Bose Condensates, Big Bang Nucleosynthesis, and Cosmological Decay of a 17 keV Neutrino

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(Received 19 December 1991; revised manuscript received 7 April 1992)

Early-Universe decay of 17 keV neutrinos is likely to happen in equilibrium for lifetime $t_{\rm free} < 10^5$ sec. For $3\times10^{-4} < t_{\rm free} < 1\times10^{-2}$ sec the primordial ^{4}He mass fraction Y_p may be *reduced* by up to 0.028 relative to the standard model. Bosons from the decay can form a Bose condensate, with interesting consequences for galaxy formation and the dark-matter problem. The effects are generic to early-Universe decay of fermions, not just of relevance for 17 keV neutrinos.

PACS numbers: 98.80.Cq, 13.35.+s, 14.60.Gh, 98.60.−a

Recent experimental evidence [1] for a 17 keV neutrino mass eigenstate with 0.8% mixing to ν_e, while still disputed, has led to extensive theoretical investigations because it is very difficult to reconcile a particle with these properties with standard particle theories, not to mention cosmology and astrophysics [2]. In particular the 17 keV neutrino (henceforth assumed to be ν_τ) must be unstable to prevent the Universe from being "overclosed" by a factor of almost 200 today.

This Letter points out that the generically simplest decay pattern

$$\nu_\tau \to B+F, \quad \bar{\nu}_\tau \to \bar{B}+\bar{F}, \qquad (1)$$

where B and F denote bosons and fermions with antiparticles $\bar{B}$ and $\bar{F}$, is severely constrained by saturation of the final-state levels, leading to an equilibrium with effective lifetime approaching 10^5 sec irrespective of the free-particle lifetime $t_{\rm free}$, provided $3\times10^{-4} < t_{\rm free} < 10^5$ sec.

Two scenarios are studied, both assuming B and F to be stable, with masses m_B and m_F less than 100 eV to avoid an "overclosure" contribution from the decay products: (1) $F \neq \nu_e$, (2) $F = \nu_e$. Both scenarios arrive in two variants, depending on B (assumed to have one internal degree of freedom) being its own antiparticle or not. Assuming a symmetric behavior of particles and antiparticles, and no other reactions than decays and the corresponding inverse reactions, *similar distributions are predicted for the particles and antiparticles, with the* same *sign for particle and antiparticle chemical potentials.* This leads to interesting consequences for big bang nucleosynthesis in scenario 2 if $3\times10^{-4} < t_{\rm free} < 1\times10^{-2}$ sec; a *reduction* of Y_p of 0.02–0.03 may occur relative to standard big bang nucleosynthesis (SBBN).

Scenario 1 (and 2 for a limited range of $t_{\rm free}$) leads to formation of a Bose condensate. This can have very interesting consequences for galaxy formation and the dark-matter problem. The possibility of nonzero chemical potentials also changes the hot dark-matter scenario for galaxy formation, and the Tremaine-Gunn phase-space limit on the mass of dark-matter particles [3,4] is significantly altered.

The generic two-particle decay of a 17 keV neutrino is simply described in the center-of-momentum frame, where both of the relativistic decay products carry 8.5 keV of energy. By application of Lorentz transformations [5] one may calculate the momentum distribution of the decay products in the cosmic frame assuming instant isotropic decay in the rest frame at temperature T_ν, taking into account the thermal Fermi-Dirac distribution of the decaying ν_τ. An example is shown in Fig. 1, for $T_\nu = 1$ MeV. The distribution function of the decay products derived in this purely kinematical fashion exceeds the thermal distributions for momenta $p < 1$ MeV. The "oversaturation" of levels proceeds down to temperatures of a few keV, corresponding to lifetimes $t_{\rm free} \approx 10^5$ sec [6]. This demonstrates that "free decay" of a ν_τ with lifetime less than 10^5 sec is impossible. The inverse reaction is crucial, and an equilibrium description is more appropriate.

