Dynamics of the QCD Phase Transition

László P. Csernai
Physics Department, University of Bergen, N-5007 Bergen, Norway

Joseph I. Kapusta
School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455
(Received 6 May 1992)

We apply a recently computed nucleation rate to a first-order deconfinement/chiral-symmetry restoring phase transition in a set of rate equations to study the time evolution of expanding quark-gluon plasma as it converts to hadronic matter. At energies of the BNL Relativistic Heavy Ion Collider, the system must supercool about 20% below T_c before nucleation of hadronic bubbles is sufficiently rapid to overcome the expansion rate. The system reheats to near T_c, nucleation turns off, and the transition is completed by growth of previously nucleated bubbles. Based on Bjorken hydrodynamics and on current parameter values, we find the transition generates 30% extra entropy.

PACS numbers: 24.85.+p, 12.38.Mh, 25.75.+r, 64.60.Qb

The primary reason for colliding large nuclei like gold or lead at high energy is to study the behavior of quantum chromodynamics (QCD) at high energy density. A standard picture [1] of a central collision at RHIC (100 GeV/nucleon in the c.m. frame for the Relativistic Heavy Ion Collider under construction at Brookhaven National Laboratory) or at LHC (3 TeV/nucleon in the c.m. frame for the Large Hadron Collider proposed at CERN) is that the two nuclei pass through each other, creating a hot plasma of quarks and gluons. This plasma subsequently cools by expanding hydrodynamically, mainly along the beam axis. Eventually the energy density becomes low enough that the quarks and gluons hadronize; the hadrons then fly off to the detectors. If there is a first-order thermodynamic phase transition [2], the associated latent heat must somehow be gotten rid of before the hadronization can be completed. Usually an idealized Maxwell construction for two-phase equilibrium is invoked as a model of the hadronization process in fluid dynamical approaches [3]. However, it is by no means clear that the QCD nucleation rate is large enough for this idealization to be anywhere near reality. For comparison, nucleation of the QCD transition has been extensively studied in the early Universe where the time scale is of order 10^{-6} to 10^{-5} sec [4]. In nuclear collisions the time scale is of order 10^{-23} sec.

Currently the initial part of this collision picture is being confirmed quantitatively. Perturbative QCD is combined with relativistic transport theory to study the early stage of nuclear collisions by computer simulation [5]. One indeed finds a nearly equilibrated and baryon-free plasma of about 150 fm^3 with an initial temperature of 300 to 350 MeV. This is about twice the expected critical temperature for the phase transition. It is the purpose of this Letter to study quantitatively what happens when the temperature drops to T_c where a phase mixture of quarks and gluons and hadrons is expected to develop.

The rate for the nucleation of the hadronic phase out of the plasma phase can be written as [6–8]

$$I = I_0\, e^{-\Delta F_*/T}, \tag{1}$$

where ΔF_* is the change in the free energy of the system with the formation of a critical size hadronic bubble and I_0 is the prefactor. In general, statistical fluctuations at $T < T_c$ will produce bubbles with associated free energy

$$\Delta F = \frac{4\pi}{3}[p_q(T) - p_h(T)]R^3 + 4\pi R^2\sigma. \tag{2}$$

Here p is the pressure of the quark or hadron phase at temperature T, and σ is the surface free energy of the quark-gluon/hadron interface. Since $p_q - p_h < 0$ it follows as usual that there is a bubble of critical radius

$$R_*(T) = \frac{2\sigma}{p_h(T) - p_q(T)}. \tag{3}$$

Smaller bubbles tend to shrink because the surface energy is too great relative to volume energy, and larger bubbles tend to grow. The free energy of the critical size bubble is therefore

$$\Delta F_* = \frac{4}{3}\pi\sigma R_*^2. \tag{4}$$

The prefactor has very recently been computed by us in a course-grained effective field theory approximation to QCD to be [8]

$$I_0 = \frac{16}{3\pi}\left(\frac{\sigma}{3T}\right)^{3/2} \frac{\sigma\eta_q R_*}{\xi_q^4(\Delta w)^2}, \tag{5}$$

where η_q is the shear viscosity in the plasma phase, ξ_q is a correlation length in the plasma phase, and Δw is the difference in the enthalpy densities of the two phases. This prefactor is very similar to that calculated by Kawasaki [9] and by Turski and Langer [10] for nonrelativistic fluids near their critical points. The nucleation rate is limited by the ability of dissipative processes to carry latent heat away from the bubble's surface, as indicated by the dependence on the viscosity. At the critical temperature, $R_* \to \infty$, and the rate vanishes. The system must supercool at least a minute amount in order that the rate attain a finite value.

