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Macroscopic Boson States Exhibiting the Greenberger-Horne-Zeilinger Contradiction with Local Realism

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It is shown that the recent Greenberger-Horne-Zeilinger "all or nothing" contradiction of quantum mechanics with local realism can be exhibited on a macroscopic scale where a large number of particles are incident on each analyzer. We present a formulation of the all or nothing paradox in terms of boson fields, and suggest how the paradox might be realized using a correlated photon triplet. The suggested experiment might be readily extended to test, for the first time to our knowledge, quantum mechanics against local realism for situations of more than one quanta per wave packet incident on each measurement apparatus.

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There has been much interest recently in the quantum states described by Greenberger, Horne, and Zeilinger [1] (GHZ) which give predictions contrary to those of all classical theories, based as they are on the Einstein-Podolsky-Rosen [2] (EPR) premises of local realism. The new unexpected feature is that the contradiction with classical theory can be accomplished in a single run or set of measurements. This is not the case with the traditional Bell inequality tests [3] where the contradiction is necessarily statistical and requires data to be collected over many runs.

In a realistic experimental situation the ideal correlations predicted by the GHZ state would not be obtainable and hence the conflict with the classical EPR arguments not in fact revealed so directly. However, Mermin [4] has recently shown that the contradiction with classical theories is still stronger than that of traditional Bell inequality tests. The violation of Bell-type inequalities derived by Mermin from classical EPR assumptions is predicted by quantum mechanics to be much greater for the GHZ experiment.

We show in this paper that the remarkable GHZ prediction can be exhibited on a macroscopic level in the following sense. We consider experiments performed using only three analyzers (six detectors) but where there is an arbitrarily large number of particles incident simultaneously on each analyzer. One thus makes measurements on three spatially separated wave packets each consisting of N (where N can be large) quanta. This is in contrast with the multiparticle states discussed recently by Mermin [4] where the individual quanta emitted are spatially separated so that there is still only one particle per analyzer. For the situations discussed in this paper, the EPR "elements of reality" are thus ascribed to a macroscopic system at each spatially separated analyzer or measurement apparatus. The strong GHZ violation of the classical predictions is even more surprising at this macroscopic level [5].

To date local realism has not been proved experimentally incorrect for any system of greater than one particle per analyzer. It is suggested here how to realize the original microscopic GHZ paradox using a single correlated photon triplet [6]. We suggest that it may be feasible to extend this proposed experiment to test quantum mechanics versus local realism for these new situations involving more than one particle per analyzer.

We choose to formulate the arguments presented by GHZ in terms of boson fields. This is relevant in view of the fact that the most successful experiments confirming quantum predictions against those of classical theory have to date used correlated photon pairs. Because of the poor efficiency of photon detectors we will later discuss the GHZ paradox in terms of the Clauser-Horne-Shimony-Holt [7] modifications of the inequalities derived by Mermin.

First we consider a simple modification of the triplecorrelation experiment suggested by Greenberger, Horne, and Zeilinger [1] and Mermin [4]. We will show initially that the predictions of the following quantum state are in contradiction with the predictions of all classical (local realistic) theories:

$$|\varphi\rangle=\frac{(a_{1+}^\dagger a_{2+}^\dagger a_{3+}^\dagger+a_{1-}^\dagger a_{2-}^\dagger a_{3-}^\dagger)^N|0\rangle}{N![\sum_{r=0}^N r!(N-r)]^{1/2}}. \quad (1)$$

Here the $a_{j+}^\dagger, a_{j-}^\dagger$ ($j=1$, 2, or 3) are boson creation operators for six orthogonal field modes. Typically the a_j are of distinct energies, while the + and − refer to orthogonal polarizations at the same energy. Alternatively the + and − might refer to quanta emitted in different directions [1]. The $|0\rangle$ symbolizes the vacuum state. Here we have N quanta generated in each of the a_1, a_2, a_3 energies. This state may describe N atoms emitting photon triplets in a cooperative fashion so that the quanta are incident on the analyzers simultaneously. The distinct energies may be spatially separated into three regions A_j. At each position A_j the measurement is made. Let us suppose that the detected outputs of the analyzers correspond to the following transformed modes:

$$d_{j\pm}(\phi_j)=(\pm a_{j+}+e^{i\phi_j}a_{j-})/\sqrt{2}. \quad (2)$$

If the + and − refer to different directions, this measurement may be realized by providing phase shifts of the a_{j-} with respect to the a_{j+}, and combining the two with a 50/50 beam splitter. If the + and − are different polarizations, the coupling would be achieved by a polarizer. Photodetectors measure how many of the N quanta incident on each analyzer are deflected "up" or "down." If a particle is detected up or down we assign to it a "spin" value of +1 and −1, respectively. We define the "spin product" $S_j^N(\phi_j)$ as the product of the spins of each of the N particles, detected at position A_j.

