Nonlinear Evolution of the Modified Simon-Hoh Instability via a Cascade of Sideband Instabilities in a Weak Beam Plasma System

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The modified Simon-Hoh instability is observed in a collisionless cylindrical plasma, produced by a weak electron beam, in which electrons are strongly magnetized and the ions are essentially unmagnetized. The nonlinear evolution of this instability occurs through a sequence of sideband instabilities, thought to be induced by trapped ions, and can lead to a chaotic state.

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Turbulence abounds in plasmas [1] as is evident from measurements on laboratory, fusion, space, and astrophysical plasmas. However, it is extremely difficult to study the evolution of turbulence in a plasma because, in addition to having a rich variety of collective modes *a* of oscillation, a plasma also has many nonlinear coupling mechanisms. In this Letter we document the nonlinear evolution of a single coherent mode (a modified Simon-Hoh instability) in a plasma produced by a weak electron beam via a sequence of sideband instabilities that are thought to arise because of ion trapping effects. While the eventual frequency-locked states observed in these experiments appear to be of the generic period-doubling type [2], the transitions are not. As in some fluid experiments [2] each transition begins with the appearance of an apparently new oscillation mode at a very low frequency after which the frequency rises to half the value of the previous lowest frequency where it meets and locks with the decreasing-frequency local lower sideband frequency. We believe that such a sequence of successive sideband instabilities and locking may be common to plasma systems in which large ion orbit dynamics are important and may lead to understanding their transition to turbulence.

To be able to exhibit the fine details of the transitions such as those shown here, the plasma must be extremely controllable. The low electron density ($n_e \sim 10^6$–10^9 cm^{-3}) plasma is generated by ionization of background gas by a very-low-density electron beam ($n_b \sim 10^4$–10^7 cm^{-3}). A Gaussian, temperature-limited, electron beam ($I_b \sim 1$ μA–1 mA, 250 V) with 5 mm (FWHM) diameter is injected axially from one end of a 10-cm-diam, 180-cm-long stainless-steel tube immersed in a dc axial magnetic field ($B \sim 50$–320 G). The beam and the plasma are terminated by a grounded collector located at 80 cm distance from the gun. The plasma density can be controlled by varying the ionization rate either by changing the gas fill pressure (5×10^{-6} to 5×10^{-5} torr) or by varying the beam current over its range. At these low values of pressure and current, the system is always below the beam-plasma discharge [3] threshold and thus can be kept remarkably free of noise.

In this experiment ($B=160$ G, Ar plasma) the electrons are strongly magnetized ($r_{Le}=0.04$ cm for $T_e \simeq 4$ eV) whereas the ions are essentially unmagnetized ($r_{Li} \sim 5.6$–17.7 cm for $T_{i\perp} \sim 1$–10 eV). Here r_{Le} and r_{Li} are the electron and ion Larmor radius, respectively, T_e is the plasma electron temperature, and $T_{i\perp}$ is the characteristic ion energy in the radial direction, as will be discussed later. The steady-state electron and ion radial density profiles were measured using a Langmuir probe. These show that the ions have a broader profile than the electrons; i.e., the center region of the chamber containing the electron beam is electron rich, whereas the outer region is seen to be ion rich [4]. These steady-state electron and ion density profiles imply the existence of a significant radial electric field E_{r0}, which was deduced by directly measuring the radial plasma potential profile $\Phi(r)$ [Fig. 1(a)] with an emissive probe. It is this potential profile that confines the ions in the radial direction. The plasma electron temperature T_e measured by a Langmuir probe is $\simeq 4$ eV, whereas in the radial direction the characteristic ion energy $T_{i\perp}$, measured by an energy analyzer, monotonically decreases as the ions bounce back and forth almost radially in the radial potential well [4].

By varying either the gas pressure P or the beam current I_b, a stable (persisting for many hours in its linear stage) mode M_1 with a frequency f_1 ($f_{ci} < f_1 < f_{pi}$) can be excited [5]. Here f_{ci} and f_{pi} are the ion cyclotron and ion plasma frequencies, respectively. This mode has four basic experimental characteristics [6]. First, it is a mode with $m=1$ azimuthal behavior, no radial variation in phase or frequency, and with a spectral amplitude peak at a radius of 3.5 mm from the beam axis [Fig. 1(b)]. By spectral amplitude we mean the peak spectral power density obtained by Fourier transforming the real time probe signal using a spectrum analyzer [5]. Second, the measured value of f_1 is consistent with a calculated effective ion $\mathbf{E}\times\mathbf{B}$ drift frequency f_{Ei}, which takes into account the large ion Larmor radius effect [Fig. 1(c)] [6]. Third, the measured value of the mean azimuthal velocity $v_{\theta i}$ of the ions (using a one-sided probe [7]) gives a frequency $f_{\theta i}=v_{\theta i}/2\pi r$, which shows a reasonable correlation with f_1 over a wide range of I_b [Fig. 1(c)]. Finally, f_1 scales as $\sqrt{M}$, where M is the ion mass for four different gases, N_2, Ar, Kr, and Xe.

