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MAGNETIC MOMENT OF POSITIVE AND NEGATIVE MUONS*

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This Letter reports on the refinement of the technique for measuring muon magnetic moments and on its application to positive and (bound) negative muons in various materials. These refinements enable us to obtain a statistical accuracy of several parts per million in reasonably short runs. In addition to the absolute measurement of g_μ, this sensitivity permits the observation of environmental effects and thus has applications to nuclear and solid-state physics.

The magnetic moment of the μ meson, like that of the electron, is of theoretical importance because of the exact predictions that can be made of its magnitude. Quantum electrodynamics enables one to calculate the moment[1] in units of the muon magneton, $e\hbar/m_\mu c$. *TCP* invariance requires the positive and negative muon, as particle and antiparticle, to have equal magnetic moments.[2]

Since the discovery of parity nonconservation, there has been a succession of increasingly accurate measurements[3-5] of the muon magnetic moment, at this laboratory and elsewhere. These experiments, as well as the present one, generally express the muon moment in units of the proton magnetic moment, and depend on separate determinations of the muon mass[6] for comparison with theory. A direct measurement of the "anomalous part" of the magnetic moment, which does not depend strongly on the mass value for its interpretation, has also recently been made.[7] The accuracy of the present experiment so far exceeds that of any muon mass determination that it must await future experiments on the latter before its significance can be realized. Alternatively one can accept present theoretical[1] or experimental[7] estimates of the g factor, and combine them with the present results to obtain a "best" value for the muon mass.

The basic method of this experiment is similar to that of reference 5, to which the reader is referred for details. The increased sensitivity of the present experiment is due to numerous improvements in the apparatus, including a higher muon precession frequency, an extremely homogeneous magnetic field, and the transistorizing of all circuitry. Recent improvements in the beam facilities at Nevis, including the long-duty-cycle beam and a new low-energy muon beam, also contributed to the increased sensitivity. Figure 1 shows the experimental arrangement.

Figure 2 shows the results of typical runs with positive and negative muons stopping in HCl solution and graphite, respectively. Early-late phase shift is plotted against magnetic field, as measured by a proton magnetic resonance[8] at the monitor position. Experimental conditions were such that for a target material that does not depolarize positive muons (e.g., bromoform), running time of three hours was sufficient to reduce the statistical error to 12 ppm.

Results of the runs with positive muons stopping in various target materials are summarized in Table I, in chronological order. Statistical errors calculated from a priori considerations are indicated.

The accuracy of the experiment is such that diamagnetic shielding of the muon by atomic elec-