Following Mermin and GHZ we restrict attention to two choices for each of the analyzer angles, $\phi=0$ and $\phi=-\pi/2$. We shall denote $d_{j\pm}(0)$ and $d_{j\pm}(-\pi/2)$ as $d_{j\pm}^x$ and $d_{j\pm}^y$, respectively, and the measurements $S_j^N(0)$ and $S_j^N(-\pi/2)$ as S_{jx}^N and S_{jy}^N, respectively. We now consider the predictions of the quantum state (1) for the following spin products: $S_{1y}^N S_{2y}^N S_{3x}^N$, $S_{1y}^N S_{2x}^N S_{3y}^N$, $S_{1x}^N S_{2y}^N S_{3y}^N$, and $S_{1x}^N S_{2x}^N S_{3x}^N$. It is convenient to rewrite $|\varphi\rangle$ in terms of the detected modes $d_{j\pm}^x$ and $d_{j\pm}^y$. To calculate $\langle S_{1x}^N S_{2y}^N S_{3y}^N\rangle$, for example, we rewrite $|\varphi\rangle$ as follows:

$$|\varphi\rangle=\frac{(d_{1+}^x d_{2-}^y d_{3+}^y+d_{1+}^x d_{2+}^y d_{3-}^y+d_{1-}^x d_{2+}^y d_{3+}^y+d_{1-}^x d_{2-}^y d_{3-}^y)^{\dagger N}|0\rangle}{N!(-\sqrt{2})^{3N}[\sum_{r=0}^N r!(N-r)]^{1/2}}. \quad (3)$$

It is apparent that if N is odd the product $S_{1x}^N S_{2y}^N S_{3y}^N$ will *always* be −1. Similar results hold for $S_{1y}^N S_{2x}^N S_{3y}^N$ and $S_{1y}^N S_{2y}^N S_{3x}^N$. To calculate $\langle S_{1x}^N S_{2x}^N S_{3x}^N\rangle$ we transform to the $d_{j\pm}^x$. One finds

$$|\varphi\rangle=\frac{(d_{1+}^x d_{2+}^x d_{3+}^x+d_{1+}^x d_{2-}^x d_{3-}^x+d_{1-}^x d_{2-}^x d_{3+}^x+d_{1-}^x d_{2+}^x d_{3-}^x)^{\dagger N}|0\rangle}{N!(-\sqrt{2})^{3N}[\sum_{r=0}^N r!(N-r)]^{1/2}}. \quad (4)$$

The product $S_{1x}^N S_{2x}^N S_{3x}^N$ is *always* +1, for all N.

Now we follow the original argument of GHZ and Mermin to establish the classical predictions for the spin products. If we select N to be odd, quantum mechanics predicts that the $S_{1x}^N S_{2y}^N S_{3y}^N$ is always −1. Because the three polarizers are spatially separated, the EPR premises of local realism apply [2,3]. In short, local realistic theories will assign to each of the states at A_j before measurement a value for S_{jx}^N and S_{jy}^N, $\tilde S_{jx}^N$ and $\tilde S_{jy}^N$ say, where this value must +1 or −1, and we have that $\tilde S_{1x}^N \tilde S_{2y}^N \tilde S_{3y}^N=\tilde S_{1y}^N \tilde S_{2x}^N \tilde S_{3y}^N=\tilde S_{1y}^N \tilde S_{2y}^N \tilde S_{3x}^N=-1$. Therefore if we examine the prediction for $S_{1x}^N S_{2x}^N S_{3x}^N$, since we can write [recalling that each $(\tilde S_{jy}^N)^2=1$]

$$\tilde S_{1x}^N \tilde S_{2x}^N \tilde S_{3x}^N=(\tilde S_{1x}^N \tilde S_{2y}^N \tilde S_{3y}^N)(\tilde S_{2x}^N \tilde S_{1y}^N \tilde S_{3y}^N)(\tilde S_{3x}^N \tilde S_{1y}^N \tilde S_{2y}^N)=-1,$$

the classical prediction must be that $S_{1x}^N S_{2x}^N S_{3x}^N$ is *always* −1. This is in strong disagreement with the quantum result predicted from (1) with N odd, that $S_{1x}^N S_{2x}^N S_{3x}^N$ is *always* +1. The contradiction is distinct for arbitrarily large N, provided N is odd. This is the startling new GHZ "all or nothing" distinction between quantum and classical which we have now shown may apply to macroscopic systems where large numbers of particles are incident on each detector.

