# QUASI-ELASTIC PEAK IN HIGH-ENERGY NUCLEON-NUCLEON SCATTERING*

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A bump in the energy spectrum of inelastically scattered protons emerging from proton-nucleus collisions for incident energies in the range 9-25 Bev, and for 20-60 milliradians (mr) scattering angles has been reported by Cocconi et al.[1] at CERN. Characteristic feature of this bump are:

1. The energy difference between the inelastic peak and the peak corresponding to elastic scattering at the same angle is ≈0.8 - 1.1 Bev and is roughly independent of the scattering energy and angle.

2. The height of the inelastic peak falls off with increasing momentum transfer as does the elastic cross section, in a manner suggestive of diffraction scattering.

It is the purpose of this Letter to suggest a mechanism giving rise to a bump with these characteristics. This mechanism is the diffraction scattering of a pion in the cloud of the target nucleon from the incident projectile nucleon. It leads to a simple physical picture with an apparently general application. We wish to discuss this here, and to give the main results of calculations for the conditions of observation at CERN. A detailed report of the calculations and especially of the relevant, and somewhat involved, relativistic kinematics is in preparation.[2]

Consider the Feynman diagram of Fig. 1 which introduces the notation $q_i$, $p_i$, and $p_f$ for the four-momenta of a target, incident, and observed final nucleon, respectively; $q_i^2 = p_i^2 = p_f^2 = m^2$; ($\hbar = c = 1$). $q$ is the recoil nucleon four-momentum and $\Delta \equiv q_i - q$ is the transferred momentum; $\Delta^2 < 0$. The meson produced in the collision carries off $k$, with $k^2 = \mu^2$. In the following, the binding of the target nucleon in the target nucleus is neglected and the mass ratio $\mu/m \to 0$.

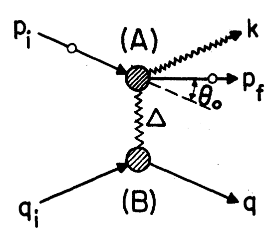


FIG. 1. Feynman diagram for scattering of a pion in the cloud of the target nucleon from the incident projectile nucleon. The circle o on the nucleon line denotes the high-energy incident and observed final nucleon.

We are interested only in a very restricted region of scattering through a laboratory angle $\theta_0 \leq 60$ mr and with small energy loss $\epsilon \equiv \delta + t/2m \approx 1$ Bev $\ll E_i$, where $t \equiv (p_i - p_f)^2 < 0$ is the invariant momentum transfer between the incident and observed nucleons of energy $E_i$ and $E_f$, respectively, in the lab system, and $\delta \equiv E_i - E_f$; $\epsilon = 0$ for elastic scattering. For such events the magnitude of the momentum transferred between the two vertices $A$ and $B$ can be small $[|\Delta|_{\min}^2 \simeq (2.4\,\mu)^2$ for $E_i = 25$ Bev, $\theta_0 = 40$ mr, and $\epsilon = 1$ Bev$]$ and we approximate the scattering amplitude to the pole term shown in Fig. 1.

This approximation has been widely discussed recently[3] and is applied here with optimism in its at least qualitative validity since there is an enhancement factor at vertex $A$ which we discuss in the following paragraph. We neglect contributions from graphs of the types in Fig. 2. Figure 2(a) is an exchange contribution and is unimportant when one of the two nucleons has almost all of the energy. Figure 2(b) was discussed previously[4] and is discarded here for two reasons: The momentum transfer between vertices is ≈1 Bev and far from the peripheral condition of an almost real pion being exchanged. If, nevertheless, the pole approximation is applied, its contribution to the inelastic peak is smaller than the observed cross section as well as smaller than

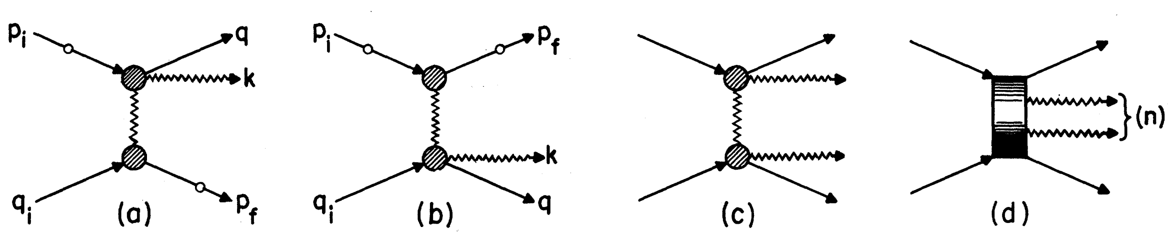


FIG. 2. Feynman diagrams for contributions neglected in the present calculation.