The assumption of equilibrium breaks down when the relevant reaction rates, depending on the actual couplings in the model, become small relative to the cosmic expansion rate. I shall assume that equilibrium is a good approximation as long as a significant fraction of the kinematically preferred phase space for the decay prod-

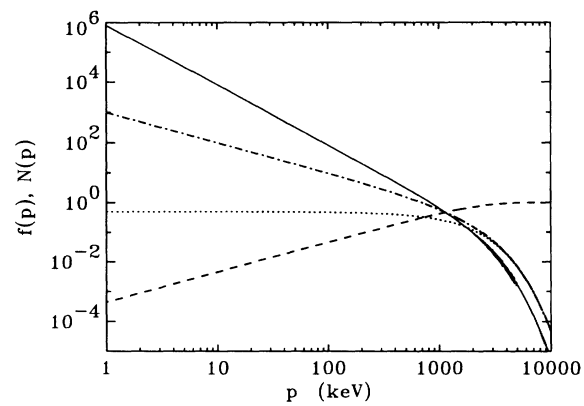

FIG. 1. Occupation number distribution for low-mass B and F if decay was purely kinematic, instantaneous, and isotropic in the rest frame at $T_\nu = 1$ MeV (solid line). Dotted and dash-dotted curves are thermal Fermi and Bose distributions at the same temperature. The dashed curve is the fraction of decay products with momentum below p.

ucts is inaccessible; that is, for $T_\nu >$ a few keV [7]. To avoid a complicated reaction network, I shall limit discussion to $t_{\rm free} > 3\times 10^{-4}$ sec, so that the decay process is unimportant prior to freeze-out of the weak interactions that keep neutrinos and antineutrinos in equilibrium at $T > 2.3$ MeV, $t < 0.13$ sec.

The occupation number distributions in equilibrium are

$$f_i(E_i) = \{\exp[(E_i - \mu_i)/T] \pm 1\}^{-1}, \qquad (2)$$

with chemical potentials constrained by

$$\mu_{\nu_\tau} = \mu_B + \mu_F, \quad \mu_{\bar{\nu}_\tau} = \mu_{\bar{B}} + \mu_{\bar{F}}. \qquad (3)$$

Equation (3) assumes that no reactions apart from decay and inverse decay take place. The total number of fermions (ν_τ plus F) must be conserved by the decay, and the number of bosons shall equal the number of ν_τ that have decayed (twice the number of ν_τ decaying if $B = \bar{B}$). Finally, the energy density must be conserved in the decay. For $F = \nu_e$, $B = \bar{B}$, this leads to the conservation equations

$$n_{17}(T_D,0) = n_{\nu_\tau}(T_A,\mu_{\nu_\tau}) + \tfrac{1}{2} n_B(T_A,\mu_B), \qquad (4)$$

$$n_{17}(T_D,0) + n_{\nu_e}(T_D,0) = n_{\nu_\tau}(T_A,\mu_{\nu_\tau}) + n_{\nu_e}(T_A,\mu_{\nu_e}), \qquad (5)$$

$$\rho_{17}(T_D,0) + \rho_{\nu_e}(T_D,0) = \rho_{\nu_\tau}(T_A,\mu_{\nu_\tau}) + \rho_{\nu_e}(T_A,\mu_{\nu_e}) + \tfrac{1}{2}\rho_B(T_A,\mu_B), \qquad (6)$$

where $n_i(T,\mu_i)$, $\rho_i(T,\mu_i)$ are number and energy densities, T_D is the neutrino temperature at decay (assumed to be instant), and T_A is the temperature immediately after decay. The distribution function of 17 keV neutrinos prior to decay is that of Eq. (2) with $\mu_{17}=0$, $E_{17}=p_{17}$ because of the relativistic decoupling. Similar conservation laws hold for the other three cases. For $\mu_B = 0$, Eqs. (3), (5), and (6) describe the equilibrium, and a critical temperature $T_c = [\pi^2 n_B/\zeta(3)]^{1/3}$ [with n_B calculated from Eq. (4)] is established. Below T_c a fraction $1-(T_A/T_c)^3$ of the bosons are in a Bose condensate with zero momentum [8]. Figure 2 shows the evolution of chemical potentials required to satisfy Eqs. (3)-(6), and Fig. 3 illustrates how ν_τ survives until it becomes nonrelativistic. This property is *independent* of the nature of the decay products [9].