We now consider how to realize the GHZ state experimentally using a correlated photon source. To account for experimental situations, where the absolute −1 prediction for $S_{1x}^N S_{2y}^N S_{3y}^N$ will never be achieved in the first place, we follow the approach of Mermin [4] and derive the following inequality based on classical (local realistic) arguments:

$$F=|\langle S_{1x}^N S_{2y}^N S_{3y}^N\rangle+\langle S_{1y}^N S_{2x}^N S_{3y}^N\rangle+\langle S_{1y}^N S_{2y}^N S_{3x}^N\rangle-\langle S_{1x}^N S_{2x}^N S_{3x}^N\rangle|\le 2. \quad (5)$$

According to local realistic or hidden-variable theories, the averages are expressible in the following form:

$$\langle S_{1x}^N S_{2y}^N S_{3y}^N\rangle=\int\rho(\lambda)d\lambda\, S_{1x}^N(\lambda)S_{2y}^N(\lambda)S_{3y}^N(\lambda), \quad (6)$$

where $S_{jx}^N(\lambda)$, $S_{jy}^N(\lambda)$ represent the spin products of the N quanta detected at A_j given the particular set of hidden variables λ describing the state. The $\rho(\lambda)$ is a probability distribution over the hidden parameters λ. Because $|S_{jx}^N(\lambda)|\le 1$ the derivation of (5) follows in a straightforward manner along the lines given by Mermin. Clearly, the quantum prediction of the state (1) for N odd violates the classical inequality, giving $|F|=4$. This is true for arbitrarily large N.

It is well known [7] that inequalities of the type (5) will not in fact be violated for optical systems, with which we are primarily concerned here, because of the very poor photodetector efficiencies. Let us detect the N quanta incident on each polarizer, assigning a value to the spin of each quanta as follows: $+1$ for up, -1 for down, and 0 if not detected. Clearly with very small detection efficiency factors, the magnitudes of the $\langle S_{1x}^N S_{2x}^N S_{3x}^N\rangle$, etc., diminish and the inequality (5) is satisfied. However, it is well known that one can violate a weaker version [7] of Bell-type inequalities even in the presence of weak detector efficiencies. Here we follow arguments similar to Clauser *et al.* [7] and present a weaker version of the Mermin inequality (5).

Let us define the modified spin product for the N quanta at detector A_j. Let $\bar{S}_{j\phi}^N(\lambda)=S_{j\phi}^N(\lambda)/P_j^N(\lambda)$, where $S_{j\phi}^N(\lambda)$ is the spin product of the N quanta for a given set of hidden variables, and where we assign a value of zero to the spin for quanta not detected. Now $P_j^N(\lambda)$ is the probability, for the set of hidden parameters λ, that the N quanta of A_j are all detected. In fact $S_{j\phi}^N(\lambda)=P_{+j}(\lambda)-P_{-j}(\lambda)$ where $P_{\pm j}(\lambda)$ is the probability of an arrangement of the N quanta at A_j giving a spin product ± 1, respectively, and $P_j^N(\lambda)=P_{+j}^N(\lambda)+P_{-j}^N(\lambda)$. Thus $|\bar{S}_{j\phi}^N(\lambda)|\le 1$. Now one makes the important auxiliary assumption that the probability $P_j^N(\lambda)$ of detecting all the N quanta is independent of the choice of analyzer angles ϕ_j. We may now rearrange (6) to obtain

$$\langle S_{1x}^N S_{2y}^N S_{3y}^N\rangle_R=\frac{\langle S_{1x}^N S_{2y}^N S_{3y}^N\rangle}{N_0}=\int \bar{\rho}(\lambda)\bar{S}_{1x}^N(\lambda)\bar{S}_{2y}^N(\lambda)\bar{S}_{3y}^N(\lambda)d\lambda\,, \qquad (7)$$

where $\bar{\rho}(\lambda)=\{\rho(\lambda)P_1^N(\lambda)P_2^N(\lambda)P_3^N(\lambda)\}/N_0$ and we define

