Electric Charges of Positrons and Antiprotons

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Tests of the electric charges carried by the positron and antiproton are derived from recent measurements of the cyclotron frequencies of these particles, and from the spectroscopy of exotic atoms in which they are constituents.

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There has recently been considerable interest in high-precision tests of the equality of particle and antiparticle masses that is required by *CPT* symmetry [1,2]. A method that has been applied to electrons and positrons [3] and protons and antiprotons [1] utilizes the cyclotron frequency, $\omega = qB/m$, of a particle of mass m and charge q in a magnetic field B. Indeed, comparisons of the cyclotron frequencies of electrons and protons (or ions that contain these particles as constituents) in the same magnetic field are regarded as tests of the particles' mass ratios, because there is independent information that the magnitudes of the electron and proton charges are equal. However, there are no analogous tests for the charges of electrons and positrons or protons and antiprotons. Therefore, cyclotron frequency comparisons for these particle-antiparticle pairs should strictly be regarded as comparisons of the particles' charge-to-mass ratios [4,5], unless the additional assumptions of either charge quantization or *CPT* symmetry for particle-antiparticle charges are made. These assumptions involve concepts that are as fundamental as the equality of particle and antiparticle masses and hence deserve their own independent tests. In this Letter we shall derive tests of the positron and antiproton charges by combining precision measurements of their cyclotron frequencies with spectroscopic measurements on positronium and antiprotonic atoms. Then we shall discuss how these tests could be improved with experiments on antihydrogen. We start by reviewing the existing experimental tests of charge quantization.

The notion that electric charge is quantized (in units of e), which dates back to the last century [6–8] can be accommodated in Kaluza-Klein theories [9], theories with magnetic monopoles [10], and grand unified theories [11]. But in 1924 Einstein suggested that a small difference in the charges carried by the proton and electron could account for the magnetic fields of the Sun and Earth [12]. This suggestion led to the first experimental test of the neutrality of matter by Piccard and Kessler [12]. Then in 1959 it was suggested that, because the electromagnetic interaction between an electron and a proton is so much stronger than the gravitational one,

$$\frac{\text{electric}}{\text{gravitational}} = \frac{e^2}{Gm_pm_e} = \alpha\frac{M^2_{\text{Planck}}}{m_pm_e} \sim 10^{+39}, \quad (1)$$

a difference between the magnitudes of the charges carried by protons and electrons as small as one part in 10^{20} could account for the expansion of the Universe [13]. This idea led to further experimental tests, and the equality of the unit of charge on the electron and the proton has now been tested to very high precision [14]. For instance, Marinelli and Morpurgo quote the limit [15]

$$\Delta q_{e\text{-}p} \leq (0.8 \pm 0.8) \times 10^{-21} e\,. \quad (2)$$

The neutron had not been discovered at the time of Einstein's original suggestion, and Piccard and Kessler's result did not constrain the neutron's charge. Therefore, there remained a possibility that because a rotating charged body has a magnetic dipole moment [16], the magnetic fields of the Sun and planets could be explained with a nonzero neutron charge [17]. Likewise, the expansion of the Universe could be accounted for by a neutron charge instead of an electron-proton charge difference [18]. However, there is now a high-precision test of the neutrality of the neutron [19],

$$q_n = (-1.5 \pm 2.2) \times 10^{-20} e\,, \quad (3)$$

which rules out these explanations.

There are also tests of the neutrality of the electron antineutrino [19]

$$q_{\bar{\nu}_e} = (1.4 \pm 1.4) \times 10^{-20} e\,, \quad (4)$$

and for 2-eV photons [20]

$$q_\gamma \leq 10^{-16} e\,, \quad (5)$$

that were derived under the assumption of charge conservation, which has itself been the subject of experimental and theoretical attention [21]. [Experimental tests of charge conservation, which are limited to strong constraints on a few forbidden reactions [21], can be viewed as tests of gauge invariance [22–24], because current nonconservation is inconsistent with the (gauge-invariant) Maxwell's equations, $\partial_\mu F^{\mu\nu} = j^\nu$, but on the other hand, can be consistent with the Proca equation, $\partial_\mu F^{\mu\nu} + m_\gamma^2 A^\nu = j^\nu$, in which the potentials attain physical significance.]