All the above observations are consistent with the notion that the basic instability being observed is a saturat-

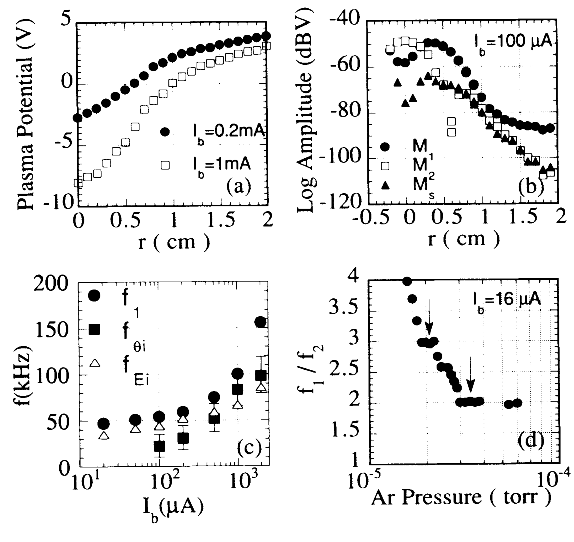

FIG. 1. (a) Radial plasma potential profile measured by an emissive probe for two different values of I_b in mA and $P=2\times10^{-5}$ torr. (b) Radial profiles of the log of spectral amplitudes (in dBV) of the M_1, M_2, and M_s modes, respectively. $I_b=100$ μA and $P=1\times10^{-5}$ torr. (c) I_b dependence of f_1, $f_{\theta i}$, and f_{Ei}. Here, $f_{\theta i}$ is the measured ion rotation frequency by the one-sided probe and expressed as $f_{\theta i}=v_{\theta i}k_\theta/2\pi$, where $k_\theta=1/r_1$ is 2 cm^{-1} and $r_1=0.5$ cm is the radius at which the spectral amplitude of the M_1 mode is maximum. f_{Ei} is the effective ion $\mathbf{E}\times\mathbf{B}$ drift frequency for the large ion Larmor radius [6]. The electron $\mathbf{E}\times\mathbf{B}$ drift frequency is nearly a factor of 20 larger than f_{Ei}. Here, $f_{ci}=6.7$ kHz and $f_{pi}=46$–460 kHz for $I_b=20$–2000 μA. $P=2\times10^{-5}$ torr, $r=0.5$ cm. (d) f_1/f_2 vs pressure P. $I_b=16$ μA, $r=0.5$ cm. The ranges of P over which f_1 and f_2 are frequency locked are shown by arrows. It is over this range that a cascade of sideband instabilities is observed. The uncertainties are smaller than the size of the symbols where no error bars are shown.

ed single mode of the modified Simon-Hoh instability (MSHI) [6], which can arise in a cylindrical, collisionless plasma if a radial dc electric field exists and if this dc electric field and the radial density gradient are in the same direction. In such a plasma, if the ions are weakly magnetized while the electrons are strongly magnetized, a velocity difference in the θ direction drift can arise between the electrons and the ions because of the finite ion Larmor radius effect. If azimuthal symmetry is broken, this leads to a space charge separation in the θ direction. The resulting azimuthal electric field $E_{\theta 1}$ and the enhancement of the density perturbation occur in the same manner as in the usual Simon-Hoh instability, seen in collisional plasmas with magnetized ions. Although the collisional Simon-Hoh instability has been investigated both theoretically [8] and experimentally [8], this is the first documentation, to our knowledge, of the MSHI dominated by kinetic effects rather than collisions.