A 17 keV ν_τ with a lifetime longer than 10^{-2} sec acts like a normal third neutrino in SBBN calculations. For lifetimes between 3×10^{-4} and 10^{-2} sec the nucleosynthesis changes if $F = \nu_e$, the yields depending on whether B is its own antiparticle. The interval of interest corresponds to decay after ν-$\bar{\nu}$ freeze-out at $T = 2.3$ MeV but prior to freeze-out of the neutron-to-proton ratio at $T \approx 0.7$ MeV, which defines the main properties of the SBBN outcome.

In scenario 1 the nucleosynthesis yields are identical to SBBN—the particles involved in the decay affect nucleosynthesis only through their energy density, and this is merely redistributed between the species (this expecta-

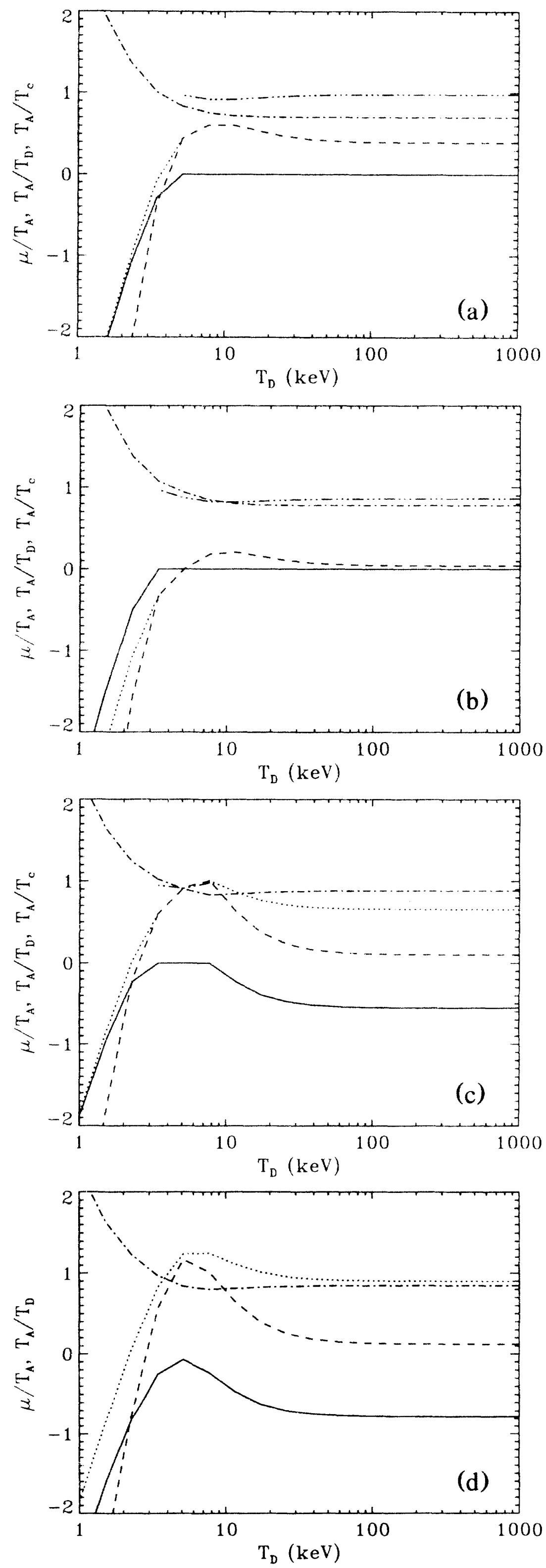

FIG. 2. Chemical potentials for decay in equilibrium. (a) $F \neq \nu_e$, $B \neq \bar{B}$; (b) $F \neq \nu_e$, $B = \bar{B}$; (c) $F = \nu_e$; $B = \bar{B}$; (d) $F = \nu_e$, $B \neq \bar{B}$. Solid lines are for B, dotted lines for F, and dashed lines for ν_τ. Dash-dotted curves are T_A/T_D, and dash-triple-dotted curves T_A/T_c.

