ratios measured at CERN at 16° lab production angle by Cocconi et al.[1] are reasonably consistent with the trend of our measurements at production angles of 13° and 20°.

A more complete account of this work will be published elsewhere.

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FAST ATOMIC TRANSITIONS WITHIN μ-MESONIC HYPERFINE DOUBLETS, AND OBSERVABLE EFFECTS OF THE SPIN DEPENDENCE OF MUON ABSORPTION*

R. Winston and V. L. Telegdi
The Enrico Fermi Institute for Nuclear Studies, The University of Chicago, Chicago, Illinois
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It has been pointed out by Bernstein, Lee, Yang, and Primakoff,[1] that the two members $F_\pm = I \pm \frac{1}{2}$ of the hyperfine doublet ground state of a μ-mesonic atom with nonzero nuclear spin I could exhibit different disappearance rates $\Lambda_\pm$ as a consequence of the possible spin dependence of the interaction responsible for muon absorption. In particular, the now currently favored "universal" V-A interaction would lead to $\Lambda_+ < \Lambda_-$. Bernstein et al.[1] made the implicit assumption that atomic processes inducing transitions between the two states $F_\pm$ were negligibly slow. With this assumption, they predicted that a logarithmic plot of the time dependence of the electron rates from negative muons stopped in a monoisotopic target of nonzero I should exhibit a positive curvature, corresponding to the superposition of the two exponentials characterized by Λ_+ and Λ_-. Experimental detection of such a curvature could serve as a proof of the spin dependence of the absorption interactions but would give no clue to its more detailed nature.

It was subsequently pointed out[2] that, at least for the targets of experimental interest, the assumption of a negligibly slow transition rate between the two states $F_\pm$ is not a valid one. It was shown that the magnetic interaction between the "core" (=nucleus + muon) of the mesonic atom and its outer s electrons provides a sizeable conversion rate R between those two states. An explicit calculation for the case of $_{13}Al^{27}$ was presented in reference 2, made under the assumption that only the $3s$ (conduction) electrons of this target would effectively contribute to R. The magnitude of R so calculated was comparable to that of the difference $|\Lambda_- - \Lambda_+| \equiv |\Delta\Lambda|$. As was emphasized in reference 2, this parent-daughter relationship would lead to a negative curvature in the experiment proposed by Bernstein et al. if the state F_- disappeared faster, which is precisely the prediction of the V-A interaction for a nucleus with positive magnetic moment μ_I.

Thus, the curvature experiment could possibly provide the as yet missing evidence that the V-A interaction governs muon capture. Some preliminary experimental evidence from Chicago, indicating the actual observation of negative curvature in Al and P, was discussed at the 1960 Rochester Conference.[3] Subsequent more refined measurements[4] performed in this laboratory indicate, however, that this preliminary "evidence" was due to spurious instrumental effects, the nature of which is now understood. This situation leads to the following questions: (a) Granting that muon capture is governed by the V-A interaction, what modification of the plausible rate calculation in reference 2 can account for the nonobservation of negative curvature? (b) Granting that a modified (and presumably reliable) rate calculation can account for this, what modification of the experiment originally proposed by reference 1 would lead to an observable negative curvature, thus confirming V-A? The present note purports to answer these questions.

(a) Conversion rate calculation. The problem of the transition rate between the states $F_\pm$ of interest, separated by an energy ϵ, is entirely analogous to that of an $M1$ transition between two nuclear states, the "core" playing here the role of the nucleus. However, the radiative transition rate R' is here readily calculable[5]:

$$R' = \tfrac{4}{3}\alpha^5 m_\mu^{-2} I/(2I+1)\epsilon^3, \tag{1}$$

while the conversion coefficients (that would yield R) are not available for the very soft ($\epsilon \lesssim 1$ kev) photons in question from standard tables.[6] We must, as was done in reference 2, estimate the rate $R_{i\to f}$ at which the magnetic hyperfine interaction between the "core" and an electron of the mesonic atom leads to electron ejection:

$$R_{ns\to ks} = \tfrac{1}{3}\pi\alpha^{-1}R'\epsilon^{-3}|u_{ns}(0)|^2|u_{ks}(0)|^2\rho_E(k), \tag{2a}$$

$$R_{np_{1/2}\to kp} = \tfrac{1}{3}\pi\alpha^{-1}R'\epsilon^{-3}(33/2)\left|\int_0^\infty u_{np}r^{-3}u_{kp}r^2dr\right|^2\rho_E(k), \tag{2b}$$

$$R_{np_{3/2}\to kp} = \tfrac{1}{3}\pi\alpha^{-1}R'\epsilon^{-3}(69/10)\left|\int_0^\infty u_{np}r^{-3}u_{kp}r^2dr\right|^2\rho_E(k). \tag{2c}$$