In an initial state with azimuthal symmetry, writing (i) the electron-fluid density perturbation as $\tilde{n}_e/n_0=(e\phi_1/T_e)\omega^*/(\omega-\omega_E)$, where ϕ_1 is the fluctuating potential, $\omega^*=-k_\theta(cT_e/eB_0)(1/n_0)dn_0/dr$ is the electron diamagnetic drift frequency, and $\omega_E=-k_\theta cE_{r0}/B_0$ is the electron $\mathbf{E}\times\mathbf{B}$ drift frequency, and (ii) the unmagnetized ion-fluid density perturbation (with azimuthal Doppler shift and neglecting radial variations) as $\tilde{n}_i/n_0=k_\theta^2 e\phi_1/M(\omega-k_\theta v_{\theta i})^2$, where $v_{\theta i}$ is the mean azimuthal ion drift speed, we obtain, assuming that perturbations in the electron and ion densities are about the same, the dispersion relation

$$\omega^*/(\omega-\omega_E)=k_\theta^2c_s^2/(\omega-k_\theta v_{\theta i})^2\,, \tag{1}$$

where $c_s=(T_e/M)^{1/2}$. This quadratic in ω is easily solved and one finds the real frequency to be very close to $v_{\theta i}k_\theta \simeq v_{\theta i}/r$, while the growth rate is nearly equal to $[c_s^2k_\theta^2(\omega_E-v_{\theta i}k_\theta)/\omega^*]^{1/2}$. This shows that when $cE_{r0}/B_0>v_{\theta i}$ and $\omega_E/\omega^*>0$, i.e., the dc electric field and the radial density gradient are in the same direction, a fluid instability (MSHI) can be excited. In fact, radial variation is important but examination of the orbit kinetics leads to the same conclusion with an appropriately modified definition for an averaged $v_{\theta i}$ [6].

Experimentally, we find that at low values of beam current I_b (<2 μA) and pressure P ($<1.5\times10^{-5}$ torr), the MSHI or M_1 mode is the only coherent mode that exists in the plasma. There are no other intermediate frequency [9,10], flutelike modes [10], or any electron plasma and cyclotron oscillations observable in this system. The typical signal-to-noise ratio before the MSHI begins to interact nonlinearly with the plasma is 10^6, at this point the perturbed ion density level of the MSHI is typically $\tilde{n}_i/n_0\simeq30\%$.

As I_b or P is increased, the spectral amplitude of the MSHI (M_1 mode) increases and the harmonics of f_1 up to the seventh harmonic can be observed. For a beam current greater than 3 μA, the nonlinear sequence begins with the appearance of a second mode M_2 with a frequency f_2 ($f_2 \ll f_1$) together with sideband modes M_s at frequencies $f_s=f_1\pm nf_2$ ($n=1,2,\ldots$). As I_b is increased f_2 increases, and f_2 and f_s migrate towards one another until they mode lock at f_1/n and $(n+1)f_1/n$, respectively. The onset of M_2 and M_s modes is characterized by a sudden decrease in the spectral amplitude of the M_1 mode. Thereafter as I_b further increases, the spectral amplitude of M_1 continues to increase monotonically whereas the spectral amplitudes of M_2 and M_s show a strong increase when f_1/f_2 are frequency locked at 3 or 2. In Fig. 1(d) the variation of f_1/f_2 vs P is presented which shows the range of P over which locking occurs. A similar frequency locking sequence can also be obtained at a given value of P by increasing I_b. The range of I_b or P over which the two frequencies are locked becomes broader when $f_1/f_2=2$ (where f_2 and the lower sideband of f_s become indistinguishable) compared to when $f_1/f_2=3$. It is over this parameter range where f_1 and f_2 are locked in this way that a whole cascade of sideband instabilities can be observed.