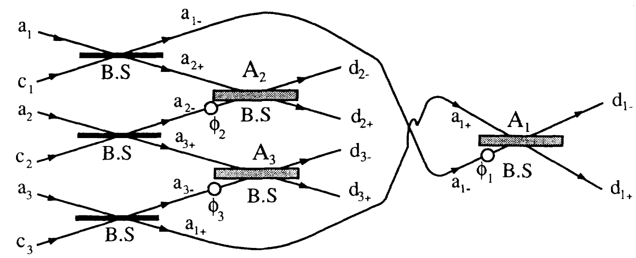

FIG. 1. A possible realization of the GHZ state. The B.S. denotes a 50/50 beam splitter or coupler. The detected modes are the $d_{j\pm}$. The a_{j-} may be phase shifted either $\phi=0$ or $-\pi/2$ relative to the a_{j+}. The diagram is a schematic depiction only, in that the real distances from the initial sources of a_1,a_2,a_3 to the final detectors of $d_{j\pm}$ are the same.

$N_0=\int\rho(\lambda)P_1^N(\lambda)P_2^N(\lambda)P_3^N(\lambda)d\lambda$ which is the proportion of runs where all $3N$ quanta are detected. Thus $\bar{\rho}(\lambda)$ is the normalized probability distribution redefined with respect to the subensemble where all $3N$ photons are detected. The $\langle S_{1x}^N S_{2y}^N S_{3y}^N\rangle_R$ is thus the product of the spin products calculated at each detector, but where the average is calculated only over those runs where all of the $3N$ quanta are detected. The derivation given by Mermin [4] now applies (as above) to the reduced $\bar{\rho}(\lambda)$ and spin products $\bar{S}_{j\phi}^N(\lambda)$. We have the weaker version of inequality (5) involving averages calculated only over the detected subensemble:

$$F_R=|\langle S_{1x}^N S_{2y}^N S_{3y}^N\rangle_R+\langle S_{1y}^N S_{2x}^N S_{3y}^N\rangle_R+\langle S_{1y}^N S_{2y}^N S_{3x}^N\rangle_R-\langle S_{1x}^N S_{2x}^N S_{3x}^N\rangle_R|\le 2\,. \qquad (8)$$

We now give a brief discussion of how a state violating $F_R\le 2$ may be prepared using correlated photon states of the type generated in parametric down-conversion. We consider a multiphoton state $a_1^{\dagger N}a_2^{\dagger N}a_3^{\dagger N}|0\rangle$ input on the apparatus sketched in Fig. 1. We thus have N quanta incident at each input port associated with a_j. The fields generated after the first set of beam splitters are

$$\begin{aligned}&a_{1-}=(ia_1+c_1)/\sqrt{2},\quad a_{2-}=(ia_2+c_2)/\sqrt{2},\quad a_{3-}=(ia_3+c_3)/\sqrt{2}\,,\\&a_{2+}=(a_1+ic_1)/\sqrt{2},\quad a_{3+}=(a_2+ic_2)/\sqrt{2},\quad a_{1+}=(a_3+ic_3)/\sqrt{2}\,,\end{aligned} \qquad (9)$$

where the c_j denote the second vacuum-state inputs to the beam splitters and the a_j are boson operators for the input number states. The outgoing state after the beam splitters is

$$|\varphi\rangle=(-ia_{1-}^\dagger+a_{2+}^\dagger)^N(-ia_{2-}^\dagger+a_{3+}^\dagger)^N(-ia_{3-}^\dagger+a_{1+}^\dagger)^N|0\rangle/N!^{3/2}(\sqrt{2})^{3N}. \qquad (10)$$

The transformations due to the second set of beam splitters and phase shifts of the a_{j-} relative to a_{j+} at the spatially separated regions A_j generate final detected modes $d_{j\pm}(\phi_j)$ given by $d_{j+}=a_{j+}+ie^{i\phi_j}a_{j-}$ and $d_{j-}=ia_{j+}+e^{i\phi_j}a_{j-}$. We define $d_{j\pm}^y=d_{j\pm}(-\pi/2)$ and $d_{j\pm}^x=d_{j\pm}(0)$ and S_{jy}^N and S_{jx}^N as the "spin" products with the appropriate choices $\phi_j=-\pi/2$ and $\phi_j=0$, respectively. The subensemble relevant to the weaker inequality (8) is where exactly N quanta are detected at each spatial region A_j. We note that the use of the beam splitters in this particular configuration requires the use of a weaker inequality even in the presence of perfectly efficient photodetectors. The terms in the expansion (10) which are relevant are

$$\sum_{r=0}^{N} \binom{n}{r}^3 i^r(a_{1-}^\dagger - a_{2-}^\dagger - a_{3-}^\dagger)^r (a_{1+}^\dagger + a_{2+}^\dagger + a_{3+}^\dagger)^{N-r}|0\rangle . \quad (11)$$