the contribution calculated from Fig. 1. (For $E_i = 16$ Bev, $\theta_0 = 40$ mr, and $\epsilon = 1$ Bev, it is smaller by a factor $\approx 4$; see also footnote 10.) Feld and Iso[5] have shown that the inelastic bump occurs at an energy corresponding to a mean of the second and third resonances in pion-nucleon scattering and suggest this isobar formation as the cause of the observed inelastic bump. Their argument is kinematical, without reference to specific (poor) field-theoretic approximations. However, it is difficult to reconcile with the absence of an inelastic bump at $\epsilon \approx 300$ Mev corresponding to the large 3-3 resonance in pion-nucleon scattering. Figure 2(c)[6] can be shown[2] to contribute insignificantly for the small values of $\epsilon \approx 1$ Bev of interest here, primarily due to phase-space limitations. Figure 2(d) for more than a single-pion exchange between $A$ and $B$ is not calculable and its neglect can be defended only by measuring the ability of Fig. 1 in reproducing the observed cross sections.

An inelastic bump at the observed $\epsilon$ and of the observed magnitude emerges from Fig. 1 in the following way. The interaction at vertex $A$ is treated as physical pion-nucleon scattering for small values of $|\Delta^2| < M^2$. The spin average of the square of the invariant matrix element for the pion-nucleon scattering, $\frac{1}{2}\sum_{\text{spins}} |M_A|^2 \equiv X_A$, can thus be related to experimental cross sections.

In the rest system of the incident nucleon we can write

$$X_A = 4(2\pi)^2 \left(1 - \frac{t}{s_1 - m^2 + t}\right)^2 \frac{d\sigma_{A,\text{lab}}(s_1, t)}{d\Omega}, \qquad (1)$$

in terms of the invariant total energy $s_1 \equiv (p_f + k)^2 > 0$ and momentum transfer $t = (p_i - p_f)^2 < 0$. We are interested in the experimental conditions at CERN which limit momentum transfers to $|t| \lesssim 2$ (Bev)$^2$. On the other hand, the scattering energy, $s_1$, can be much larger for inelasticities $\epsilon \approx 1$ Bev. It is this possibility that leads to the enhancement at the vertex $A$: For very large energies $s_1$, the pion-nucleon differential cross section develops a large forward diffraction peak. For forward scattering, $t = 0$, and we have in the approximation that there is mainly diffraction scattering at high energies:

$$\frac{d\sigma_{A,\text{lab}}(s_1, 0)}{d\Omega} \simeq \frac{\sigma_{\text{tot}}^2(s_1)}{4(4\pi)^2} \frac{(s_1 - m^2)^2}{m^2}. \qquad (2)$$

Here $\sigma_{\text{tot}}(s_1)$ is the total cross section for the pion-nucleon cross section and is nearly constant[7] [$\sigma_{\text{tot}} \approx 25$-$30$ mb at high energies, $s_1 \gtrsim (2$ Bev$)^2$]. The observed diffraction peak in high-energy $\pi$-$N$ scattering suggests that we put in (1)

$$\frac{d\sigma_{A,\text{lab}}(s_1, t)}{d\Omega} = \frac{d\sigma_{A,\text{lab}}(s_1, 0)}{d\Omega} g(t). \qquad (3)$$