Given the nucleation rate one would like to know the (volume) fraction of space $h(t)$ which has been converted from QCD plasma to hadronic gas at the proper time t, which is the time as measured in the local comoving frame of an expanding system. This requires kinetic equations which use I as an input. Langer and Schwartz [11] have discussed such kinetic equations and compared predictions of their theory to cloud-point data in near-critical fluids. Guth and Weinberg [12] proposed a formula for $h(t)$ and applied it to cosmological first-order phase transitions. One may find other kinetic equations in the literature. It does not seem possible to derive such kinetic equations from first principles. The nonrelativistic kinetic equations are inadequate because in relativistic systems the latent heat per unit volume can be a significant fraction of the total energy density, which is obviously not the case in ordinary atomic systems. We will motivate what we believe is a more accurate kinetic equation than the one used by Guth and Weinberg below.

The nucleation rate I is the probability to form a bubble of critical size per unit time per unit volume. If the system cools to T_c at time t_c then at some later time t the fraction of space which has been converted to hadronic gas is

$$h(t) = \int_{t_c}^{t} dt' I(T(t'))[1-h(t')]V(t',t). \quad (6)$$

Here $V(t',t)$ is the volume of a bubble at time t which had been nucleated at the earlier time t'; this takes into account bubble growth [13]. The factor $1-h(t')$ takes into account the fact that new bubbles can only be nucleated in the fraction of space not already occupied by hadronic gas. Equation (6) can most easily be "derived" by discretizing time. This conservative approach does not take into account collisions and fusion of bubbles, which would tend to decrease the time needed to complete the transition [14].

Next we need a dynamical equation which couples the time evolution of the temperature to the fraction of space converted to hadronic gas. For this we use the longitudinal scaling hydrodynamics of Bjorken [1]. The time evolution of the energy density e is

$$\frac{de}{dt} = -\frac{w}{t}. \quad (7)$$

This assumes kinetic but not phase equilibrium, and is basically a statement of energy conservation. We express the energy density as

$$e(T) = h(t)e_h(T) + [1-h(t)]e_q(T), \quad (8)$$

where e_h and e_q are the energy densities in the two phases at the temperature T, and similarly for w.

We also need to know how fast a bubble expands once it is created. This is a subtle issue since by definition a critical size bubble is metastable and will not grow without a perturbation. We shall only attempt a crude description of this growth process here. The growth of bubbles has been studied numerically with relativistic hydrodynamics by Miller and Pantano [15]. After applying a perturbation, the bubble begins to grow. As the radius increases, the surface curvature decreases, and an asymptotic interfacial velocity is approached. The asymptotic radial growth velocity was determined numerically. Their results are consistent with the growth law

$$v(T) = v_0[1-T/T_c]^{3/2}, \quad (9)$$

where v_0 is a model-dependent constant. For numerical purposes we shall use $v_0 = 3c$, which corresponds to their parameter $\alpha = 1$. This expression is intended to apply only when $T > \frac{2}{3}T_c$ so that the growth velocity stays below the speed of sound of a massless gas, $c/\sqrt{3}$. Actually, the interior of the bubble is at a slightly higher temperature than the exterior; we neglect this small temperature difference. Our simple illustrative model for bubble growth then is

$$V(t',t) = \frac{4\pi}{3}\left(R_*(T(t')) + \int_{t'}^{t} dt''\, v(T(t''))\right)^3. \quad (10)$$

This at least has the expected qualitative behavior: the closer T is to T_c the slower the bubbles grow. At T_c there is no motivation for bubbles to grow at all since one phase is as good as the other.

In this Letter we avoid a detailed discussion of complicated equations of state. We simply model the hadronic phase by a massless gas of pions, and the plasma phase by a gas of gluons and massless quarks of two flavors,

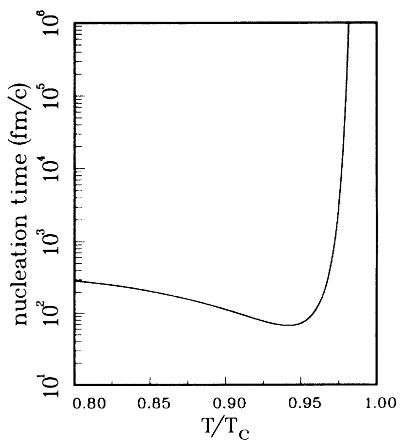

FIG. 1. The characteristic nucleation time as a function of temperature corresponding to Eq. (11). In a nucleus-nucleus collision the onset of nucleation is impeded until $T/T_c \approx 0.95$. This characteristic time of about 100 fm/c should be compared to the typical hadronic time scale of 1 fm/c.

with a bag constant B to simulate confinement dynamics. We use the same parameters as in [8], namely, $\sigma = 50$ MeV/fm^2, $B^{1/4} = 235$ MeV, $\xi_q = 0.7$ fm, and $\eta_q = 14.4T^3$. This gives $T_c = 169$ MeV.