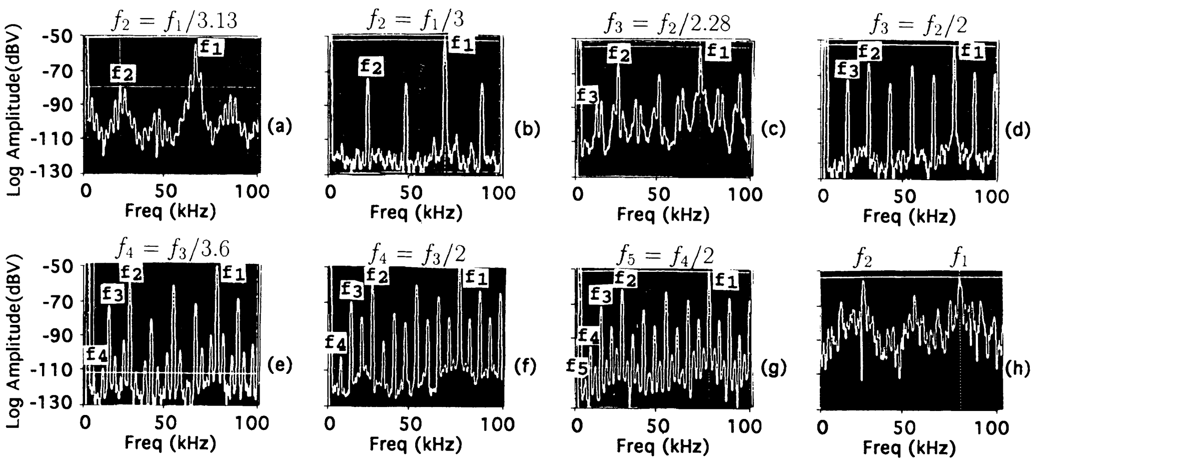

FIG. 2. Observation of a cascade of sideband instabilities at $r=0.5$ cm when f_1/f_2 is locked at 3 and P is gradually increased from (a) 1.9×10^{-5} to (h) 2.15×10^{-5} torr. The vertical scale in (a)-(h) is $\log_{10}$ of spectral amplitude, in dBV. 0 dBV = 1 V. We identify the state observed in (h) as the chaotic state.

One such sequence, showing a great range of cascading, is shown in Fig. 2, where $f_1/f_2 \simeq 3$ [Figs. 2(a) and 2(b)]. An increase in P leads to the appearance of a third low-frequency mode M_3 which has a frequency f_3, which is less than f_2 and the beat components $mf_1+nf_2+pf_3$ $(m,n,p=\pm1,\pm2,\ldots)$ [Fig. 2(c)]. With further increase in P, f_3 migrates towards f_2 until f_3 locks with f_2 such that $f_3=f_2/2=f_1/6$ [Fig. 2(d)]. At this point with a very slight increase in P yet another mode M_4, which has a frequency f_4 (which is less than f_3), appears [Fig. 2(e)], migrates toward f_3, and locks with its lower sideband at $f_3/2$ [Fig. 2(f)]. We have seen clearly up to five cascades of decays by extremely fine control of the pressure which gives frequencies $f_5=f_4/2=f_3/4=f_2/8=f_1/24$ (i.e., 3×2^3 discrete frequency subharmonic components) [Fig. 2(g)]. Any further increase in the control parameter (P or I_b) leads to frequency unlocking of all the modes as shown in Fig. 2(h) and the spectrum becomes quite chaotic where various peaks are no longer identifiable as a linear combination of the modes, their harmonics, and their beat frequencies. Here we point out that the nonlinear sequence described above is also applicable to the harmonics of f_1 which are present over the entire pressure range shown in Fig. 1(d).

Now we discuss characteristics of the M_2 and the M_s modes. While M_1 is an $m=1$ mode with an off-axis spectral amplitude peak, M_2 is an $m=0$ mode whose spectral amplitude peaks on axis [Fig. 1(b)] and has a rather deep null at about the beam edge, together with a rapid radial phase change of 180°, across the near-null with nearly uniform radial phase elsewhere. Like M_1, M_s is also an $m=1$ mode whose spectral amplitude peaks near the beam edge [Fig. 1(b)]. It is also found that after the onset of the M_2 mode, its frequency f_2 is proportional to the square root of the electron-density fluctuation level associated with M_1 which is $\propto \tilde{n}_e^{1/2}$. Furthermore, the frequency f_2 of the M_2 mode shows a reasonable correlation (Fig. 3) with the calculated value of the azimuthal $(m=1)$ bounce frequency of the ions. Here the bounce or trapping frequency f_b is given by $(1/2\pi)(e\phi_1/Mr^2)^{1/2}$ and ϕ_1 is the fluctuating potential of the M_1 mode measured at $r=0.2$ cm and at $r=0.5$ cm by using an emissive probe. We also found that, while the dc plasma potential showed no dependence on the ion mass, ϕ_1 was found to be $\propto M$. Since $f_b \propto (\phi_1/M)^{1/2}$, and $f_2 \simeq f_b$, f_2 was thus found to be independent of the ion mass.