Another challenge to the notion of charge quantization has come from recent discussion of the possibility of "millicharged particles" ($q \sim 10^{-3}e$) [25], which could provide an explanation of the orthopositronium lifetime puzzle [26]. The existence of such particles is poorly constrained experimentally.

From this brief survey it is clear that there are fundamentally important cosmological, metrological, and field-theoretical reasons to test the charges of elementary particles. Therefore, it is quite unsatisfactory that there are no tests of the charges on the positron or antiproton, particularly because without such tests the *CPT* symmetry of particle and antiparticle charges remains untested.

Separate tests of the charges and masses of positrons and antiprotons can be obtained by combining measurements of their cyclotron frequencies with additional experimental data that have a different functional dependence on their charges and masses, such as the Rydberg for exotic atoms that have the antiparticle as a constituent. To see this, suppose that we are concerned with a particle of mass m_1 and charge e_1, and a second particle of m_2 and (opposite) charge e_2. The cyclotron frequencies (in SI units) of the particles in a magnetic field of strength B are given by

$$\omega_i = e_i B/m_i, \quad i = 1,2, \tag{6}$$

whereas the Rydberg of the hydrogenic atom formed from the bound state of the pair would be

$$R_{21} = \mu e_1^2 e_2^2 / 8\varepsilon_0^2 c h^3, \tag{7}$$

where $\mu = m_1 m_2/(m_1+m_2)$ is the reduced mass of the pair. (We have assumed that the charges are unaltered by being bound into an atomic state.) For instance, particles 1 and 2 might be an electron and a positron, in which case the exotic atom would be positronium, or a proton and an antiproton where the atom would be the proton-antiproton bound state, unaffected by strong interaction effects, or a positron and an antiproton, in which case the atom would be antihydrogen, etc. The cyclotron frequencies can now be used to eliminate the particles' masses from the Rydberg.

We next introduce the cyclotron frequency ratio

$$\omega_{21} \equiv \frac{\omega_2}{\omega_1} = \frac{m_1}{m_2}\frac{e_2}{e_1}, \tag{8}$$

and the ratio of Rydberg values

$$\bar{R}_{21} \equiv \frac{R_{21}}{R_\infty/2} = \frac{2\mu}{m_e}\left(\frac{e_1^2 e_2^2}{e^4}\right), \tag{9}$$

where e is the electron or proton charge, m_e is the electron mass, and $R_\infty = m_e e^4/8\varepsilon_0^2 c h^3$ is the Rydberg of the hydrogen atom for infinite proton mass, which has recently been measured to a precision of 1.7 parts in 10^{10} [27] (limited by the existing optical frequency standard).

For the case where particle 1 is an electron and particle 2 is a positron we can eliminate their mass ratio between Eqs. (8) and (9) and obtain the equation

$$\left(\frac{e_{\bar{e}}}{e}\right)^3 - \frac{\bar{R}_{\bar{e}e}}{2}\left(\frac{e_{\bar{e}}}{e}\right) - \frac{\omega_{\bar{e}e}}{2}\bar{R}_{\bar{e}e} = 0, \tag{10}$$

for the charge ratio, where $e_{\bar{e}}$ is the positron charge. For clarity, we assume to a first approximation that the charge ratio is approximately equal to 1, $e_{\bar{e}}/e = 1 + \varepsilon_{\bar{e}}$, and so obtain an equation for the charge equality:

$$\varepsilon_{\bar{e}} = \tfrac{1}{5}(\Delta\omega_{\bar{e}e} + 2\Delta\bar{R}_{\bar{e}e}), \tag{11}$$

where we have used $\bar{R}_{\bar{e}e} = 1 + \Delta\bar{R}_{\bar{e}e}$ and $\omega_{\bar{e}e} = 1 + \Delta\omega_{\bar{e}e}$, with $\Delta\bar{R}_{\bar{e}e}$ and $\Delta\omega_{\bar{e}e}$ small.