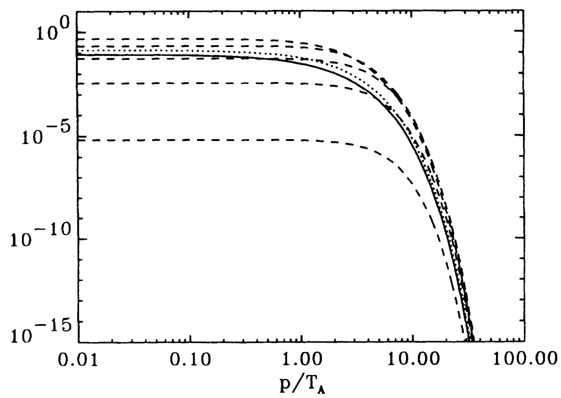

FIG. 3. Solid and dotted curves are the occupation number distributions for B and F at $T_D=1$ keV. Dashed curves from top to bottom show the evolution for ν_τ at $T_D=100$, 10, 5, 3, and 1 keV for $F\neq\nu_e$, $B\neq\bar{B}$. The other scenarios are similar; ν_τ disappears after 10^5 sec, at temperatures of a few keV.

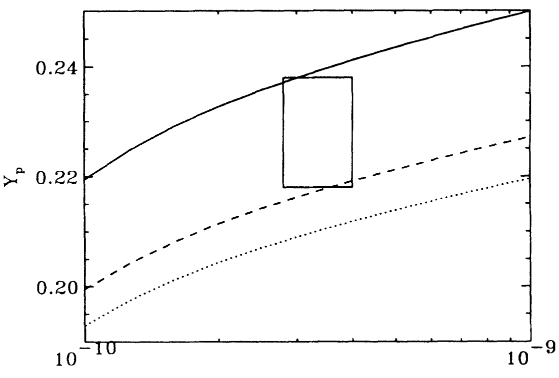

FIG. 4. Primordial mass fraction of ^{4}He as a function of baryon-to-photon ratio. The solid curve is the standard model. Dashed and dotted curves correspond to Figs. 2(c) and 2(d). The box is allowed by observations.

tion was confirmed to 0.1% accuracy by numerical nucleosynthesis calculations).

In scenario 2 the energy is again redistributed without changing the cosmic expansion rate. But now the weak interactions giving the neutron-to-proton ratio at freeze-out are pushed to the proton side since *both* μ_{ν_e} *and* $\mu_{\bar{\nu}_e}$ take identical positive values of $+0.899$ for $B\neq\bar{B}$ ($+0.657$ for $B=\bar{B}$) in units of T_A [which is 0.844 (0.884) times T_D]. This results in a total *reduction* of 0.028 (0.021) in Y_p as illustrated in Fig. 4. The results in Fig. 4 were calculated using Kawano's [10] version of the Wagoner code [11], modified to accommodate equal instead of opposite values for the chemical potentials of ν_e and $\bar{\nu}_e$, as well as $T_A<T_D$ [12]. Minor but observationally indistinguishable changes resulted for D, ^{3}He, and ^{7}Li [a (10–15)% reduction in ^{7}Li being the only potentially observable exception]. The neutron half-life was taken to be 10.3 min.

Fitting the observed abundances of D, ^{3}He, and ^{7}Li [13] leads to an allowed range of $2.8\leq\eta_{10}\leq4.0$ ($\eta_{10}=10^{10}n_{\rm bar}/n_{\rm ph}$, where $n_{\rm bar}$ and $n_{\rm ph}$ are the baryon and photon number densities), with a corresponding ^{4}He production of $0.236\leq Y_p\leq0.241$ in SBBN. This is marginally consistent with the primordial production deduced from observations: $Y_p=0.228\pm0.010$ [14]. Scenario 2 predicts $0.208\leq Y_p\leq0.212$ for $B\neq\bar{B}$, and $0.215\leq Y_p\leq0.219$ for $B=\bar{B}$. The latter is marginally consistent with observations. For lifetimes very close to 3×10^{-4} or 1×10^{-2} sec the ^{4}He yield approaches that of SBBN. There has been a tendency of reducing observational estimates of Y_p in recent years. Should this tendency continue, it is worth noting that the scenario presented here is one of the only mechanisms known that leads to a reduction in the primordial ^{4}He production without noticeable effects on the other light elements.

