It is my great pleasure to thank Professor William L. Kraushaar for suggesting and guiding this work. I would like to thank many people for their contributing efforts, in particular, W. B. Smith, E. Mangan, R. Mudge, and Dr. M. S. Webster.

*Supported in part by the U. S. Atomic Energy Commission and by the Office of Naval Research.

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SELF-CONSISTENT CALCULATION OF THE MASS AND WIDTH OF THE $J=1$, $T=1$, $\pi\pi$ RESONANCE

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(Received June 28, 1961)

The existence of a $J=1$, $T=1$ resonance in $\pi\pi$ scattering now seems to be very probable, though there are still rather large uncertainties in its position and width.[1] From a theoretical point of view, one may believe this resonance to be due to the existence of an unstable vector meson,[2] called ρ, or one may say the resonance is dynamical.[3] If the ρ meson has an infinite bare mass, there is essentially no difference in the results of these two points of view.[4,5] The principal distinction between the two philosophies comes from the fact that if the ρ meson is really a new particle, its mass and coupling constant must be thought of as new independent parameters which can be chosen to be anything, while if the ρ is a dynamical resonance, the mass and coupling constant are determined by other parameters already in the theory.

Within the "dynamical" philosophy, attempts have been made to calculate the mass and coupling constant of the ρ meson.[3,6] These have not been entirely successful; furthermore, the most complete and careful attempt to do this,[3] based on the Mandelstam representation, involves the solution of a very complicated set of coupled integral equations on a computing machine, and is therefore not very transparent. One qualitative feature which has been emphasized by Chew and Mandelstam,[3] however, is that the existence of the resonance seems to follow from the operation of a "bootstrap mechanism," in which the strong force between two pions in a P state, which is needed to produce the resonance, is provided by the exchange of a pair of resonating pions.

In this way, the basic underlying source of the pion-pion interaction (which could, for example, be the $\lambda\phi^4$ interaction or a force produced by the exchange of a strongly interacting S-wave pion pair) does not seem to play a large quantitative role in the final results, but merely provides the spark which sets the bootstrap off. One would, therefore, expect to be able to obtain the ρ meson's properties as the result of a self-consistent calculation with no parameters.

It is our purpose here to apply the bootstrap mechanism in a very simple-minded and trivial calculation, which yields quantitative values for the ρ mass and coupling constant. The approximation is the following: a ρ meson, of mass m_ρ and coupled to the pion with a coupling constant $\gamma_{\rho\pi\pi}$, is exchanged between two pions as shown in Fig. 1.

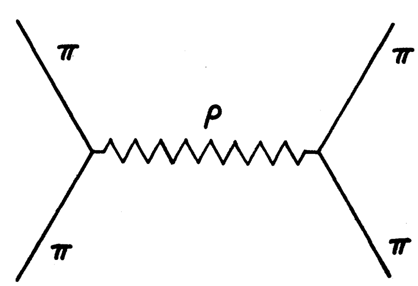

FIG. 1. The one ρ-meson exchange diagram.

The force thereby produced in the $J=1$, $T=1$ channel is attractive and depends on m_ρ and $\gamma_{\rho\pi\pi}$. If m_ρ and $\gamma_{\rho\pi\pi}$ are adjusted properly, this force may be made to produce a resonance at m_ρ with coupling constant $\gamma_{\rho\pi\pi}$. Thus we obtain two relations between m_ρ and $\gamma_{\rho\pi\pi}$ from which both may be determined.

In practice, the calculation is most easily carried out using the determinantal method.[7] One computes the determinantal function for the $J=1$, $T=1$ channel from the equation[8]

$$D\left(\frac{s}{m_\pi^2}, \gamma_{\rho\pi\pi}, \frac{m_\rho^2}{m_\pi^2}\right) = 1 + \frac{s-s_0}{\pi}\int_{4\mu^2}^{\infty}\frac{\mathrm{Im}D(s/m_\pi^2, \gamma_{\rho\pi\pi}, m_\rho^2/m_\pi^2)}{(s'-s_0)(s'-s-i\epsilon)}ds', \tag{1}$$

where s is the total c.m. energy squared,

$$\mathrm{Im}D = -(\sin\delta e^{i\delta})_1, \tag{2}$$

and $(\sin\delta e^{i\delta})_1$ is calculated from the graph of Fig. 1. Then we require

$$\mathrm{Re}D(m_\rho^2/m_\pi^2, \gamma_{\rho\pi\pi}, m_\rho^2/m_\pi^2) = 0, \tag{3}$$

and[4]

$$\frac{\mathrm{Im}D(m_\rho^2/m_\pi^2, \gamma_{\rho\pi\pi}, m_\rho^2/m_\pi^2)}{(d/ds)\mathrm{Re}D\big|_{s=m_\rho^2}} = \frac{1}{3}\left(\frac{\gamma_{\rho\pi\pi}^2}{4\pi}\right)\left(\frac{(m_\rho^2-4\mu^2)^3}{m_\rho^2}\right)^{1/2}. \tag{4}$$

Equations (3) and (4) together fix m_ρ and $\gamma_{\rho\pi\pi}$. The computation is explicit and straightforward, and an approximate numerical evaluation of the necessary two integrals yields

$$m_\rho \approx 950\ \mathrm{Mev}, \quad \gamma_{\rho\pi\pi}^2/4\pi \approx 2.8.$$

There are no parameters to be adjusted in obtaining these results, other than the pion mass which only provides a dimension. The numbers are in fair agreement with the present experimental data,[1] which indicate something like

$$m_\rho \approx 750\ \mathrm{Mev}, \quad \gamma_{\rho\pi\pi}^2/4\pi \approx 1.$$

The most serious deficiency of a calculation like this based on the determinantal method lies in the violation of crossing symmetry. One may hope, however, in analogy to other more or less successful calculations,[7] that this lack is not of great quantitative importance; in any case, it may be of value to illustrate by a simple method what it is that the complicated and more complete calculations are supposed to do if they can ever be successfully carried out.

In conclusion, it may be remarked that the "bootstrap" mechanism may be responsible for the existence of many, if not all, other particles as well; for example, the strange vector meson called M by Gell-Mann,[9] with isotopic spin $\frac{1}{2}$, may be a P-wave resonance in $K\pi$ scattering. There may, in addition, be an S-wave resonance or pseudoresonance, corresponding to a K' meson like the K but with opposite parity. In a complete calculation of the bootstrap in the $\pi\pi$ system, all of these particles, or resonances, should of course be included. For a more complete discussion of these points, see reference 5.

I should like to thank M. Gell-Mann for several comments and suggestions, and D. Sharp for the numerical calculations.

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