$g(t)$ must fall off faster than $t^{-1}$ at large space-like $t$ so that $\sigma_{A,\text{lab}}(s_1)$ remains constant as observed[7] in the several-Bev region, $\approx 5$-$8$ mb. We choose

$$g(t) = 1 \Big/ \left(1 - \frac{t}{10\mu^2}\right)^2, \qquad (4)$$

for simplicity.[8] The results are insensitive to the specific form assumed here.[2] Inserting (2), (3), and (4) into (1) and approximating $|s_1/t| \gg 1$, since the second factor (2) weights heavily to high scattering energies, we obtain

$$X_A \cong \tfrac{1}{4}\sigma_{\text{tot}}^2(s_1) \frac{(s_1 - m^2)^2}{m^2} \Big/ \left(1 - \frac{t}{10\mu^2}\right)^2. \qquad (5)$$

This strong weighting of the amplitude at $A$ to large energies $s_1$ is what leads to a bump as observed in the CERN experiments. Of the nine final momentum components $\vec{p}_f$, $\vec{q}$, and $\vec{k}$, only five are kinematically independent. If we observe a final nucleon at fixed $\vec{p}_f$ this leaves a two-dimensional integral,

$$\int d^3q d^3k\, \delta^4(q + k + p_f - p_i - q_i) = \int d^3\Delta d^3k\, \delta^4(k + p_f - p_i - \Delta),$$

to be carried out. What (5) tells us is that the contribution from large values of $s_1 = (p_f + k)^2 = (p_i + \Delta)^2$ is greatly enhanced in the phase-space integrals. This means that $\vec{p}_i$ and $\vec{\Delta}$ tend to be antiparallel, and the dominant contribution to the observed inelastic scattering comes when the target nucleon is hit head-on in the lab system. If the angles of $\vec{\Delta}$ are constrained in this way, we are then led to a unique relation between the magnitude of the momentum transfer $\Delta^2 \approx -|\vec{\Delta}|^2$ for $|\Delta^2| < M^2$, and the final nucleon energy $E_f$, or inelasticity,[9] $\epsilon$:

$$\epsilon = \frac{|\vec{\Delta}|}{2} - \frac{t}{2}\left(\frac{1}{|\vec{\Delta}|} - \frac{1}{m}\right). \qquad (6)$$

Integrating over the range of possible $|\vec{\Delta}|$ for a given $E_i$ and $\theta_0$ leads to a corresponding range of preferred $\epsilon$ values according to (6). For the conditions of the CERN experiments these preferred inelasticities occur at $\frac{1}{2}$ Bev $\lesssim \epsilon \lesssim 2$ Bev and in the range of the observed quasi-elastic peaks.

The more accurate results of detailed calculations reproduce these general features. Inserting (5) into the calculation of the differential cross section for Fig. 1 gives

$$\frac{d^2\sigma}{dE_f d\Omega_f} \simeq \frac{3}{8}\frac{1}{(2\pi)^4}\frac{f^2}{\mu^2}\sigma_{\text{tot}}^2(s_1)g(t)\frac{E_f}{E_i}\frac{1}{|\vec{p}_i - \vec{p}_f|}\int_{a-b}^{a+b} d(-\Delta^2)\frac{(-\Delta^2)}{(-\Delta^2+\mu^2)^2}F^4(\Delta^2)\int_0^{2\pi} d\varphi(s_1 - m^2)^2, \qquad (7)$$

where

$$a \approx \frac{2m^2 - (m+\epsilon)t}{m+2\epsilon} \text{ and } b \simeq |\vec{p}_i - \vec{p}_f|\frac{2m\epsilon}{m+2\epsilon}, \qquad (8)$$

and $\varphi$ is the azimuthal angle of $\vec{q}$ relative to the plane of $\vec{p}_i$ and $\vec{q}_i$, which determines the polar axis. The integral $d\varphi$ is best carried out in the system in which the pion and recoil nucleon come off back to back; i.e., $\vec{k}+\vec{q}=0$. The form factor $F(\Delta^2)$ is introduced at each vertex to cut off the integral $d(-\Delta^2)$ at maximum values of $-\Delta^2 \approx (3\mu$ to $4\mu)^2$ compatible with the peripheral collision assumption of one-pion exchange with a physical amplitude inserted at vertex $A$. The observed quasi-elastic bump comes about because the emerging nucleon wants to steal most of the energy as indicated by (6). On the other hand, the integration interval in (7) vanishes when $b \propto \epsilon \to 0$ and this leads to a very sharp dropoff of the cross section at the inelastic threshold $\epsilon = 0$.