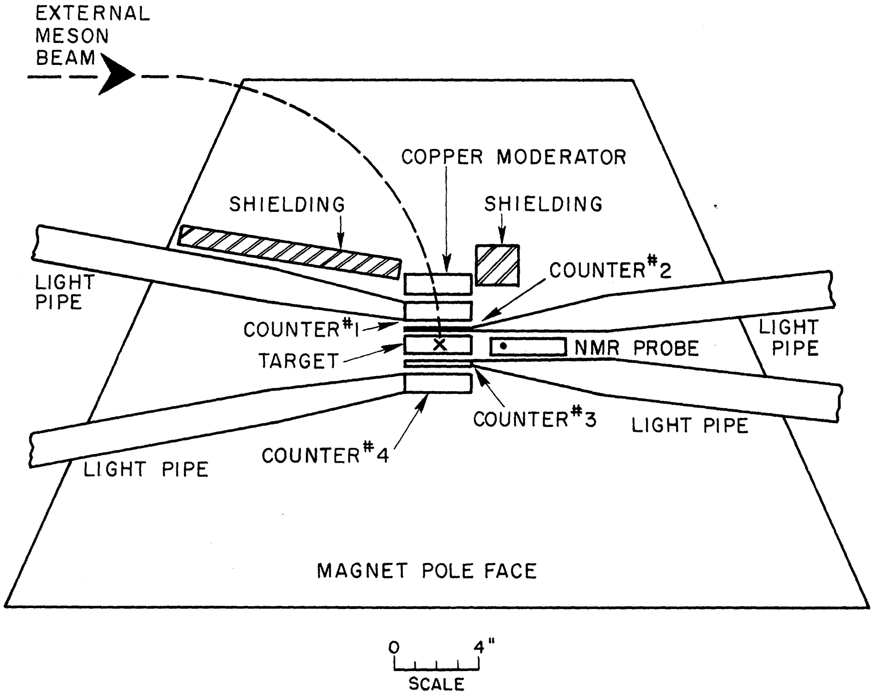


FIG. 1. Experimental arrangement. The meson beam from the Nevis cyclotron enters at the left, and is brought to rest in the target. Decay electrons emitted into counters No. 3 and No. 4 are analyzed.

Table I. Results for positive muons. Errors indicated are statistical only.

Stopping material	Proton frequency at resonance (kc/sec)	
Methylene iodide – CH_2I_2	55 824.45	±0.49
Distilled water – H_2O	55 823.41	±0.49
HCl	55 822.07	±0.35
H_2O – repeat	55 821.39	±0.94
Methylene iodide – repeat	55 823.33	±0.59
Bromoform	55 823.30	±0.69
Average	55 823.01	±0.43
Monitor-to-center field difference	-4.85	±0.10
	55 818.16	
Muon precession frequency at resonance	177 688.4	±0.1
f_μ/f_p	3.18334	±0.00005
μ^+ in graphite (99.995 % pure electromolded)	55 801.8	±1.8

trons must be considered. If the positive muon substitutes immediately for a proton and experiences the same shielding, the shifts are expected to be less than 3 ppm (relative to a proton in water) for all liquid targets used.[9] The proton resonance for measuring the field is also taken in aqueous solution, so essentially no net correction has to be made for the ratio of the moments.

It is noted that the substituted hydrocarbons yield consistently higher values than the aqueous targets, beyond the predictions of chemical shifts. Moreover, there is a downward trend with time for runs repeated with the same type of target. Rigorous checks of the system gave no evidence for a systematic drift in this direction. Therefore, it is assumed that both these effects are statistical fluctuations and the error is increased to account for the observed spread.

The average proton resonance frequency thus obtained is 55823.01 ± 0.43 kc/sec. From this