The approximation in the above expressions consists in taking the proper nonrelativistic limits, neglecting retardation effects and dropping contributions from nuclear magnetic moments. The rates in (2) are already summed over the electrons of the pertinent atomic shells, assumed to be filled; the total conversion rate is clearly

$$R = \sum R_{nlj\to klj'}, \tag{3}$$

where the sum is taken over all ionizable shells (nlj); ionizable means that the "edge" E_{nlj} lies below the available energy

$$\epsilon = E(F_+) - E(F_-) \simeq \tfrac{2}{3}\alpha^2 m_p^{-1} m_\mu^{2}(\mu_I/\mu_B)(2I+1)/IZ_{\text{eff}}^{3}, \tag{4}$$

where $Z_{\text{eff}}^3 = Z_{\text{eff}}^4/Z$ taken from μ capture.[7] The replacement of Z^3 by Z_{eff}^3 corrects for the finite nuclear size. For a completely general discussion, see Bohr and Weisskopf.[8]

We estimate R_n, the conversion rate for a given shell n, by the following approximate expressions valid for $|\epsilon - V_0| \ll Z'^2/2$:

$$|u_{ns}(0)|^2 \simeq 4Z'(Z'-S)^2/n^3, \quad |u_{ks}(0)|^2\rho_E(k) \simeq 4Z', \tag{5a}$$

where $Z' \equiv Z-1$, the charge of the mesonic atom core, and

$$\left|\int_0^\infty u_{np}r^{-3}u_{kp}r^2dr\right|^2\rho_E(k) \simeq Z'^2(Z'-S)^2/9n^3, \tag{5b}$$

so that

$$R_{np\to kp} \simeq 0.16 R_{ns\to ks}. \tag{5c}$$

Here, S = inner, V_0 = outer Slater screening constants. These estimates are straightforward generalizations to off-diagonal matrix elements of Goudsmit's semiclassical hyperfine formula, justified by Fermi and Segrè via the JWKB method.[9] For the shielding of the emitted s electron, the remarks of Rose[10] are pertinent. In reference 2, R was calculated from (2a), assuming that only the 3s electrons of Al (magnetic M_I shell conversion) could be ejected. Inasmuch as $\epsilon(\text{Al}) \simeq 260$ ev while $E_{2s_{1/2}} = 88.60$ ev (exp. L_I-edge of Mg[11]), this assumption was in error. Using (3), (4), and (5), one finds:

$$R_2 \simeq 3.2\times 10^7 \text{ sec}^{-1} \text{ for } {}_{13}\text{Al}^{27} \tag{6}$$

$$(Z'=12,\ S=4.15,\ V_0=120 \text{ ev}).$$

The numerical value (6) far exceeds that calculated in reference 2, viz., $R = 6.2\times10^5$ sec^{-1}. It is two orders of magnitude larger[12] than $|\Delta\Lambda|$ and this fact, as we shall show later, accounts for the absence of observed negative curvature in Al. For $_{15}P^{31}$ the situation is entirely similar.

$R \gg |\Delta\Lambda|$ is not restricted to Al and P but holds quite generally for odd-Z nuclei of interest throughout the periodic table. This is due to the fact that whenever s conversion can occur, it does so at a rate that greatly exceeds $|\Delta\Lambda|$ (specifically disregarding nuclei with anomalously small μ_I). Since R_n increases with Z more rapidly than does $|\Delta\Lambda| \sim Z_{\text{eff}}^3$, it is sufficient to show this for the lowest Z at which a particular n, s shell can first be ionized. Thus, for $Z < 9$, conversion occurs first at $Z = 5$; for $_5B^{11}$, $R_2 \simeq 1.7\times10^5$ sec^{-1}, $\Delta\Lambda \simeq 5\times10^3$ sec^{-1}. For $9 \leq Z \leq 15$, $2s$ conversion always occurs; for $_9F^{19}$, $R_{2S} \simeq 2.9\times10^6$ sec^{-1}, $\Delta\Lambda \simeq 1.1\times10^5$ sec^{-1}. For $17 \leq Z < 39$, $3s$ conversion always occurs; for $_{17}Cl^{35}$, $R_{3S} \simeq 7.5\times10^6$ sec^{-1}, $\Delta\Lambda \simeq 2.3\times10^5$ sec^{-1}. For $39 \leq Z \leq 75$, $4s$ conversion always occurs; for $_{39}Y^{89}$, $R_4 \simeq 4.1\times10^7$ sec^{-1}, $\Delta\Lambda \simeq 2.8\times10^5$ sec^{-1}. Since the observable effects will be shown to decrease as $1/Z$, we do not consider very high values of Z.