In Fig. 1 is plotted the nucleation time as conventionally defined, namely,

$$\tau_{\rm nuclea}^{-1} = \frac{4\pi}{3} R_*^3 I. \tag{11}$$

This characteristic time scale obviously neglects bubble growth. It is quite clear that on the time scale of nuclear collisions the matter must supercool by at least 5% before nucleation can begin.

One must distinquish between the nucleation time and the time it actually takes to complete the transition, as emphasized by Langer and Schwartz [11]. In Fig. 2 we show the results of numerically integrating the coupled dynamical equations. If the plasma is first equilibrated at a temperature $T_0 = 2T_c$ at time $t_0 = 3/8$ fm/c as suggested by the uncertainty principle and by detailed simulations [5], then the plasma will cool according to the law $T(t) = T_0(t_0/t)^{1/3}$ until the time $t_c = 8t_0 = 3$ fm/c. The matter continues to cool below T_c until T falls to about $0.95T_c$, when noticeable nucleation begins. When the temperature has fallen to about $0.8T_c$, bubble formation and growth is sufficient to begin reheating the system, due to the release of latent heat. When the temperature exceeds about $0.95T_c$ nucleation of new bubbles shuts off. We remark that during this stage of the transition the radius of critical sized bubbles is on the order of 1 fm [8]; this is a nontrivial result since bubbles much larger than several fermi would not be contained within the nuclear diameter. The transition continues only because of the growth of previously created bubbles. However, the temperature must remain somewhat below T_c in order for these bubbles to grow. Compared to the idealized adiabatic Maxwell construction of phase equilibrium at T_c the finite transition rate delays completion of the transition by about 11 fm/c. In the Bjorken hydrodynamics the proper volume of the system increases linearly with time, $V(t) = V(t_c)t/t_c$. Since completion of the phase transition is delayed from 37 to 48 fm/c, and the entropy density at completion is the same, 30% extra entropy is generated.

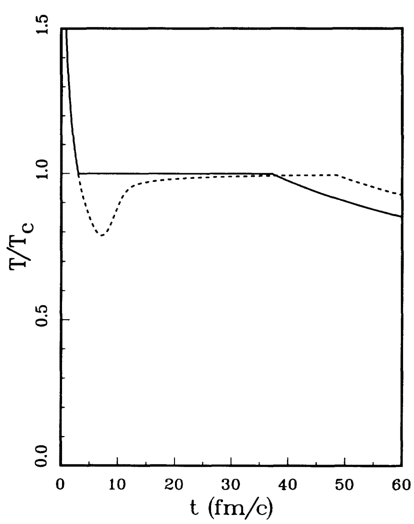

FIG. 2. The temperature as a function of time in a central high-energy nucleus-nucleus collision. The solid curve is the idealized adiabatic Maxwell construction corresponding to complete phase equilibrium. The dashed curve is a result of solving the coupled rate equations described in the text. The matter reheats due to the release of latent heat, and approaches T_c as the transition completes.

In Fig. 3 we show the fraction of space occupied by hadronic matter as a function of time. Again the time delay compared to the adiabatic Maxwell idealization is apparent.

In conclusion, we have demonstrated that if QCD undergoes a first-order phase transition then it is quite likely (with present parameter values and current understanding of phase transition dynamics) that the hot matter created in an ultrarelativistic nuclear collision will pass through a phase mixture not unlike the idealized Maxwell construction. Clearly the input parameters and the equation of state in each of the two phases can be improved upon with time. Also, improvements to the dynamics can be made, such as the incorporation of transverse expansion [16] and bubble interactions. It is possible to study the space-time evolution of the phase transition with hadron interferometry [17] and/or correlations [18]. It would be quite exciting to decide the issue of the existence of a QCD phase transition experimentally.

We are grateful to Keijo Kajantie and Keith Olive for interesting comments. L.P.Cs. thanks the School of Physics and Astronomy and the Theoretical Physics Institute at the University of Minnesota for hospitality. This work was supported by the U.S. Department of Energy under Grant No. DOE/DE-FG02-87ER40328 and by the Norwegian Research Council for Science and Humanities.