On substituting the $a_{j\pm}$ for the detected modes $d_{j\pm}^x$ and $d_{j\pm}^y$, it is revealed that the spin products $S_{1y}^N S_{2y}^N S_{3x}^N$, $S_{1x}^N S_{2y}^N S_{3y}^N$, and $S_{1y}^N S_{2x}^N S_{3y}^N$ are always $+1$. This is seen upon careful examination by noting that the $r=N-x$ term in the expansion (11) may be obtained from the $r=x$ term by simply replacing the d_{j-} with $-d_{j-}$. The result is that terms of odd powers of d_{j-} will not contribute, and the spin product must thus be $+1$. Similar substitutions and examination for the $S_{1x}^N S_{2x}^N S_{3x}^N$ case reveal that for N odd the product $S_{1x}^N S_{2x}^N S_{3x}^N$ is always -1. Thus quantum mechanics predicts a violation of the weaker inequality (8) for the system depicted in Fig. 1, provided N is odd.

Of course for $N=1$ we have the original microscopic version of the GHZ paradox. This may be a useful realization of the GHZ state, since photon experiments provide good analyzer efficiencies and sources of correlated particles. Highly correlated photon pairs generated via parametric down-conversion have already been predicted [8] and used [9] to demonstrate violation of traditional (weaker) Bell inequalities using schemes similar to that of Fig. 1 [6]. Here we require a single correlated photon triplet. The paradox would be as follows: Given that N quanta are detected simultaneously at each A_j, the spin products $S_{1y}^N S_{2y}^N S_{3x}^N$, $S_{1y}^N S_{2x}^N S_{3y}^N$, $S_{1x}^N S_{2y}^N S_{3y}^N$ would be observed to always be -1; the product $S_{1x}^N S_{2x}^N S_{3x}^N$ is predicted to be always $+1$ according to quantum mechanics, but always -1 according to the classical EPR premises.

Although three-photon-emitting atomic systems seem more natural, generation of the photon triplet may be possible using parametric down-conversion in a way that is readily extended to test quantum mechanics for the new situation discussed here of more than one quanta per wave packet. A single photon state has been prepared experimentally using down-conversion by Hong and Mandel [10]. This is possible because detection of a photon in an idler field $a(k)$ implies that a photon is present in the corresponding signal field $a(k')$. Thus one can prepare a correlated triplet $|1\rangle|1\rangle|1\rangle$ in the three signal fields k_j by detection of a photon in each of the corresponding idler fields k_j'. We point out, however, that the calculations presented correspond to situations where the three photons are incident on the apparatus simultaneously. Recent work by Yurke and Stoler [11] suggests one to use three independent parametric amplifiers, the three-photon state being prepared conditional on the simultaneous detection of a photon in each idler mode k_j'. One can then test for the situations of N (N is odd) photons per analyzer by preparing N photons in each of the three k_j directions. This is done by detecting N photons in each of the corresponding idler fields $a(k_j')$. The feasibility of the experiment for larger N values is currently limited by the effect of poor detector efficiencies. If N_0 photons are detected by an inefficient detector, one cannot exclude the possibility that for example there are actually N_0+1 photons incident on the analyzer. The presence of the N_0+1 state will tend to destroy the GHZ effect anticipated with just N_0 photons incident. Thus if one chooses to do the above experiment with $N=N_0$, one needs to operate the parametric down-conversion at sufficiently low intensities to ensure that the N_0+1 state is not generated with appreciable probability. This makes generation of the N_0 state itself difficult, if N_0 is large [12]. Nevertheless, $N_0=3$ may be possible in the near future and would signify a new test of quantum mechanics.

To conclude we have presented new aspects of the all or nothing GHZ contradiction of quantum mechanics with local realism. The paradox is formulated in terms of boson fields and is shown to hold even for situations where large numbers of particles are incident on a single analyzer. An appropriate experiment is suggested.

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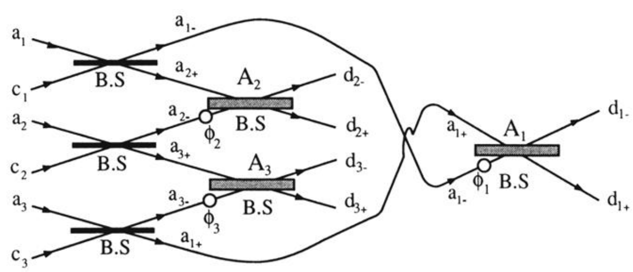

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