STRUCTURE OF THE PROTON AND NEUTRON*

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We have previously analyzed our data[1] to show that the proton and neutron have a common core of positive charge of about $0.35e$, whose radius might be about 0.2 fermi. This core is surrounded by a meson cloud of radius 0.8 f, with an average charge which changes from $+0.5e$ for the proton to $-0.5e$ for the neutron. Additionally, we found it necessary to postulate another cloud, the same for the neutron and proton, with a positive charge of $0.15e$ and an rms radius of about 1.4 f, thus extending out beyond the meson cloud. These clouds of charge take on meaning in the simple A model of the nucleon put forward by Bergia, Stanghellini, Fubini, and Villi,[2] referred to herein as BSFV. Their model is based on dispersion theory and on a two-pion interaction similar to that used by Frazer and Fulco.[3] In this model they identify the meson cloud with a $T=1$, $J=1$ two-pion resonant state while the extended cloud might correspond to a three-meson resonant state, most probably with $T=0$ and $J=1$. Our new measurements,[4] when examined in the light of the above model of the nucleon, imply that the resonant energy of the two-pion state is $4.0m_\pi$ and that of the three-pion state is $2.9m_\pi$.

Let us now analyze our form factors into isoscalar and isovector partial form factors[5]:

$$F_{1p}=F_{1S}+F_{1V};\quad F_{1n}=F_{1S}-F_{1V},$$
$$F_{2p}=F_{2S}+F_{2V};\quad F_{2n}=F_{2S}-F_{2V}. \tag{1}$$

In making our simple core model analysis, we got a vanishing rms radius for the neutron charge distribution by resolving the isoscalar form factor F_{1S} of Eq. (1) further into two terms, i.e., $F_{1S}=F_{1S}^{\mathrm{core}}+F_{1S}^{\mathrm{cl}}$, where we associated F_{1S}^{core}, the term with the smaller radius, with the core of the nucleon, and F_{1S}^{cl}, with an extended cloud. Our new results can best be fitted by partial form factors corresponding to exponential density distributions. Partial form factors of Gaussian or other shape without a singularity give a worse fit. We write therefore

$$F_{1S}=e_s^{\mathrm{core}}(1+a_{s,\mathrm{core}}^2q^2/12)^{-2}+e_s^{\mathrm{cl}}(1+a_{s,\mathrm{cl}}^2q^2/12)^{-2},$$
$$F_{1V}=e_v(1+a_v^2q^2/12)^{-2}, \tag{2}$$

where the a's are the rms radii of the respective charge distributions. In order to obtain the right total charges and rms radii for the proton and neutron, the following conditions must hold:

$$e_s^{\mathrm{core}}+e_s^{\mathrm{cl}}+e_v=1,$$
$$e_s^{\mathrm{core}}+e_s^{\mathrm{cl}}-e_v=0,$$
$$e_s^{\mathrm{core}}a_{s,\mathrm{core}}^2+e_s^{\mathrm{cl}}a_{s,\mathrm{cl}}^2+e_va_v^2=a_p^2,$$
$$e_s^{\mathrm{core}}a_{s,\mathrm{core}}^2+e_s^{\mathrm{cl}}a_{s,\mathrm{cl}}^2-e_va_v^2=a_n^2. \tag{3}$$

Assuming that $a_{s,\mathrm{core}}^2 \ll a_{s,\mathrm{cl}}^2$ and because $a_n^2=0$, one obtains

$$e_v=\tfrac{1}{2},\quad e_s^{\mathrm{core}}+e_s^{\mathrm{cl}}=\tfrac{1}{2},$$
$$a_v^2=2e_s^{\mathrm{cl}}a_{s,\mathrm{cl}}^2=a_p^2. \tag{4}$$

These conditions restrict the six adjustable parameters so that only two are left to fit the experimental data at higher q values, e.g., e_S^{core} and $a_{S,\mathrm{core}}$.

To describe our newly determined magnetic form factors, we now assign partial magnetic moments μ_S^{core}, μ_S^{cl}, μ_v to the same core and clouds that we have used to describe the charge distributions and we write

$$F_{2S}=\mu_s^{\mathrm{core}}(1+b_{s,\mathrm{core}}^2q^2/12)^{-2}+\mu_s^{\mathrm{cl}}(1+b_{s,\mathrm{cl}}^2q^2/12)^{-2},$$
$$F_{2V}=\mu_v(1+b_v^2q^2/12)^{-2}, \tag{5}$$

where $b_{s,\,\mathrm{core}}$, $b_{s,\,\mathrm{cl}}$, and b_v are the rms radii of the partial form factors for the magnetic moment and all magnetic moments are measured in nuclear magnetons (nm).

