THEORETICAL VALUES FOR MAGNETIC MOMENTS OF μ-MESONIC ATOMS*

Kenneth W. Ford
Brandeis University, Waltham, Massachusetts

Vernon W. Hughes
Yale University, New Haven, Connecticut

and

John G. Wills†
Los Alamos Scientific Laboratory, Los Alamos, New Mexico
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Precise measurements of the magnetic moments of negative μ mesons bound in μ-mesonic atoms are reported in the preceding Letter. Theoretical calculations of the magnetic moments of μ-mesonic atoms are outlined and summarized in this Letter.

It has been pointed out that the magnetic moment (or g value) of a negative muon bound in an atomic orbit about a nucleus will be slightly less than that of a free muon and that the magnitude of this alteration is sensitive to the size of the nucleus.[1] The corrections to the gyromagnetic ratio of a Dirac particle ($g_0 = 2$) bound in the field of a zero-spin nucleus and surrounded by a zero-spin electronic cloud may be listed as follows:

1. radiative correction, g_1;
2. binding correction to radiative correction, g_2;
3. direct binding correction, g_3;
4. nuclear polarization correction, g_4;
5. electronic polarization correction, g_5;
6. electronic diamagnetic shielding correction, g_6;
7. center-of-mass correction, g_7.

We consider that the muon is bound in a 1s orbit about the nucleus.

The radiative correction[2] and the binding correction[3] thereto have already been calculated:

$$g_1/g_0 = \alpha/2\pi + 0.75(\alpha/\pi)^2 = 0.001165, \qquad (1)$$

$$g_2/g_0 = (26/15\pi)\alpha\langle V\rangle/mc^2, \qquad (2)$$

where $\langle V\rangle$ is the expectation value of the potential energy and m is the muon mass. The direct binding correction, g_3, is computed from a relativistic treatment of the magnetic moment of a Dirac particle in a central field.[4] The result is

$$g_3/g_0 = -(4/3)\int F^2 dr, \qquad (3)$$

where F is the small component of the radial wave function of the muon in the Coulomb field of a nucleus with finite size. Both the corrections g_2 and g_3 have been obtained from the relativistic bound-state muon wave functions calculated on an IBM-704 computer. The nuclear charge distribution is taken to be

$$\rho(r) = (Ze/4\pi r_1^3 N_0)\left(1 - \tfrac{1}{2}e^{-n(1-x)}\right) \text{ for } x < 1,$$
$$= (Ze/4\pi r_1^3 N_0)\left(\tfrac{1}{2}e^{-n(x-1)}\right) \text{ for } x \geq 1, \qquad (4)$$

where $x = r/r_1$ and

$$N_0 = \tfrac{1}{3} + 2n^{-2} + n^{-3}e^{-n}.$$

The parameters r_1 and n, which characterize the nuclear radius and surface thickness, were chosen to be those which have been used to fit high-energy electron scattering results.[5] The effect of the nuclear size on g_3 is significantly larger than the experimental error for $Z \geq 12$.

A significant nuclear polarization correction term, g_4, arises in second order perturbation theory from the magnetic interaction between the muon and the nucleus together with the magnetic interaction of the nucleus in the external static magnetic field. In an approximate calculation the term g_4 can be related to $M1$ nuclear gamma transition rates. An electronic polarization correction term, g_5, arises in second order perturbation theory from the magnetic interaction between the muon and the electrons together with the magnetic interaction of the electrons in the external static magnetic field.[6] The correction term g_5 is negligible unless the ground electronic state has fine-structure levels.

The usual diamagnetic shielding[7] by the atomic electrons will be very nearly the same for the muon as it is for the nucleus. This effect can be expressed by a correction term, g_6:

$$g_6/g_0 = -\tfrac{1}{3}\alpha^2\sum_i\langle a_0/r_i\rangle,$$

in which the sum is taken over all the electrons. The center-of-mass correction g_7 is negligibly small.

Table I. Calculated corrections to the g values of selected μ-mesonic atoms.

Element	g_1/g_0	g_2/g_0	g_3/g_0 [a]	g_4/g_0	g_6/g_0	g/g_0 [b]	$(g-g_{\mu^+})/g_0$
$_6$C	0.001165	-0.000008	-0.000629(0)	+0.000004	-0.00019	1+0.00034	-0.00082
$_8$O	0.001165	-0.000013	-0.001104(1)	+0.000012	-0.00032	1-0.00026	-0.00143
$_{12}$Mg	0.001165	-0.000029	-0.002379(6)	+0.000053	-0.00062	1-0.00181	-0.00298
$_{14}$Si	0.001165	-0.000040	-0.003172(10)	+0.000090	-0.00079	1-0.00275	-0.00391
$_{16}$S	0.001165	-0.000051	-0.004035(15)	+0.00014	-0.00096	1-0.00374	-0.00491

[a]Numbers in parentheses are uncertainties arising from 2% uncertainty in nuclear radius.
[b]The quantity $g \equiv \sum_{i=0}^{6} g_i$.

Table I lists the significant corrections for the elements measured experimentally. The terms g_5 and g_6 depend on the electronic state of the μ-mesonic atom. For the elements listed, binding-energy considerations suggest that the μ-mesonic atoms will be positive ions with Z-2 electrons and the electronic states will be $^1S_0$ except for oxygen and sulfur which will be $^3P_0$. The diamagnetic shielding term g_6 is obtained from Dickinson's value[8] for an atom with Z-1 electrons together with a small estimated correction to give g_6 for the positive ion. The term g_5 will be negligible except for oxygen and sulfur where it amounts to about 5% and 2%, respectively; it is not included in the table since the large theoretical values for O and S are clearly inconsistent with the experimental values, thus indicating that the atomic states are not simple $^3P_0$ states.

In view of uncertainties about the electronic state of the μ-mesonic atom (the state might even have electronic angular momentum J nonzero and then the hfs interaction between muon and electrons would be important), about chemical effects,[9] and about solid-state effects such as the Knight shift,[10] the agreement between theory and experiment is close. The chemical shift and the Knight shift are paramagnetic effects, having the same sign as that required to account for the small remaining discrepancies. There is no indication for any anomalous behavior of the muon such as its polarization in the strong electric field of the nucleus.

A more detailed report on these theoretical calculations will be submitted to The Physical Review.

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†Now at University of Washington, Seattle, Washington.

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