T=(2\pi)^4(G/\sqrt{2})\delta^4(K-p-k-q)(4K_0q_0)^{-1/2}\bar{u}(k)\{\tfrac{1}{2}(\gamma\cdot K+\gamma\cdot q)f_+(Q^2)+\tfrac{1}{2}(\gamma\cdot K-\gamma\cdot q)f_-(Q^2)\}(1+\gamma_5)v(p). \quad (1)$$

We use units in which $\hbar=c=m_\pi=1$. The mass and four-momentum of K, π, e, ν are (M,K), $(1,q)$, (m_e,p), and $(0,k)$, respectively;

$$Q^2=(K-q)^2=(K-q)_0{}^2-(\vec{K}-\vec{q})^2; \quad (2)$$

f_+ and f_-, the K-π form factors, are analytic except for a branch on the real axis for $Q^2>(M+1)^2$; they are real in the range of Q^2 which occurs in K_{e3} decay: $(M^2-1)/2M<Q<M$. The equations

$$\bar{u}(k)\gamma\cdot k=0,\quad \gamma\cdot pv(p)=m_ev(p) \quad (3)$$

can be used to write Eq. (1) in the form

$$T=(2\pi)^4(G/\sqrt{2})(4K_0q_0)^{-1/2}f_+(Q^2)\delta^4(K-p-q-k)$$
$$\times\ \bar{u}(k)\gamma\cdot K(1+\gamma_5)v(p). \quad (4)$$

In Eq. (4) and in the following, we set $m_e=0$. This neglect of m_e is justified as long as $|f_-/f_+|\ll M/m_e\sim10^3$; the near equality of the K_{e3} and $K_{\mu3}$ rates[2] shows that this latter condition is satisfied.

We find for the number of electrons per unit time in the energy interval dp emitted by K's decaying at rest

$$N(p)dp=dp[G^2/2(2\pi)^3]M(M-2p)F(x), \quad (5)$$

where

$$x=\frac{2p(p_{\max}-p)}{M-2p},\quad p_{\max}=\frac{M^2-1}{2M}, \quad (6)$$

$$F(x)=\int_0^x dy\,(x-y)f_+^2(2My). \quad (7)$$

This energy distribution for the electrons occurs (with different normalization) for any combination of vector and axial vector in the lepton current.

Note that p is a double-valued function of x. Therefore, $N(p)/(M-2p)\propto F(x)$ must have the same value for the two values of p corresponding to each value of x. This is, therefore, a simple condition that the electron spectrum must satisfy if leptons are universally coupled via a vector-axial vector current. This condition is independent of the form factor which occurs in $F(x)$ and depends only on the assumption of V-A lepton coupling. The data available at present are not sufficiently accurate to allow any conclusion as to whether or not this condition is satisfied.

$F(x)$ also has the properties

$$F(x)>0,$$
$$F'(x)>0,\quad F'(x)\geqslant x^{-1}F(x),$$
$$F''(x)=f_+^2(2Mx). \quad (8)$$

From these properties, it follows that $N(p)$ cannot have minima or maxima unless

$$p=0,\ p=p_{\max},\ \text{or}\ (M^2-1)/4M\leqslant p\leqslant(M-1)/2. \quad (9)$$

Thus, except for the end points, maxima and minima must lie in the range

$$114\ \text{Mev}<p<180\ \text{Mev}. \quad (10)$$

The histogram given by Furuichi et al.[3] does not appear to satisfy this last condition, although improved statistics are needed before a definite conclusion can be drawn.

With sufficient data, it will be possible to determine $F(x)\propto N(p)/(M-2p)$. Besides the single-valuedness of $F(x)$, the conditions (7) can be verified and, in principle, $f_+^2(2Mx)$ can be determined from the curvature of $F(x)$. Note that $F(x)$ can be determined more accurately than $N(p)$, since each value of $F(x)$ comes from both values of p corresponding to that x and therefore has an accuracy greater than either of the corresponding values of $N(p)$.