In exactly the same manner as for the partial electric charges, we can infer the conditions

$$\mu_s^{\mathrm{core}} + \mu_s^{\mathrm{cl}} + \mu_v = \mu_p = 1.793 \text{ nm},$$
$$\mu_s^{\mathrm{core}} + \mu_s^{\mathrm{cl}} - \mu_v = \mu_n = -1.913 \text{ nm},$$
$$\mu_s^{\mathrm{core}} b_{s,\,\mathrm{core}}^2 + \mu_s^{\mathrm{cl}} b_{s,\,\mathrm{cl}}^2 + \mu_v b_v^2 = \mu_p b_p^2,$$
$$\mu_s^{\mathrm{core}} b_{s,\,\mathrm{core}}^2 + \mu_s^{\mathrm{cl}} b_{s,\,\mathrm{cl}}^2 - \mu_v b_v^2 = \mu_n b_n^2. \tag{6}$$

From these one readily obtains with $b_{s,\,\mathrm{core}}^2 \ll b_{s,\,\mathrm{cl}}^2$:

$$\mu_v = 1,$$
$$\mu_s^{\mathrm{core}} + \mu_s^{\mathrm{cl}} = (\mu_p + \mu_n)/2 = -0.060 \text{ nm}, \tag{7}$$
$$2\mu_s^{\mathrm{cl}} b_{s,\,\mathrm{cl}}^2 = \mu_p b_p^2 + \mu_n b_n^2,$$
$$2\mu_v b_v^2 = \mu_p b_p^2 - \mu_n b_n^2. \tag{8}$$

A consequence of these conditions is again that only two parameters, e.g., μ_s^{core} and $b_{s,\,\mathrm{core}}$, are left to fit the data at high q values, if b_p^2 and b_n^2 are obtained from the slope of the form factors at small q values.

Table I. Best-fit parameters for core model with exponential density distributions. See text for definition of symbols.

$e_s^{\mathrm{core}} = 0.25$	$a_{s,\mathrm{core}} = 0.2$ f
$e_s^{\mathrm{cl}} = 0.25$	$a_{s,\mathrm{cl}} = 1.13$ f
$e_v = 0.5$	$a_v = 0.80$ f
$\mu_s^{\mathrm{core}} = -0.22$	$b_{s,\mathrm{core}}$ undetermined
$\mu_s^{\mathrm{cl}} = 0.16$	$b_{s,\mathrm{cl}} = 1.30$ f
$\mu_v = 1.853$	$b_v = 0.89$ f
$a_p = 0.80$ f	$b_p = 0.98$ f
$a_n = 0$	$b_n = 0.79$ f

At large values of q^2, the effect of the extensive isoscalar cloud will be negligible and we can fit the core parameters. A point moment of -0.19 nm would give a satisfactory fit, but let us ascribe to the core, arbitrarily, the same rms radius of 0.2 f which fits the charge core. In this case, the core moment must be -0.22 nm. In order to make the static moments correct, we must then ascribe a magnetic moment of +0.16 nm to the extended isoscalar cloud so that the total isoscalar moment $= \frac{1}{2}(\mu_p + \mu_n) = -0.060$ nm, as required by (7).

Table I summarizes the parameters we have found for the partial form factors; these are plotted in Fig. 1, and the corresponding spatial distributions are shown in Fig. 2.

The qualitative result then of these new measurements is that we find it necessary to attribute a magnetic moment of negative sign to the charge core that our previous measurement had revealed. It is not unreasonable that the angular momentum of the meson cloud be unity; then necessarily the core will have a spin of $-\frac{1}{2}$, which might rather

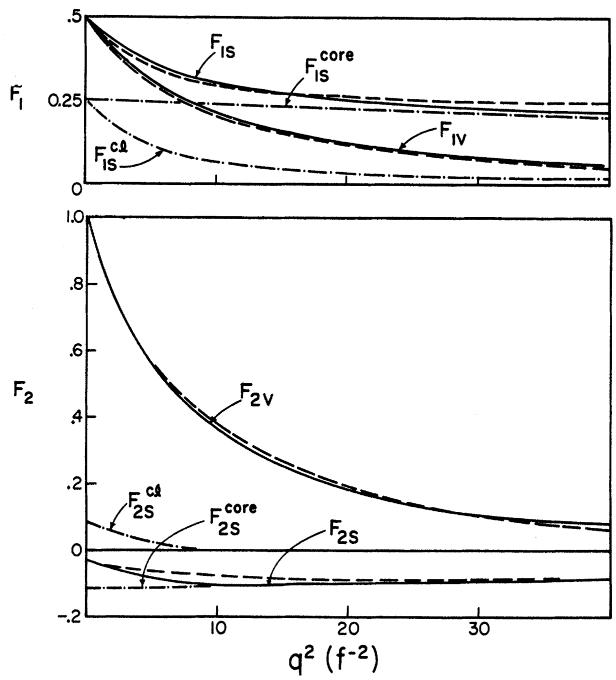

FIG. 1. Partial form factors for F_1 and F_2. Each form factor is resolved into isoscalar and isovector parts, whose sum and difference give, respectively, the neutron and proton form factors. The solid curves indicate the fit according to the core model, where the scalar partial form factors have been further split into terms corresponding to a core and an extended cloud, each of exponential distribution. The dashed curves indicate the best fit obtained by the Clementel-Villi form.