The high conversion rates ($R \gg |\Delta\Lambda|$) discussed exclude practically observable effects in the time distribution of decay events, quite independently of the nature of the μ-capture interaction. Assuming $\mu_I > 0$, the time dependence of the populations in the $F_\pm$ states is governed by the equations

$$n_+(t) = n_+(0)e^{-(\Lambda_+ + R)t},$$

$$n_-(t) = [n_-(0) + n_+(0)R(R-\Delta\Lambda)^{-1}]e^{-\Lambda_- t} - n_+(0)R(R-\Delta\Lambda)^{-1}e^{-(\Lambda_+ + R)t}, \quad (7)$$

where $n_\pm(0)$ are the statistical weights of the states $F_\pm$. In the limit that $R \gg |\Delta\Lambda|$, the time distribution of the decay events (i.e., the electron rate) to order $\Delta\Lambda/R$ is

$$N^{\text{dec}}(t) \sim e^{-\Lambda_- t}(1 - Ae^{-Rt}), \quad (8)$$

where $A \equiv n_+(0)\Delta\Lambda/R$. The smallness of the amplitude A together with its rapid decay (in $_{13}Al^{27}$, $A \simeq 2\times10^{-3}$, $R^{-1} \simeq 3\times10^{-8}$ sec) accounts for the absence of observed negative curvature in $_{13}Al^{27}$ and $_{15}P^{31}$. As emphasized above, this smallness is not restricted to these nuclides.

(b) Experiments leading to observable negative curvature. Contrary to the study of the time distribution of the decay events, as originally proposed in reference 1, a study of the time distribution of capture events (capture neutrons and gamma rays) can lead to practically observable spin-dependence effects and, in particular, exhibit negative curvature. One has for these events from (7), again in the limit $R \gg \Delta\Lambda$,

$$N^{\text{cap}}(t) \sim e^{-\Lambda_- t}(1 - A'e^{-Rt}), \quad (9)$$

where $A' \equiv [n_+(0)\Delta\Lambda/\bar{\Lambda}^{\text{cap}}]/[1 + n_+(0)\Delta\Lambda/\bar{\Lambda}^{\text{cap}}]$; $\bar{\Lambda}^{\text{cap}} \equiv n_+(0)\Lambda_+^{\text{cap}} + n_-(0)\Lambda_-^{\text{cap}} \equiv$ mean capture rate. The striking difference between (8) and (9) is that A' may be a large amplitude of order unity. It is easy to see how this comes about:

When $|\Delta\Lambda|/R \ll 1$, the muon population is for $t \gg R^{-1}$ captured almost entirely from the F_- state at a rate $\Lambda_-^{\text{cap}}e^{-\Lambda_- t}$. Extrapolating back to $t = 0$, this would require an initial capture rate Λ_-^{cap}, whereas the true initial capture rate is $\bar{\Lambda}^{\text{cap}}$. Therefore, $1 - A' = \bar{\Lambda}^{\text{cap}}/\Lambda_-^{\text{cap}}$, where $A' = [n_+(0)\Delta\Lambda/\bar{\Lambda}^{\text{cap}}]/[1 + n_+(0)\Delta\Lambda/\bar{\Lambda}^{\text{cap}}]$ as in (9). The same argument applied to decay events shows that $A \approx 0$ to zeroth order in $\Delta\Lambda/R$, since the decay probability is the same in both F states. Since A' decreases as $1/Z$, the effect is best studied in low-Z elements. For statistical efficiency, R should not greatly exceed Λ_-. Several elements (e.g., $_9F^{19}$) appear to be suited in practice for such experiments.