In the low-temperature limit the end products of the equilibrium decay in scenario 1 are a new fermion (and its antiparticle), plus a boson (and its antiparticle, assuming $B\neq\bar{B}$), all with $\mu\rightarrow-\infty$. More likely, equilibrium is broken in the Bose condensate regime, leaving as many as 36% of the bosons in the zero-momentum ground state. The final number densities of F and B each equal that of a relativistic zero-chemical-potential neutrino flavor. In scenario 2 one again gets $\mu\rightarrow-\infty$ in the low-T limit, with a more likely loss of equilibrium when the chemical potentials are $-1<\mu/T_A<1$. For $B=\bar{B}$ and T_D near 5 keV there is a chance of pushing up to 26% of the bosons into a condensate.

These distributions may change the hot dark-matter galaxy formation scenarios with eV-mass fermions [15] or bosons [16]. If $F=\nu_e$, half the usual ν_e mass is needed to account for a given value of the density parameter, Ω. This improves the possibilities for ν_e to be the dark matter if the mass is close to the experimental upper bound of 10 eV, but it also doubles the free-streaming scale, if $\mu_{\nu_e}\approx0$, which may intensify the problems in fitting simulations of structure formation. For $T_D<5$ keV free streaming is further increased due to heating of the ν_e distribution. The lower mass limit for neutrino dark matter from the simplest applications of the Tremaine-Gunn phase-space constraint [3,4] is strengthened by a significant factor for low μ_{ν_e} since the maximum occupation number $f_{\rm max}$ is increased from 0.5 to $1/\{\exp[(m_{\nu_e}-\mu_{\nu_e})/T_\nu]+1\}$, and the mass limit is proportional to $f_{\rm max}^{-1/4}\approx\exp(-\mu_{\nu_e}/4T_A)$. Thus ν_e cannot cluster in galaxies for very negative μ_{ν_e}.

If the bosons are massive, they will contribute like one light neutrino flavor to the density but have more power on small scales due to higher occupation numbers at low momenta. For small negative chemical potential the distribution will be somewhat intermediate between the zero-chemical-potential neutrino and boson spectra shown in Fig. 2 of Ref. [16]. The Tremaine-Gunn limit for the boson masses [4] is strengthened by a factor approaching $\exp(-\mu_B/4T_A)$ for very negative μ_B. If a fraction of the bosons form a condensate, one has the fascinating possibility of a dark-matter candidate which at the *same* time acts as hot dark matter [16] (the $\approx 65\%$ of particles not in the condensate) *and* cold dark matter (the $\approx 35\%$ of particles in the condensate). Such hybrid models have been shown to work well with massive neutrinos plus an unknown cold dark-matter component [17]. The boson model will be rather similar, but with only one particle involved.

In summary, two-particle decay of a 17 keV ν_τ in the early Universe is forced into equilibrium with the inverse decay. The effective lifetime is thereby extended to 10^5 sec for free-particle lifetimes 3×10^{-4}-10^5 sec. If the fermion produced in the decay is ν_e and the lifetime is 3×10^{-4}-10^{-2} sec, this may give one of the only known mechanisms to reduce primordial ^{4}He without significantly changing the production of other elements [18]. If either of the decay products have eV masses, galaxy formation may proceed via a variant of the hot dark-matter model, or (in the case of Bose condensation) like a hybrid hot+cold dark-matter model, with the *same* particle responsible for both components. Phase-space constraints on the masses of dark-matter particles in galaxy halos are strengthened significantly due to the nonzero chemical potentials (except in the hybrid case).

I have attempted to keep the discussion very general without reference to specific models for the 17 keV neutrino or its decay properties. Details may change in particular models, but the main inferences seem to be generic, and will prevail even for decays into three or more particles, for models identifying the 17 keV neutrino with a fermion distinct from ν_τ, and for other fermion decays in the early Universe.

It is a pleasure to thank Georg Raffelt for useful discussions and hospitality during a visit to MPI, München. This work was supported in part by the Danish Natural Science Research Council.

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