naturally give rise to the negative magnetic moment—for example, by dissolution of the core into a $K^+\Lambda$ system in direct analogy to pion emission. It is also perhaps significant that the radius of the extended isoscalar cloud of magnetic moment comes out so close to that of the isoscalar charge cloud—also true of the isovector clouds.

BSFV have proposed aesthetically simple form factors for the nucleon based on the model already referred to above. In our notation their form factors are

$$F_2^{\,v} = \tfrac{1}{2}\left[(1-\alpha_v) + \frac{\alpha_v}{1+q^2/q_v^{\,2}}\right], \quad F_2^{\,v} = 1.853\left[(1-\beta_v) + \frac{\beta_v}{1+q^2/q_v^{\,2}}\right], \tag{9}$$

$$F_1^{\,s} = \tfrac{1}{2}\left[(1-\alpha_s) + \frac{\alpha_s}{1+q^2/q_s^{\,2}}\right], \quad F_2^{\,s} = -0.060\left[(1-\beta_s) + \frac{\beta_s}{1+q^2/q_s^{\,2}}\right], \tag{10}$$

where one minus the constants α or β give the fraction of the charge or magnetic moment in a point core, and α_s or α_v give the corresponding fraction in a Yukawa meson cloud of mean square radius $6/q_s^{\,2}$ or $6/q_v^{\,2}$. It appears that BSFV have, not unreasonably, assumed that the radii of the Yukawa clouds that appear in F_1 and F_2 are the same. They show that the radius of the Yukawa part of the isovector partial form factor is due to a $T=1$, $J=1$ two-pion resonant state with the resonance at q_v, while the isoscalar radius will correspond to a three-meson resonant state, most probably with $T=0$, $J=1$, at q_s. The six parameters in (9) and (10) are reduced to five because the experimental requirement that the neutron charge radius is zero leads to the relation

$$a_s/q_s^{\,2} = a_v/q_v^{\,2}. \tag{11}$$

Table II gives values of the five independent parameters that best fit our data plus the rms radii of the neutron and proton. The same parameters have also been determined at Stanford[6] and their values in our nomenclature are shown for com-

Table II. Parameters for best fit according to BSFV[a] form.

	Cornell	Stanford
α_v	1.10	1.20
β_v	1.14	1.20
α_s	0.58	0.56
β_s	-1.5	-3.0
a_v	0.85 f	0.77 f
a_s	1.16 f	1.13 f
a_p	0.88 f	0.85 f
b_p	0.95 f	0.94 f
b_n	0.87 f	0.76 f
$q_v^{\,2}$	$8.3\ f^{-2} = 16(m_\pi/c)^2$	$10\ f^{-2} = 19.6(m_\pi/c)^2$
$q_s^{\,2}$	$4.4\ f^{-2} = 8.5(m_\pi/c)^2$	$4.7\ f^{-2} = 9(m_\pi/c)^2$

[a]See reference 2.

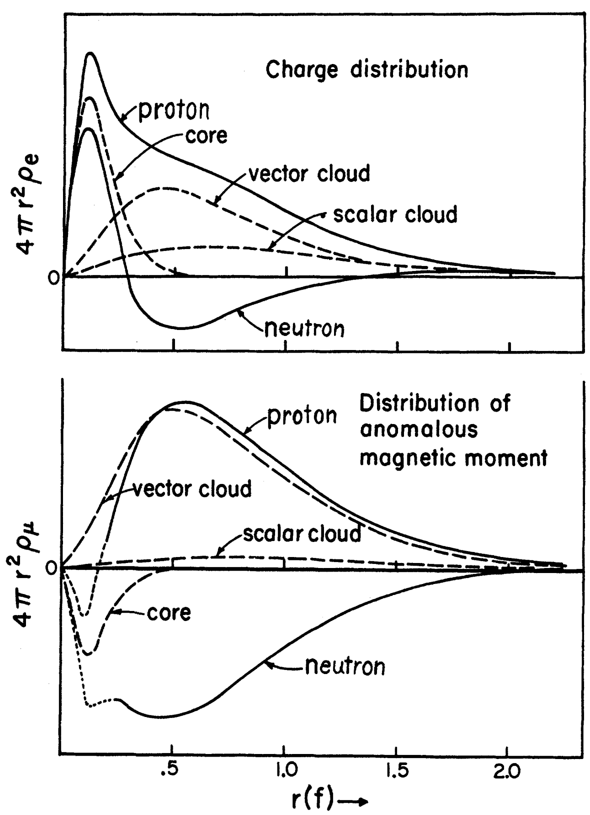

FIG. 2. Spatial distribution of charge and anomalous magnetic moment for proton and neutron, according to the core model.

parison; the general agreement is good. Slight discrepancies arise mainly because our experimental values for F_{2p} at high q^2 values are close to zero whereas an extrapolation of the Stanford form factors gives negative values. Our experiments indicate also a somewhat smaller difference between F_{2p} and F_{2n}.