We conclude with two remarks:

(i) If one had $\Lambda_+/R \gg 1$, targets with $I = \frac{1}{2}$ (e.g., $_9F^{19}$ and $_{15}P^{31}$) should exhibit[13] a μ-decay asymmetry parameter $\alpha(I)$ half as large as $\alpha(0)$, the corresponding parameter for $I = 0$. With $\Lambda_+/R \approx 1$, one should see a "damping" of the precession in the F_+ state, while $\Lambda_+/R \ll 1$ would imply $\alpha(\frac{1}{2}) \simeq 0$. In the case of $_{15}P^{31}$ ($\Lambda_+/R \simeq 0.03$), both $\alpha(\frac{1}{2})/\alpha(0) \approx 0.5$[14] and $\alpha(\frac{1}{2})/\alpha(0) \approx 0$[15] have been reported, while recent Liverpool results on $_9F^{19}$ ($\Lambda_+/R \approx 0.25$) are consistent with $\alpha(\frac{1}{2})/\alpha(0) \approx 0$.[16]

(ii) With $\Lambda_+/R \ll 1$, virtually all muons should be absorbed from the F_- state (assuming $\mu_I > 0$). The V-A interaction predicts $\Lambda_- > \Lambda_+$; thus on a plot of the reduced capture rates $\Lambda^{\text{cap}}/Z_{\text{eff}}^4$ vs neutron excess $(A-Z)/2A$, such nuclei as $_9F^{19}$, $_{11}Na^{23}$, $_{13}Al^{27}$, and $_{15}P^{31}$ should lie above the "Primeakoff line."[17] Some evidence for this has been obtained recently,[4] but a quantitative comparison with theory does not appear possible at the present time.

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MEASUREMENTS OF MUON DISAPPEARANCE RATES vs TIME IN C, Mg, Al, Si, AND P†

J. L. Lathrop, R. A. Lundy, V. L. Telegdi, R. Winston, and D. D. Yovanovitch

The Enrico Fermi Institute for Nuclear Studies, The University of Chicago, Chicago, Illinois

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At the 1960 Rochester Conference we reported[1] some preliminary evidence for the observation of a hyperfine structure (i.e., spin dependence) effect in the capture of muons by $_{13}Al^{27}$ and $_{15}P^{31}$. This effect[2] consists in a (negative) curvature[3] K of the plot of $f(t)$, the logarithm of the electron rate from these targets, vs time,

$$K = K(0) = f''(1 + f'^2)^{-3/2} \text{ at } t = 0, \qquad (1)$$

and depends on the physical rates as follows:

$$K = K(\Delta\Lambda, R) = -2^{-3/2} n_+ (\Delta\Lambda/\bar{\Lambda})(R - n_-\Delta\Lambda)/\bar{\Lambda}, \qquad (2)$$

where $n_\pm$ are the statistical weights of the hyperfine states $F_\pm = I \pm \frac{1}{2}$, $\bar{\Lambda} = n_+\Lambda_+ + n_-\Lambda_-$ the mean disappearance rate, $\Delta\Lambda = \Lambda_- - \Lambda_+$, and R is the conversion rate from F_+ to F_- (assuming a positive nuclear magnetic moment).[4]

We have now repeated these "curvature" measurements under greatly improved conditions, and have found that, to within experimental error, $K = 0$ for Al and Mg. We used the experimental setup schematically indicated in Fig. 1. The most essential improvements were the following ones:

(a) Comparison of the Al target ($I = \frac{5}{2}$) data with Mg data (mostly $I = 0$) obtained under essentially identical conditions. In the past, C reference

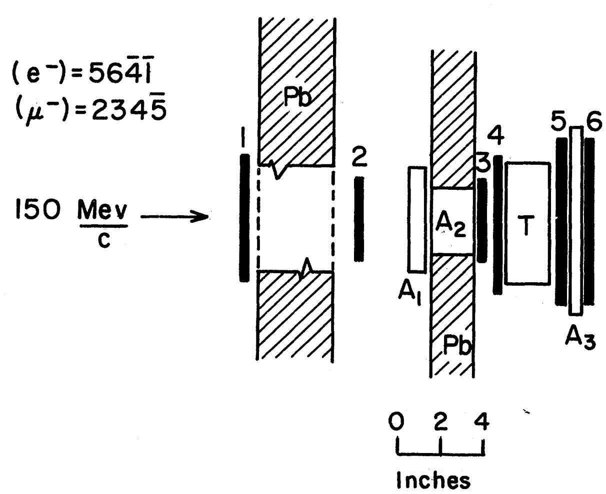

FIG. 1. Experimental setup. 1, 2, 3, 4, 5, and 6, square scintillation counters; $A_1 = \frac{3}{4}$-in. Cu moderator, $A_2 = 2$-in. graphite moderator, $A_3 = \frac{1}{2}$-in. polythene absorber. Pb = Pb collimators. $(\bar{1}456)$ = electron coincidence, $(234\bar{5})$ = muon coincidence.