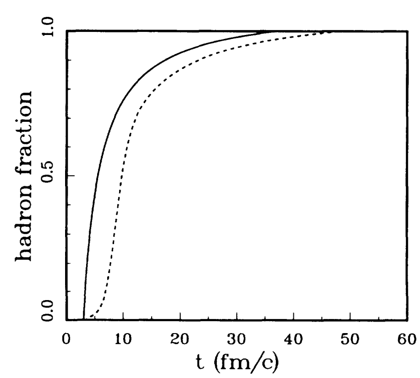

FIG. 3. The volume fraction of space occupied by hadronic matter as a function of time. The solid and dashed curves correspond to those in Fig. 2.

[1] J. D. Bjorken, Phys. Rev. D **27**, 140 (1983).
[2] See the proceedings of Quark Matter '91, to appear as a special issue of Nucl. Phys. A, for the current status of the order of the phase transition, especially the report by N. Christ.
[3] See the proceedings of previous quark matter meetings: Nucl. Phys. **A418** (1984); in *Proceedings of the Fourth International Conference on Ultra-Relativistic Nucleus-Nucleus Collisions*, edited by K. Kajantie, Lecture Notes in Physics 221 (Springer-Verlag, Berlin, 1985); Nucl. Phys. **A461** (1987); Z. Phys. C **38** (1988); Nucl. Phys. **A498** (1989); **A525** (1991).
[4] C. J. Hogan, Phys. Lett. **133B**, 172 (1983); K. Kajantie and H. Kurki-Suonio, Phys. Rev. D **34**, 1719 (1986); G. M. Fuller, G. J. Mathews, and C. R. Alcock, Phys. Rev. D **37**, 1380 (1988).
[5] K. Geiger and B. Müller, Nucl. Phys. **B369**, 600 (1992); K. Geiger, "Thermalization in Ultra-Relativistic Nuclear Collisions," University of Minnesota (to be published).
[6] R. Becker and W. Döring, Ann. Phys. (N.Y.) **24**, 719 (1935).
[7] J. S. Langer, Ann. Phys. (N.Y.) **54**, 258 (1969).
[8] L. P. Csernai and J. I. Kapusta, Phys. Rev. D (to be published).
[9] K. Kawasaki, J. Stat. Phys. **12**, 365 (1975).
[10] L. A. Turski and J. S. Langer, Phys. Rev. A **22**, 2189 (1980).
[11] J. S. Langer and A. J. Schwartz, Phys. Rev. A **21**, 948 (1980).
[12] A. H. Guth and E. J. Weinberg, Phys. Rev. D **23**, 876 (1981).
[13] The Guth-Weinberg (GW) formula assumes that the bubble centers are fixed in position. Growth ceases where two bubbles come into contact. We assume that bubbles are mobile, and that growth continues when bubbles touch. The former is more appropriate for crystal growth; the latter should be more appropriate for real bubbles and droplets. See M. Avrami, J. Chem. Phys. **7**, 1103 (1939). The GW formula corresponding to Eq. (6) is $h_{\rm GW}(t) = 1 - \exp[-\int_{t_c}^{t} dt'\, I(t')V(t',t)]$. It is a solution of Eq. (6) if bubbles do not grow, or when $h \ll 1$. Of course, when $h \approx 0.5$ bubble interactions cannot be neglected, but at least our expression allows for completion, $h = 1$; the GW formula does not.
[14] Since the bubbles are created in an expanding system, we assume that the matter inside the bubble expands at the same rate as on the outside; that is, in the local comoving frame we assume that the interface is created at rest. Hence there is no dilution factor under the integration symbol. Inclusion of a dilution factor would increase the time needed to complete the transition.
[15] J. C. Miller and O. Pantano, Phys. Rev. D **40**, 1789 (1989); **42**, 3334 (1990).
[16] H. von Gersdorff, L. McLerran, M. Kataja, and P. V. Ruuskanen, Phys. Rev. D **34**, 794 (1986); K. Kajantie, M. Kataja, L. McLerran, and P. V. Ruuskanen, Phys. Rev. D **34**, 811 (1986).
[17] S. Pratt, Phys. Rev. D **33**, 1314 (1986); S. Pratt, P. J. Siemens, and A. P. Vischer, Phys. Rev. Lett. **68**, 1109 (1992). For a review see D. H. Boal, C.-K. Gelbke, and B. K. Jennings, Rev. Mod. Phys. **62**, 533 (1990).
[18] D. Seibert, Phys. Rev. Lett. **63**, 136 (1989).