The experimental data are fit equally well by the BSFV form factors or by those following from our simple core model (see Fig. 1 and the curves in the preceding Letter[4]).

According to the interpretation of BSFV, our values of q_v and q_s would imply that the resonant energy of the two-pion state is $4.0\,m_\pi$ and that of the three-pion state is $2.9\,m_\pi$.

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[1]D. N. Olson, H. F. Schopper, and R. R. Wilson, Phys. Rev. Letters 6, 286 (1961).

[2]S. Bergia, A. Stanghellini, S. Fubini, and C. Villi, Phys. Rev. Letters 6, 367 (1961).

[3]W. Frazer and J. Fulco, Phys. Rev. 117, 1609 (1960).

[4]R. M. Littauer, H. F. Schopper, and R. R. Wilson, preceding Letter [Phys. Rev. Letters 7, 141 (1961)].

[5]Our normalization, following BSFV, is such that the value of the form factor at $q^2=0$ gives either the static charge or magnetic moment as the case may be. Thus F_1 normalizes to unity or zero for the proton or neutron, and F_2 normalizes to the appropriate anomalous magnetic moment measured in nuclear magnetons. The Rosenbluth scattering formula then must be written in the form

$$\frac{d\sigma}{d\Omega}=\sigma_0\left\{F_1^{\,2}+\frac{q^2}{4M^2}\left[2(F_1+F_2)^2\tan^2(\theta/2)+F_2^{\,2}\right]\right\},$$

i.e., the different proton and neutron cross sections result directly from inserting the different values of F_1 and F_2 that are given by (1). We can speak of a partial charge or a partial magnetic moment whose value is given by the value of the corresponding partial form factor at $q^2=0$, the total charge or moment being the sum of the partial charges or partial moments.

[6]R. Hofstadter and R. Herman, Phys. Rev. Letters 6, 293 (1961); see, however, the reinterpretation of some of these results by L. Durand, III, Phys. Rev. Letters 6, 631 (1961).

X-RAY YIELDS IN THE K AND L SERIES OF μ-MESONIC ATOMS*

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Since the experimental work of Stearns and Stearns,[1] much theoretical speculation has been presented[2-7] to account for the low yields of x-rays in the K and L series of light μ-mesonic atoms reported by these authors. Their results, reproduced in Figs. 1(a) and 1(b) (filled circles), require that the Auger rates[8] competing with radiative transitions be, respectively, ~300 times (K series) or ~30 times (L series) as fast as predicted by conventional theory, or else that one postulate some hithertofore unknown competitive transition mechanism. The situation for the yields of π-mesonic x rays of comparable energies is entirely similar,[9] but need not be discussed here.

In view of this situation, we decided to remeasure the yields in the π-mesonic K and L series, in particular for those light elements for which low (or even experimentally unobservable) yields were reported in reference 1. In view of our findings, we think it useful to present here our preliminary results. They are displayed in Figs. 1(a) and 1(b) (open circles).

The experimental arrangement used by us is shown in Fig. 2 and is essentially the same as that used by Stearns and Stearns.[1] Some relevant differences are the following: (a) use of thinner (1/16 in.) plastic scintillators, wrapped in 1-mil Al foil, just before and just behind the x ray target T, to improve the transmission of soft x rays and to decrease the background of carbon mesonic x rays; (b) use of an improved window (20 mils of Be) on the 1/16-in., 2-in.-diameter NaI detector employed for x rays up to 75 kev; (c) use of a longer effective resolving time (100 nsec vs the 50 nsec of Stearns and Stearns) for NaI-induced coincidences; (d) use of a 100-channel pulse-height analyzer (RIDL 34).

The running conditions differed more markedly from those available to earlier workers in this field, in the following respects: (a) use of a μ^- beam of low e^- content and of a Čerenkov counter (No. 3 in Fig. 2) for further electron rejection; (b) use of a good duty-cycle (~5) meson beam, obtained by means of a vibrating cyclotron target.[10] These conditions enabled us to use rather slow NaI coincidences without generating large accidental backgrounds. As is well known, hard clipping of NaI pulses of small amplitude (from, say,