Intermittency in Second-Order Phase Transitions

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It is shown that self-similar behavior in multiplicity fluctuations exists in the Ginzburg-Landau description of second-order phase transitions. Furthermore, there exists a numerical exponent that characterizes the intermittency properties in the hadronic phase and is independent of the specific values of the coefficients in the Ginzburg-Landau potential. Current data on intermittency are only 2σ away from the critical exponent.

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In a high-energy nuclear collision, which is the only feasible way in the laboratory to possibly create a thermalized quark phase, the remnants of the phase transition to hadrons would be copious and readily measurable. The usual connection between correlation functions and the Ginzburg-Landau description of phase transitions in conventional statistical physics [1] has been applied to the study of multiparticle final state a long time ago [2], and were revived more recently in consonance with the development of interest in multiparticle production [3–5]. The introduction of intermittency to particle physics [6] has at the same time stimulated considerable activities in the study of self-similarity behavior of multiplicity fluctuations in varying sizes of resolution cells [7,8]. In relating intermittency to the quark-hadron phase transition there is so far only a speculation on the behavior of the intermittency index [9], based partly on the results of studies of the Ising model [10,11]. In this paper we integrate the various concepts mentioned above and determine the properties of intermittency in the Ginzburg-Landau (GL) theory. What emerges is a critical exponent that is independent of the precise values of the coefficients in the GL potential, so long as they allow the hadronic phase to develop.

To simplify our problem let us focus our attention on one small cell in phase space of size δ, ignoring all other parts of the phase space. We shall take the variables to be the rapidity y and transverse momentum $\mathbf{p}_T$, and denote them collectively by z; thus δ represents $\delta y\,\delta\mathbf{p}_T$. Let the number of particles observed in δ in an event be n, and let the multiplicity distribution in n after many events be $P_n(\delta)$. The scaled factorial moments [6] are defined by

$$F_q(\delta)=\frac{\langle n(n-1)\cdots(n-q+1)\rangle}{\langle n\rangle^q}, \tag{1}$$

where $\langle\cdots\rangle$ denotes (vertical) averaging with weight $P_n(\delta)$. Intermittency refers to the power-law behavior

$$F_q(\delta)\propto\delta^{-\varphi_q} \tag{2}$$

over a range of small δ. There exists abundant evidence for such behavior in e^+e^-, μp, pp, pA, and AA collisions [12].

If there is no dynamical contribution to the multiplicity fluctuation, such as due to a phase transition or any other production mechanism, P_n should be just the Poisson distribution, P_n^0, reflecting statistical fluctuation only. In that case F_q would have no dependence on $\langle n\rangle$, and therefore on δ. Thus our interest is in the deviation of P_n from P_n^0. It suggests the use of the coherent-state representation, since the multiplicity distribution of a pure coherent state $|\phi\rangle$ is Poissonian, i.e., $|\langle n|\phi\rangle|^2=P_n^0$, with

$$\langle n\rangle=\langle\phi|\int_0^\delta dz\,a^\dagger(z)a(z)|\phi\rangle=\int_0^\delta dz\,|\phi(z)|^2, \tag{3}$$

where the property $a(z)|\phi\rangle=\phi(z)|\phi\rangle$ has been used. It is the fluctuation from such a pure state that characterizes our problem. The use of coherent states in the study of multiparticle production has been considered before [13–15]. We find it ideally suited for the formulation of our problem in intermittency.

In general the system need not be in a pure state $|\phi\rangle$. Let $F[\phi]$ be the free-energy functional of ϕ that prescribes the probability that the system is in the state $|\phi\rangle$. Then we have

$$P_n=Z^{-1}\int\mathcal{D}\phi\,P_n^0e^{-F[\phi]} \tag{4}$$

and

$$Z=\int\mathcal{D}\phi\,e^{-F[\phi]}. \tag{5}$$

Using (3) in $P_n^0=(n!)^{-1}\langle n\rangle^ne^{-\langle n\rangle}$, which is then substituted into (4), we can apply the resultant P_n to the calculation of the fractorial moments

$$f_q\equiv\sum_{n=q}^{\infty}\frac{n!}{(n-q)!}P_n=Z^{-1}\int\mathcal{D}\phi\left(\int dz\,|\phi(z)|^2\right)^qe^{-F[\phi]}. \tag{6}$$