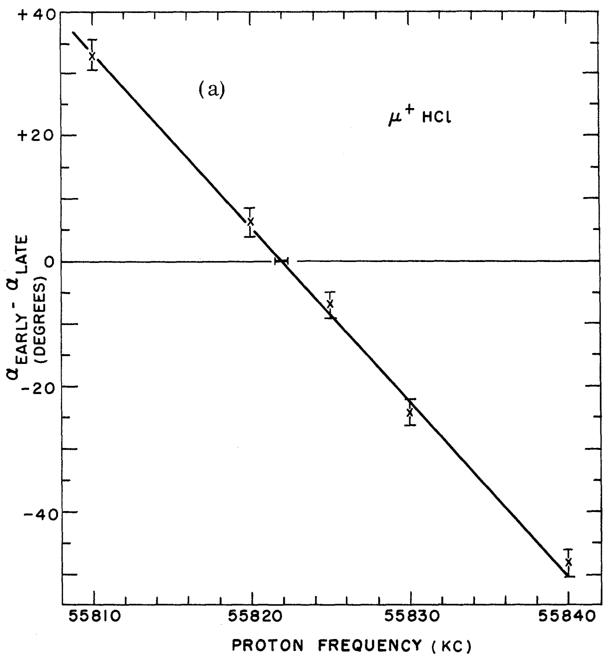
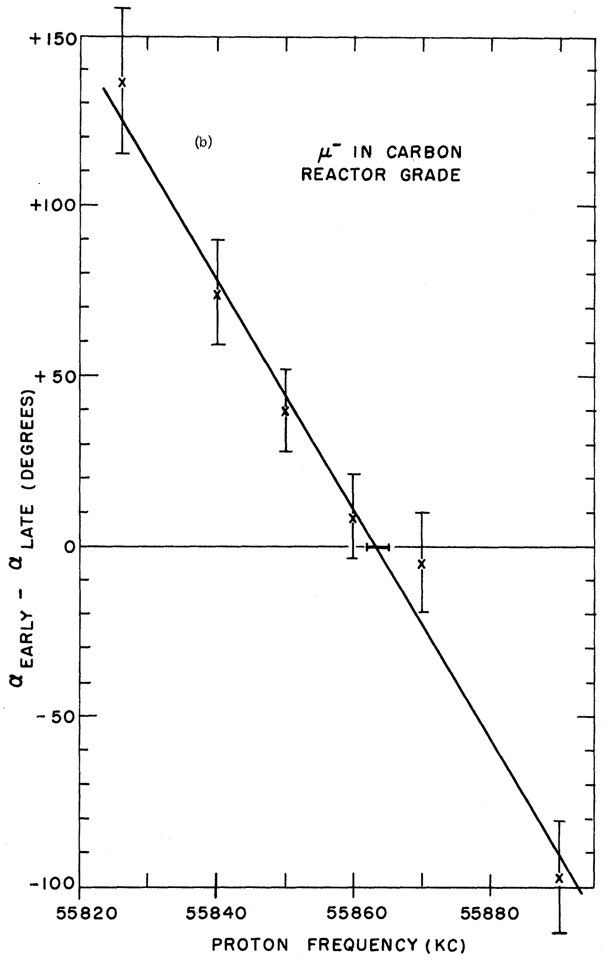


FIG. 2. Early-late phase shift vs proton resonance frequency at monitor position. The straight line is the least-squares best fit, crossing the x axis at the field setting where the muon precession frequency equals the reference value. The horizontal flag at this point is a calculated a priori statistical error. (a) Positive muons in HCl solution; (b) negative muons in graphite (reactor grade).



must be subtracted 4.85 ± 0.10 kc/sec as the difference between the field measured at the monitor position and at the center of the target. The muon precession frequency at "resonance" is 177 688.4 ± 0.1 kc/sec. An improved transistor regulator[10] permits maximum excursions of less than 5 ppm in the field setting (~13.4 kgauss) during each one-hour run. Field spatial homogeneity is such that the maximum deviation from the average over 95 % of the target volume (2 in. × 3 in. × 1 in.) is not more than 10 ppm. The result for positive muons is $f_\mu/f_p = 3.18334 \pm 0.00005$, where the error has been increased to allow for possible systematic errors.

Similar measurements have been made on the magnetic moment of the electron. The most accurate experimental result to date is[11] $f_e/f_p = 658.22759 \pm 0.00004$, as the ratio of the spin precession frequency of free electrons to that of protons in water. Since in the present experiment both muon and proton are bound, a correction[12] for diamagnetic shielding of 25.6 ppm must be made to one of these results before they can be compared. Making it to the electron experiment yields f_e/f_p(both free) = 658.2107, with an error small compared to that of the muon result. These frequencies are proportional to the g factors of the particles, and inversely to their masses. For the ratio of the g factors, one can use either the theoretical value, $g_\mu/g_e = 1.000006$;[1] or the experimentally determined g factors, $g_\mu = 1.001145 \pm 0.000022$,[7] and $g_e = 1.0011609 \pm 0.0000024$.[13] Using the theoretical g factor, one obtains for the ratio of the masses $m_\mu/m_e = 206.768 \pm 0.003$. Using the experimental numbers, one has $m_\mu/m_e = 206.763 \pm 0.005$. The larger error in the latter case reflects the effect of folding in the error in the result of Charpak et al. with that of this experiment. The average of the mesonic x-ray determinations of the mass[6] yields $m_\mu/m_e = 206.76 \pm 0.02$, in excellent agreement with the above, but not sufficiently

Table II. Results for negative muons. Errors indicated are statistical only.