The second characteristic feature, of a decrease in peak height with increasing momentum transfer, is assured both by the diffraction factor $g(t)$ in (7) and by the integration limits, (8). As $-t$ increases, so does $a$ and, therefore, $(-\Delta^2)_{\text{min}}$; this reduces the integral $d(-\Delta^2)$ and further decreases $d^2\sigma/dE_f d\Omega_f$. The first characteristic feature, that the quasi-elastic peak is separated from the elastic one by a constant $\epsilon$ independent of momentum transfer $t$ in the range $\frac{1}{2}$ (Bev)$^2 \lesssim -t \lesssim 2$ (Bev)$^2$, is also established. In order to understand this result, note that, according to (8), $(-\Delta^2)_{\text{min}}$ increases with $-t$; this leads to a corresponding increase in the mean value of $|\vec{\Delta}|$ contributing to (7). As suggested by (6), an appropriate increase of $|\vec{\Delta}|$ with $-t$ can lead to a constant $\epsilon$ independent of $t$. This is what we find to occur to a good approximation.

Calculated curves are shown in Fig. 3 together with the data. The calculated numbers correspond to a cutoff in the integral $\int d(-\Delta^2)$ at a maximum[10] of $(4\mu)^2$. Were we to run the integrals out to the limits with $F(\Delta^2) \to 1$, the characteristic bump would remain but its height increases by a factor ≈6 for the 40-mr points and for the calculations at $E = 16.0$ Bev and $\theta_0 = 56$ mr. For the points with the largest momentum transfers, at $E_i = 26.10$ Bev and $\theta_0 = 56$ mr where $-t \approx 2$ (Bev)$^2$, the calculations are much less reliable. We are farther from the one-pion exchange pole, since, for $\epsilon \approx 1$ Bev, $|\Delta_{\text{min}}| \approx 4\mu$ and the calculated cross sections are cutoff-sensitive and reduced by an order of magnitude. The neglected processes of Fig. 2 may then be relatively important and not negligible.

From the above arguments it appears that the process of Fig. 1 is the dominant peripheral contribution to quasi-elastic small-angle nucleon-nucleon scattering at high energies. The contribution of Fig. 2(b), which was discussed as a

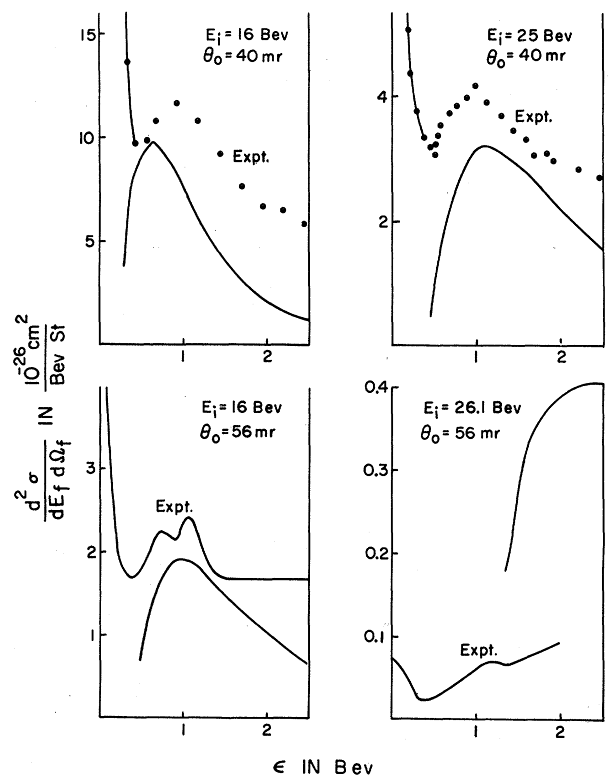


FIG. 3. Comparison of observed[1] cross sections in the region of the quasi-elastic bump for different values of the experimental parameters with calculations from Eq. (7).

test of the peripheral approximations,[4] is less important for the very small inelasticities, $\epsilon$, considered here. This conclusion is also true as $\theta_0 \to 0$ and we approach the one-pion exchange pole.[2] It wins out only for larger $\epsilon$ which keep the amplitude of Fig. 1 from developing its diffraction peak. In an interesting letter which helped motivate this work, Selleri[11] has shown that at much lower energies both processes are important.