Now we may use the experimental results for the ratio of electron and positron cyclotron frequencies [3]:

$$\omega_{\bar{e}e} = 1 \pm 1.3 \times 10^{-7}, \tag{12}$$

and the ratio of the Rydbergs in positronium and hydrogen [28]:

$$|\Delta\bar{R}_{\bar{e}e}| < 4 \times 10^{-8} \quad (90\%\ \text{C.L.}). \tag{13}$$

We conclude that

$$e_{\bar{e}}/e = 1 \pm 4 \times 10^{-8}. \tag{14}$$

Improvements in the precision of the cyclotron frequency comparison to the level of one part in 10^{11} have been suggested [29], and the ultimate precision on measurements of the 1*S*-2*S* interval in positronium (from which the Rydberg for positronium can be deduced) is also 1 part in 10^{11} [30]. Therefore we can anticipate an improvement of this test of the electron-positron charge equality by roughly 3 orders of magnitude, and hence the equality of their masses could be tested with comparable precision by comparison of their cyclotron frequencies.

For the case in which particle 1 is a proton and particle 2 an antiproton we have [1]

$$\omega_{\bar{p}p} = 1 \pm 4 \times 10^{-8}, \tag{15}$$

and we shall assume that the uncertainty of 5×10^{-5} quoted on the antiproton mass measured from the spectroscopy of antiprotonic atoms [31] reflects the uncertainty in the "antiprotonic Rydberg," $\bar{R}_{\bar{p}p}$. We can derive an equation for the ratio of the charge of the antiproton, $e_{\bar{p}}$, to the electron's charge, e,

$$\left(\frac{e_{\bar{p}}}{e}\right)^3 - \frac{1}{2}\left(\frac{m_e}{m_p}\right)\bar{R}_{\bar{p}p}\left(\omega_{\bar{p}p} + \frac{e_{\bar{p}}}{e}\right) = 0, \tag{16}$$

and hence, putting $e_{\bar{p}}/e = 1 + \varepsilon_{\bar{p}}$, we find the equation

$$\varepsilon_{\bar{p}} = \tfrac{1}{5}(\Delta\omega_{\bar{p}p} + 2\Delta\bar{R}_{\bar{p}p}), \tag{17}$$

for the uncertainty in the antiproton's charge relative to the electron's, where $\omega_{\bar{p}p} = 1 + \Delta\omega_{\bar{p}p}$ and $\bar{R}_{\bar{p}p} = (1 + \Delta\bar{R}_{\bar{p}p})m_p/m_e$. The error here is clearly dominated by the antiprotonic Rydberg, and we find

$$e_{\bar{p}}/e = 1 \pm 2 \times 10^{-5}. \tag{18}$$

[Therefore, the proton-antiproton mass ratio can strictly be said to be equal to 1 only with a precision of 2 parts in 10^5, in spite of the very much higher precision of the comparison of their cyclotron frequencies, (15).] So, although the precision of proton and antiproton cyclotron frequency comparisons is projected to increase to 1 part in 10^{11} [29], this would not lead to any improvement in this charge comparison. However, spectroscopic measurements on antihydrogen could yield an improved test of the antiproton's charge.

Although antihydrogen has not yet been produced in the laboratory, plans to do so and to perform precision spectroscopic measurements on it exist [5,32,33]. By comparison of the $1S$-$2S$ two-photon transitions in hydrogen and antihydrogen [33] the Rydberg ratio

$$\bar{R}_{e\bar{p}}=\frac{m_{\bar{e}}}{m_e}\frac{m_{\bar{p}}}{m_p}\frac{1+m_p/m_e}{m_{\bar{e}}/m_e+m_{\bar{p}}/m_e}\left(\frac{e_{\bar{p}}e_{\bar{e}}}{e^2}\right)^2 \tag{19}$$