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[1]See N. Brene, L. Egardt, and B. Qvist, Nuclear Phys. 22, 553 (1961) for a complete list of references.

[2]K. Bøggild, K. H. Hansen, J. E. Hooper, M. Scharff, and P. K. Aditya, Nuovo cimento 19, 621 (1961).

[3]S. Furuichi, S. Sawada, and M. Yonezawa, Nuovo cimento 10, 541 (1958).

POSSIBLE EXPLANATION OF THE HIGHER PION-NUCLEON AND K^--p RESONANCES IN TERMS OF INELASTIC THRESHOLDS

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We have found by means of partial-wave dispersion relations that a large, rapidly rising inelastic cross section, such as is observed in the vicinity of the second and third pion-nucleon resonances,[1] can account for a sharp peak in the elastic scattering. We have calculated the inelastic contribution to the higher partial waves by means of the strip approximation to the Mandelstam representation,[2,3] in which the principal mechanism is the production of the $J=1$, $I=1$ pion-pion resonance. Assigning to this resonance a position and width in accord with recent experimental evidence,[4] we find strong inelastic scattering in angular momentum and isotopic spin states which phenomenological analyses have suggested as accounting for the resonances.[5,6] Moreover, resonances seem to be implied in approximately the correct positions, although we are unable at this stage to make really quantitative calculations. The same strip approximation calculation seems to imply a K^--p resonance, primarily in the $D_{3/2}$ state, at the observed energy,[7] provided the recently observed K^* has $J=1$ and $I=\frac{1}{2}$.[8]

In order to find the effect of strong, rapidly rising inelastic scattering in a given partial wave, consider the amplitude $f(\nu)$, which assumes the following form in the physical region:

$$f(\nu)=(e^{2i\delta}-1)/2i. \qquad (1)$$

Above the inelastic threshold ν_i, $\delta=\delta_R+i\delta_I$, with $\delta_I>0$ according to unitarity. The variable ν is chosen appropriate to each particular reaction; for equal-mass scattering $\nu=k^2$, the square of the momentum in the center-of-mass system. Note that

$$\operatorname{Im}f=|f|^2+(1-\eta^2)/4, \qquad (2)$$

where $\eta=e^{-2\delta_I}$. Thus the inelastic contribution to the cross section is proportional to $(1-\eta^2)/4$, so that δ_I is determined if the inelastic scattering in the partial wave is known. Let us assume for the moment that this is the case.

In order to keep the argument from being obscured by inessential complications, we shall now make two simplifying assumptions which could be relaxed without difficulty. We assume that $A(\nu)=f(\nu)/k$ is the function which is analytic in the ν plane except for the singularities given by the Mandelstam representation, thereby neglecting some relativistic effects. Since the point we wish to make concerns the effect of the inelastic scattering on the elastic at rather high energies, we also neglect the unphysical cuts.

We now have the following mathematical problem: Find a function $A=f/k$ with the properties: (a) It is analytic in the entire ν plane, except for a cut along the positive real axis. (b) It is real on the negative real axis. (c) The function $f(\nu)$ assumes the form of Eq. (1), with given δ_I, when the positive real axis is approached from above.

The solution is remarkably simple; construct

$$\delta(\nu)=\frac{k}{\pi}\int_{\nu_i}^{\infty}d\nu'\frac{\delta_I(\nu')}{k'(\nu'-\nu)}. \qquad (3)$$

Using this formula to define δ in the entire ν plane, we assert that Eq. (1) provides a solution to the problem posed.[9,10] It can easily be verified that $A(\nu)$ has only the desired branch cuts, provided k is defined to have its cut along the positive real axis.

Let us now consider the effect upon the elastic amplitude of a rapidly rising inelastic cross section which rises to a value near its unitarity limit, such as is shown by the solid line in Fig. 1. Equation (3) implies that δ_R will be a sharply peaked function in the neighborhood of the rise