Our model offers no explanation of the camel's hump structure suggested in the recent experiments in progress at CERN.[1] Its existence, if confirmed, may indicate final-state interactions or other effects not included in the simple picture.

Finally we note that similar considerations apply to high-energy $\pi$-$N$ scattering cross sections. For the same kinematics as in the CERN measurements, there should be a quasi-elastic diffraction peak in the process $\pi + N \to \pi + \pi + N$, whose height measures the square of the total $\pi$-$\pi$ cross section in place of the $\pi$-$N$ cross section $\sigma_{\text{tot}}^2(s_1)$, in (7).

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[1]G. Cocconi, A. N. Diddens, E. Lillethun, and A. M. Wetherell, Phys. Rev. Letters 6, 231 (1961). We wish to thank Dr. Wetherell for sending us results of further experiments on C and $CH_2$ targets carried out with higher resolution by Cocconi, Diddens, Lillethun, Manning, Taylor, Walker, and Wetherell. The same general features are observed with the added suggestion of a possible structure in the inelastic peak in the form of camel humps. The results of this study were presented at the CERN Conference on High-Energy Processes, Geneva, Switzerland, June, 1961 (unpublished) by Professor Marshall Baker of Stanford to whom we are grateful for a valuable report and discussion of the proceedings of the Conference especially as relevant to this work.

[2]K. Hiida and S. D. Drell (to be published). A similar mechanism was considered by D. Amati and J. Prentky (private communication).

[3]For a general discussion as well as detailed references to the literature contributed by many authors, see S. D. Drell, Revs. Modern Phys. 33, 458 (1961).

[4]S. D. Drell, Phys. Rev. Letters 5, 342 (1960).

[5]B. T. Feld and C. Iso, Nuovo cimento (to be published). We wish to thank these authors for a copy of their paper. Feld has also shown (to be published) that this isobar model leads to a diffraction angular distribution also in accord with the observed general features.

[6]I. M. Dremin and D. S. Chernavskii, J. Exptl. Theoret. Phys. (U.S.S.R.) 38, 229 (1960) [translation: Soviet Phys.–JETP 11, 167 (1960)]; D. S. Chernavski, I. M. Dremin, I. M. Gramenitski, and V. M. Maksimenko, Report A-27, and D. S. Chernavski and I. M. Dremin, Report A-28 of the Lebedev Physical Institute, Academy of Sciences of the U.S.S.R., Moscow, 1960 (unpublished); F. Salzman and G. Salzman, Phys. Rev. Letters 5, 377 (1960), and Phys. Rev. 121, 1541 (1961).

[7]O. Piccioni, Proceedings of the 1958 Annual International Conference on High-Energy Physics at CERN (CERN Scientific Information Service, Geneva, 1958); P. Falk-Vairant, G. Valladas, and G. Cocconi, Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester (Interscience Publishers, New York, 1960).

[8]In the region of $t$ values of interest this is numerically similar to the dependence deduced by Amati (report to La Jolla Conference) from the Mandelstam representation by considering the two-pion exchange contribution to elastic $\pi$-$N$ scattering. This matches the data at several Bev.

[9]Equation (6) is insensitive to a variation in angle between $\vec{\Delta}$ and $-\vec{p}_1$ such that $(S_1 - m^2)^2$ falls to $\frac{1}{2}$ maximum value.

[10]Very similar numbers are obtained if we use a smooth cutoff such that $F^4(\Delta^2 = (3.5\mu)^2) \approx \frac{1}{2}$. The effect of such a cutoff on the isobar excitation calculation for Fig. 2(b) is to reduce the computed cross section to less than 4% of the observed value in typical parameters $E_i = 25$ Bev, $\theta_0 = 40$ mr.

[11]F. Selleri, Phys. Rev. Letters 6, 64 (1961). We find that the numerical results of this calculation should be multiplied by a factor of two due to an incorrect normalization procedure in computing the incident flux. See also T. Kobayashi, Progr. Theoret. Phys. (Kyoto) 18, 318 (1957); F. Bonsignori and F. Selleri, Nuovo cimento 15, 465 (1960).