It then follows from (1) that

$$F_q=f_q/f_1^q. \tag{7}$$

We now let $F[\phi]$ take the Ginzburg-Landau form [16]

$$F[\phi]=\int dz\,[a|\phi(z)|^2+b|\phi(z)|^4+c|\partial\phi/\partial z|^2], \tag{8}$$

where a, b, and c are constants that characterize the system, commonly regarded to depend on temperature T in specific ways. The range of integration can be over any volume in the phase space of interest, just as it is for the integration over z in (6). If it is restricted to the y axis over an interval Y as is usually done, then one obtains, for $b=0$, the correlation length $\xi=\sqrt{c/|a|}$ [2-5]. Our interest here, however, is to consider only a small cell of size δ, within which we claim no power for further resolution. That is, within δ we cannot subdivide the cell and examine the variation of $\phi(z)$ within δ. The consequence is that $F[\phi]$ becomes

$$F[\phi]=\delta(a|\phi|^2+b|\phi|^4) \qquad (9)$$

for vertical analysis at small δ. The $c|\partial\phi/\partial z|^2$ term is relevant to the horizontal analysis over the entire phase space, which is currently under investigation.

As is well known, the transition point is at $a=0$; b is always positive for a stable system. For $a>0$, the potential minimum is at $|\phi|^2=0$, corresponding to the quark phase without condensate. For $a<0$, the minimum is at $|\phi|^2=|\phi_0|^2\equiv -a/2b$, corresponding to the hadron phase with condensate. If there were no fluctuation in ϕ from that value, then we would have from either (3) or (6) with $q=1$ the average hadron density

$$\langle n\rangle/\delta=|\phi_0|^2=-a/2b\,. \qquad (10)$$

Thus $-a/2b$ has immediate physical interpretation for $a<0$. Of course, more interesting physics follows from the fluctuations allowed by (9). Nothing about $F[\phi]$, for $a>0$, can be identified with measurable quantities in terms of hadrons. In the following we shall do computations only for $a<0$.

Putting (9) in (6) and requiring ϕ to be constant inside δ, but allowing it to vary over the whole complex plane, we have

$$f_q=Z^{-1}\delta^q\pi\int_0^\infty d|\phi|^2|\phi|^{2q}e^{-\delta(b|\phi|^4-|a||\phi|^2)}\,. \qquad (11)$$

This can be integrated exactly so that from (7) we get

$$F_q=q!D_{-q-1}(s)D_{-2}^{-q}(s)D_{-1}^{q-1}(s)\,, \qquad (12)$$

where $s=-|a|\sqrt{\delta/2b}$, and $D_{-a}(s)$ is the parabolic cylinder function. We shall use the variable

$$x=s^2=(a^2/2b)\delta \qquad (13)$$

and examine the x dependence of $F_q(x)$.

Our first remark is that for the scaled moments F_q the dependences on δ, a, and b do not occur separately but only through the one variable x. That implies a connection in the δ dependences for various cases involving different values of a and b; we shall return to this point below. If δ were allowed to be large, then the exponential in (11) would be sharply peaked at $|\phi_0|^2$ and we would have the trivial Poissonian result without the dynamical effect due to the $c|\partial\phi/\partial z|^2$ term. If, on the other hand, we let δ become infinitesimal, then the fluctuation in ϕ can be so large that there would be no sensitivity to the value of a in (11). Thus because of (13) we expect F_q to become independent of x in that limit, which means no intermittency, i.e., $\varphi_q=0$. Evidently, the interesting region that can possibly reveal some nontrivial δ dependence in F_q is for some intermediate values of x.

In Fig. 1 we show $\ln F_q$ vs $-\ln x$ for various values of q. Evidently, the relationship is not linear over the whole range $-4<-\ln x<4$. Thus we do not have the strict power-law behavior of (2). If one restricts the $-\ln x$ variable to, say, two narrower ranges so that there is approximate linearity in each, then because of (13) the intermittency index φ_q at fixed q is smaller at the upper range (smaller $|a|$), corresponding to less multiplicity fluctuation. Note that this is the result of vertical analysis in a fixed cell δ, and is indirectly related to what one expects in horizontal analysis, where small $|a|$ implies long correlation length ξ and therefore long-range order.