				Theoretical corrections ($\times10^4$)					
Z	Stopping material	Proton frequency at resonance (kc/sec)	$\left(\frac{g-g^+}{g^+}\right)\times10^4$	Direct binding[a]	Binding effect on radiative correction[a]	Nuclear polarization[a]	Diamagnetic shielding[b]	Knight shift[c]	Total
μ^+	(corrected for diamagnetism)	55 821.5 ± 0.5							
6	Graphite, reactor grade	55 863.4 ± 1.7	- 7.5 ± 0.3	- 6.29	-0.08	+0.04	-1.99	?	- 8.32
6	Graphite, 99.995% pure, 300°K	55 860.5 ± 3.1	- 6.9 ± 0.6	- 6.29	-0.08	+0.04	-1.99	?	- 8.32
6	Graphite, 99.995% pure, 77°K	55 865.7 ± 2.7	- 7.9 ± 0.5	- 6.29	-0.08	+0.04	-1.99	?	- 8.32
8	Oxygen in water	55 873.7 ± 5.7	- 9.3 ± 1.0	-11.04	-0.13	+0.12	-3.25	0	-14.30
12	Magnesium	55 968.5 ± 3.7	-26.3 ± 0.7	-23.79	-0.29	+0.53	-6.29	?	-29.84
14	Silicon 300 ohm cm, *n* type	56 023.5 ± 6.0	-36.1 ± 1.1	-31.72	-0.40	+0.90	-7.95	?	-39.17
16	Sulfur	56 090.0 ± 9.1	-48.1 ± 1.6	-40.35	-0.51	+1.4	-9.70	0	-49.16

[a] See reference 17.
[b] See reference 19.
[c] See reference 15.

accurate to afford a real comparison.

A run was made with positive muons stopping in graphite. As noted in Table I, there is a paramagnetic shift of about 400 ppm from the average of the liquid targets. A similar (but smaller) shift was earlier[14] noted in copper and aluminum, and was then interpreted as a Knight shift.[15] No other explanation of this effect is known to the authors; further investigations are being made. The graphite result was not used in the average for the positive muons.

The results for negative muons stopping in various target materials are tabulated in Table II. These muons are observed in the ground state of mesonic atoms formed with the nuclei listed, and corrections must be made in order to compare with positive muons. The largest correction is due to binding.[16] The statistical error on the data requires that the finite nuclear radius be taken into account in calculating this correction for $Z \geqslant 12$. This experiment thus offers another way of probing the nucleus. The numbers quoted in Table II are from Ford, Hughes, and Wills.[17]

The next largest correction to be applied to negative muons is due to diamagnetic shielding at the nucleus by the bound electrons.[18] Because of uncertainty in the final electronic states, Dickinson's[19] results for atomic number Z-1 have been quoted without modification, although his results are for neutral isolated atoms. The magnitude of the effect is sufficient for this experimental method to provide a determination of the absolute shielding.

Several smaller corrections taken from Ford, Hughes, and Wills[17] are also listed in Table II. No entries have been made under "Knight shift" for carbon, magnesium, and silicon. We hope to have soon a measurement of Knight shift in our carbon samples. The Knight shift in magnesium, expected to be a few parts in 10^3, would predict a result in great disagreement with the data. A search was made for a "resonance" in the predicted region, with negative results. There exists a result for the Knight shift in silicon.[20] However, the shift may be sensitive to the doping of the sample, and therefore this result has not been used.

The negative-muon results are less accurate than those for positive muons because of depolarization and, for higher Z, shorter lifetime.

Table II shows reasonable agreement between predicted and measured muon moments in all the atoms investigated. It is concluded that, allowing also for theoretical uncertainties, the magnetic moment of positive and negative free muons are equal to within five parts in 10^4. It may be that the various shifts mentioned will become a subject for investigation in their own right, the muon serving as a tool for probing them.

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