could be formed, which would be equal to 1 if *CPT* symmetry is exact. This ratio could in principle be measured with a precision of 1 part in 10^{15} [33]. The $1S$-$2S$ interval in hydrogen has already been measured with a precision of 5 parts in 10^{10} [34], which is limited by the precision of the existing optical frequency standard, but improvements are anticipated. Independent information on the antiproton and positron charge-to-mass ratios can be introduced from the electron-positron cyclotron frequency comparison (12) [3], an electron-antiproton cyclotron frequency comparison, $\omega_{\bar{p}e}=1836.152680(88)$, with a precision of 5 parts in 10^8 [1], and the proton-antiproton cyclotron frequency comparison (15) [1].

Introducing $\bar{R}_{\bar{e}\bar{p}}=1+\Delta\bar{R}_{\bar{e}\bar{p}}$ and $\omega_{\bar{p}e}=(1+\Delta\omega_{\bar{p}e})m_e/m_p$, we find after dropping small terms

$$\varepsilon_{\bar{p}}=\tfrac{1}{2}(\Delta\bar{R}_{\bar{e}\bar{p}}+\Delta\omega_{\bar{e}e}+\Delta\omega_{\bar{p}p}-\Delta\omega_{\bar{p}e}-3\varepsilon_{\bar{e}})\,. \tag{20}$$

Hence, the precision with which the charge of the antiproton could be tested is limited by the cyclotron frequency comparisons and the Rydberg measurement on positronium (via the positron charge), i.e., of the order of 1 part in 10^{11}. However, both the proton-antiproton mass and charge ratios would be tested to this precision.

At present three routes to antihydrogen synthesis, capture at low temperatures, and subsequent spectroscopic measurements are under consideration. These are the reactions between laser-excited positronium and cooled, trapped antiprotons [5,35]; positrons and antiprotons at 4 K in nested ion traps [36]; and positronium and antiprotons that are bound in an exotic helium atom [37].

Our results, (14) and (18), represent the first tests of charge quantization for the positron and antiproton, which, with the available experimental results, are at a very much lower precision than the tests for electrons, protons, and neutrons. These results also provide the first tests of *CPT* symmetry for particle and antiparticle charges, with precisions of 4 parts in 10^8 for the electron and positron, and 2 parts in 10^5 for the proton and antiproton. The ultimate precision of our method is set by the projected precisions of cyclotron frequency measurements and the measurement of the Rydberg in positronium, namely, about 1 part in 10^{11}, which therefore also represents the limit to which separate tests of *CPT* for particle and antiparticle charges and masses can be derived from such measurements. As tests of charge quantization this precision would still be 9 orders of magnitude less than for electrons, protons, or neutrons, and as tests of *CPT* only the electron and positron gyromagnetic ratio comparison [38] and the difference of the neutral kaon masses [2] would exceed this precision (by 1 and 6 orders of magnitude, respectively).

The tests that we have derived are "direct" in the sense that they make no assumption about charge conservation, nor electrical neutrality of the photon. However, because the photon (at least at 2 eV) is known to be neutral to 1 part in 10^{16} [20], it might be argued that very much more precise tests of the charges carried by positrons and antiprotons could be deduced from the observed phenomenon of electron-positron annihilation into photons (or proton-antiproton annihilation). But this argument would require the assumptions of charge conservation and the absence of millicharged particles. Alternatively, because we now have direct tests of the charges of the initial and final states of the decays positronium → photons, we can derive a new test of charge conservation and hence gauge invariance [22–24] (if we assume that there are no millicharged particles in the final state). We conclude that charge is conserved with a precision of at least 4×10^{-8} in this reaction. This should be contrasted with other tests of charge conservation in which attention has focused on electron decay: $e\rightarrow\nu\gamma$ [21].

Finally, if we assume that charge is conserved in β^+ decay, we can deduce that the charge of the electron neutrino is less than $4\times10^{-8}e$, because we now have information about the charges of the other initial- and final-state particles.

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