From Fig. 1 we also see that for fixed x the degree of intermittency increases with q. That is, of course, a general property, since only events with $n\geq q$ can contribute to (1), so high q selects events with high multiplicity in the cell under consideration, i.e., large fluctuations from $\langle n\rangle$.

In a heavy-ion collision the temperature T decreases as the system expands. Assuming that a thermalized quark-gluon plasma is formed at high T, the system goes through both confinement and chiral phase transitions, as T is lowered. In lattice-gauge calculations it is found that they may be weak first-order or second-order transitions [17]. We assume that the GL description is adequate for our purpose here. Since T decreases with time, the values of a and b in $F[\phi]$ would vary with time accordingly. Thus it seems that we cannot predict the values of φ_q to be checked by experiment, since it is uncertain what values of a and b are relevant to the observation.

However, the power-law behavior of F_q can be general-

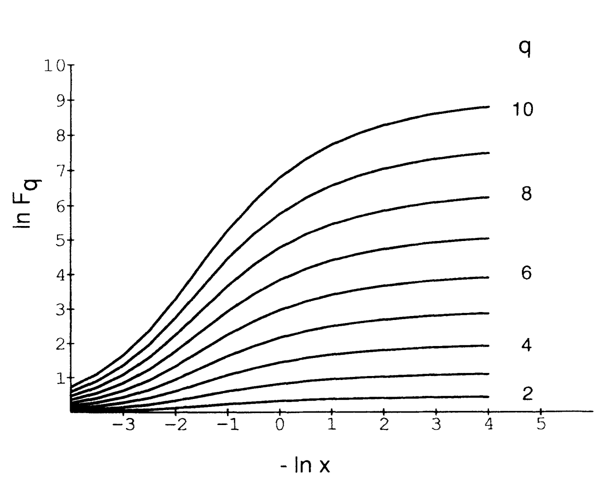

FIG. 1. $\ln F_q$ vs $-\ln x$ for various values of q.

ized from (2) to $[g(\delta)]^{\varphi_q'}$, where $g(\delta)$ can be any function of δ [18]. More specifically, we can consider the Ochs-Wosiek plot for our F_q. We show in Fig. 2 a plot of $\ln F_q$ vs $\ln F_2$, which reveals remarkable linearity for all q over nearly the entire range of $-\ln x$ exhibited in Fig. 1. Thus we have found strict power-law behavior in

$$F_q \propto F_2^{\beta_q}, \qquad (14)$$

where $\beta_q = \varphi_q/\varphi_2$ are independent of a and b, even though φ_q are not. The linearity in Fig. 2 is lost at large x for which either a^2 is too large to describe a system near phase transition or δ is too large to be regarded as an indivisible cell to justify (9); however, in the opposite extreme the linearity persists even in the limit $x \to 0$.

It should be recognized that the strict power law (14) implies that the indices β_q are not only independent of δ but also of the dimension of the cell in phase space. If we had integrated over one or two dimensions first before making the intermittency analysis, our treatment of the problem would still have been the same, so the same values for β_q would result. This independence of the number of dimensions has been noted by Ochs [19] as a phenomenological regularity in the experimental data of e^+e^-, μp, pp, and AA collisions, in none of which has the formation of quark matter been claimed. Here we have proven it to be true in the GL description of second-order phase transitions, at least for vertical analysis with only the potential term in (9).

For heavy-ion collisions aimed at creating quark matter, what is most significant about (14) is that it is independent of the parameters a and b, provided $a<0$ and $b>0$. Since they are T dependent, β_q is therefore independent of T for $T<T_c$. This is an important property because we do not know the time (or T) at which a detected hadron is decoupled from the system. Thus the T independence elevates β_q to play a fundamental role in the characterization of the phase-transition process. Of course, there are phenomenological details that may complicate the verification of the theoretical values of β_q. But at least for an ideal system describable by the GL theory we have a clean prediction here on quantities that are directly measurable.

For a quantitative description of the result we show in Fig. 3 the values of β_q as a function of q, as determined from the slopes in the straight-line portion in Fig. 2 for $0.15 < \ln F_2 < 0.4$. It can be extremely well fitted by

$$\beta_q = (q-1)^\nu, \quad \nu = 1.304, \qquad (15)$$

as shown by the solid line. Thus we have obtained an exponent ν that describes the general consequences of the GL phase transition independent of the details of the GL potential after T goes below T_c.