Now we propose the following interpretation of the above-mentioned observations. When the M_1 mode attains a sufficiently large amplitude, it can trap a significant number of ions in the wave potential. These trapped ions are not, however, resonant with the M_1 mode in the usual sense of executing only small oscillations around the mode angular velocity. As a result of the large radial excursions, the trapping lies in the fact that these particles are resonant over the whole cycle and

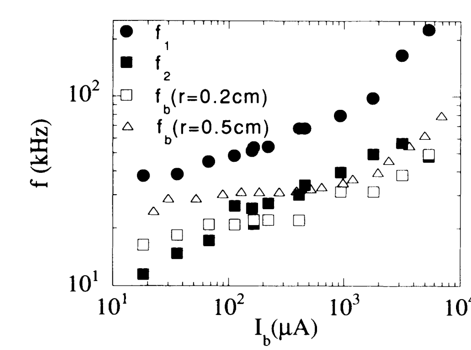

FIG. 3. Dependences of f_1, f_2, and f_b on I_b. $P=2\times10^{-5}$ torr, $r=0.2$ cm. Values of f_b calculated from ϕ_1 measured at two different radii are shown.

slowly gain or lose energy and angular momentum as a result, even when making large excursions about their mean velocity and radius. A proper calculation of f_b should take the details of ion orbits in the dc as well as fluctuating fields into account. However, given that there is a reasonable correlation between the measured f_2 and calculated f_b, we postulate that, as a result of trapping, the M_1 mode may now be driven modulationally unstable to sideband modes M_s [11]. The sideband modes will have Doppler-shifted frequencies in the laboratory frame of $f_1 \pm f_b = f_s$, via a nonlinear coupling introduced by these trapped particles. Indeed, the real time probe signals do show an amplitude-modulated wave form at the beat frequencies, $f_1 \pm f_2$. Simultaneously, the M_2 mode with a frequency $f_2 = f_1 - f_s$ and azimuthal mode number $m=0$ is excited in the plasma. Although the physical origin of the M_2 mode is at present not understood, there can exist a whole continuum of sound wave modes with finite $k_\parallel$ which can be simultaneously excited with the strong sideband modes. However, in the experiments we were unable to measure the necessary $k_\parallel$ for these modes. An alternate explanation for the M_2 mode is that it is some sort of kinetic hybrid mode between the electron-rich inner core and the ion-rich (kinetic) outer halo, all driven by the azimuthally averaged modulation of the ion orbits.

As ϕ_1 increases, f_2 migrates towards f_1 ($f_2 = f_b \propto \phi_1^{1/2}$) and the final state can be a new periodic oscillation, the mode-locked state with $f_2 = f_1/n$ ($n = 1,2,\ldots$). Once f_2 is mode locked with f_1, this new periodic state is likely to be further modulationally unstable to a low-frequency mode M_3 with a frequency f_3 which will then migrate and mode lock at $f_2/2$ with the local lower sideband frequency and so on. Thus, what at first sight appears to be the subharmonic route to a chaotic state, follows a rather complex path starting at the low-frequency modulational end and finally mode locking at the subharmonic frequency.

In conclusion, we have observed a modified Simon-Hoh instability in a collisionless plasma column that is produced by a weak electron beam. The nonlinear evolution of this instability occurs through a sequence of sideband instabilities, thought to be induced by trapped ions, and can lead to a chaotic plasma state.

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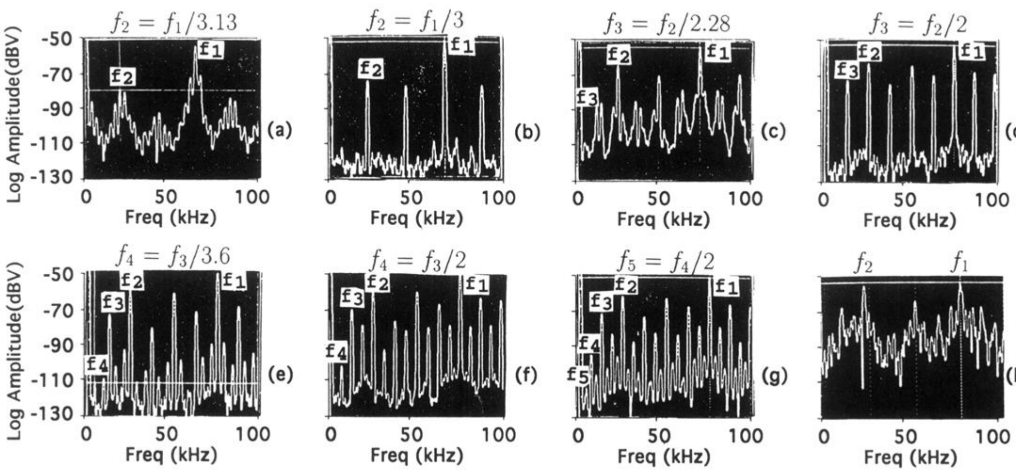

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