The exponent ν is not a critical index in the conventional sense as to how such an index describes the behavior of an order parameter in the neighborhood of T_c. Nevertheless, ν shares the same significance in that it is universal (i.e., independent of the details of the system) and completely characterizes the behavior of a measurable quantity at T just below T_c.

If we use the "anomalous fractal dimension" defined by $d_q = \varphi_q/(q-1)$ [20,21], then we have

$$d_q/d_2 = \beta_q/(q-1) = (q-1)^{\nu-1}, \qquad (16)$$

which is not 1 ($\nu=1$), as obtained by Satz for the 2D Ising model [11], nor any other constant for a monofractal conjectured in [9] as a signature of quark-matter formation. The critical behavior of β_q in (15) does not belong to the Levy stable law [22] for any μ in the formula

$$\beta_q = (q^\mu - q)/(2^\mu - 2). \qquad (17)$$

Ochs [19] found that nearly all data on φ_q for e^+e^-, μp, hh, pA, and AA fall in the range $1.3<\mu<1.6$. Representative points from the nuclear data [23,24] are shown in Fig. 3 for comparison. Interestingly, they also follow the power-law behavior (15). The value of ν for the data is $\nu = 1.55 \pm 0.12$, 2σ away from the critical value 1.304. Thus one may conclude that quark-matter formation has not yet been achieved, although a definitive

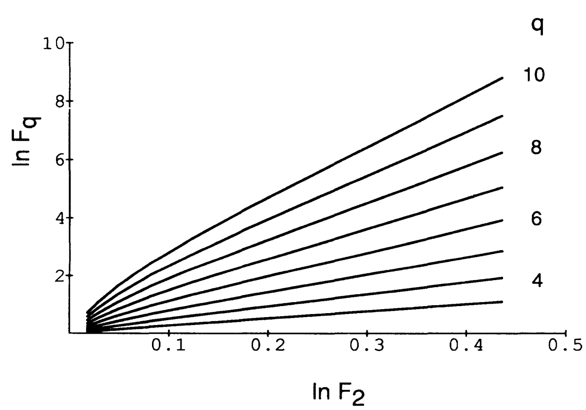

FIG. 2. $\ln F_q$ vs $\ln F_2$ for various values of q.

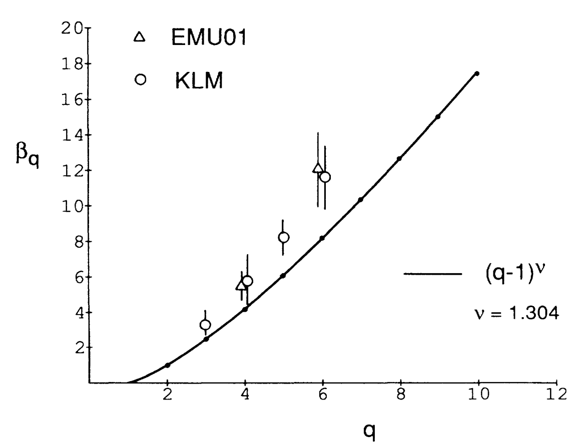

FIG. 3. β_q vs q. Dots are determined from Fig. 2; the solid line is a fit. The data points are from Refs. [23,24].

criterion still awaits a more extensive treatment that includes the last term of (8), not considered in (9).

The application of our result to realistic heavy-ion collisions is not straightforward. While it is very interesting that the value of ν is independent of the coefficients a and b, it does not by itself indicate whether the system has gone through a phase transition from quark matter at $T > T_c$ to hadrons at $T < T_c$, or has never reached T_c from below. The two scenarios may have other features that are distinctive, such as $\langle p_T \rangle$, which is expected to be higher in the former case due to the extra time available for collective flow through the transition point. If the system has never entered a thermalized quark phase, then the chiral phase transition has never occurred with a change of vacuum, and the GL description would seem inappropriate. What this work indicates is that, if the system can be described by the GL theory, there is an expected value for ν. Thus if the measured value of ν is significantly different from the critical value, then obviously the GL description is inappropriate, and second-order phase transitions can most likely be ruled out. On the other hand, if it is close, then what is gained is that the system acquires the GL formalism for its description and phase transition may well be close at hand. In either case we have shown the importance of studying intermittency to characterize dense systems.

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