Excess Conductance of Superconductor-Semiconductor Interfaces Due to Phase Conjugation between Electrons and Holes

B. J. van Wees,[(1)] P. de Vries,[(2)] P. Magnée,[(1)] and T. M. Klapwijk[(1)]
[(1)]*Department of Applied Physics and Materials Science Centre, University of Groningen, Nijenborgh 4, 9747 AG Groningen, The Netherlands*
[(2)]*Institute for Theoretical Physics, University of Groningen, Nijenborgh 4, 9747 AG Groningen, The Netherlands*
(Received 31 March 1992)

A semiclassical description is given of charge transport through a superconductor-semiconductor interface. As a result of the presence of a potential barrier both Andreev and normal reflection occur. Elastic scatterers in the semiconductor generate multiple reflections at the interface. The constructive quantum interference which results from the phase conjugation between electrons and holes enhances the (differential) conductance above its classical value. This excess conductance is suppressed by a magnetic field, or by a finite energy. The latter can be due to a finite voltage bias or a finite temperature.

PACS numbers: 73.20.Fz, 73.40.−c, 74.50.+r

Andreev reflection [1] is a phenomenon which occurs at the interface between a superconductor and a normal conductor. An electron from the normal conductor with an energy E (relative to the Fermi energy E_F) below the superconducting gap Δ cannot enter the superconductor. Instead, it is retroreflected [1] as a hole, a particle carrying a charge $+e$.

In the absence of a magnetic field electrons and holes at the Fermi energy can be considered as each other's time-reversed particles. For the classical dynamics this implies that the Andreev-reflected hole will trace back the path of the incoming electron [2]. Another consequence of the time-reversal symmetry between electrons and reflected holes is related to their wave functions. The wave function of the reflected hole is identical to that of the incoming electron: $\psi_h(\mathbf{r})=\psi_e(\mathbf{r})=\exp(i\mathbf{k}_F\cdot\mathbf{r})$. This means that the phase changes which are accumulated by the incoming electron are canceled by opposite phase changes of the reflected hole, when it traces back the path of the incoming electron.

In this Letter we will show that the quantum interference which arises from this phase conjugation [3] can result in an excess conductance of superconductor-semiconductor (S-Sm) interfaces, and can provide an explanation for the recent experimental results of Kastalsky *et al.* [4], and Van der Post *et al.* [5].

We model the S-Sm interface region by a geometry depicted in Fig. 1. It has three sections. In the reservoir the particle distribution is assumed to be in equilibrium [6], with an electrochemical potential $\mu=-eV$ relative to the superconductor. The boundary of the reservoir is put an inelastic scattering length $l_{\mathrm{in}}(D\tau_{\mathrm{in}})^{1/2}$ away from the S-Sm interface, with the diffusion constant D, and the inelastic scattering time τ_{in}. In the middle region elastic scatterers are present, which result in an elastic mean free path l_e ($l_e \ll l_{\mathrm{in}}$). Inelastic scattering is absent in this region, and both electron and hole waves propagate in a phase-coherent way.

At the S-Sm interface between this region and the superconductor an electrostatic potential barrier [7] is present. The scattering at this barrier is described by an S matrix [8]. In the absence of a magnetic field time-reversal invariance holds, and the most general expression is

$$S_e=\begin{pmatrix} |t|\exp(i\alpha) & |r|\exp[i(2\alpha-\beta)] \\ |r|\exp[i(\pi+\beta)] & |t|\exp(i\alpha) \end{pmatrix}, \tag{1}$$

with reflection probability at the barrier $R=|r|^2$ and transmission probability $T=|t|^2=1-|r|^2$. These, together with the phases α and β, are determined by the specific shape of the barrier. At E_F the S matrix for holes is given by $S_h=S_e^*$.

The amplitudes $r_{ee}(1)$ and $r_{eh}(1)$ for normal and Andreev reflection of incoming electrons, as well as the amplitudes $r_{hh}(1)$ and $r_{he}(1)$ for normal and Andreev reflection of incoming holes can now be calculated [9]:

$$r_{ee}(1)=\frac{-2|r|}{1+|r|^2}\exp(i\beta),\quad r_{eh}(1)=\frac{i(1-|r|^2)}{1+|r|^2}, \tag{2a}$$

$$r_{hh}(1)=\frac{-2|r|}{1+|r|^2}\exp(-i\beta),\quad r_{he}(1)=\frac{i(1-|r|^2)}{1+|r|^2}. \tag{2b}$$

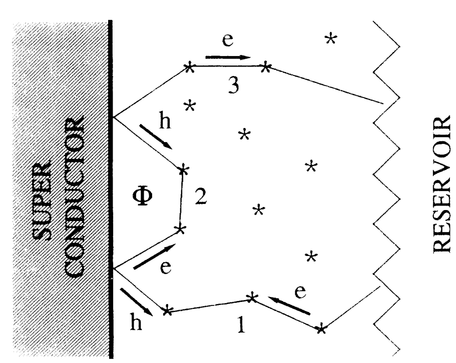

FIG. 1. Geometry of the model, consisting of three sections (see text).

Note that $r_{he}(1)=r_{eh}(1)$, and that $r_{ee}(1)=r_{hh}^*(1)$.

Figure 1 illustrates a typical trajectory of an electron emitted by the reservoir. The electron reaches the S-Sm interface, after being scattered several times (path 1). At the interface it is either reflected as an electron, with a probability $P_N=|r_{ee}(1)|^2$, or it is retroreflected as a hole, with a probability $P_A=|r_{eh}(1)|^2=1-P_N$. In the latter case the hole will trace back the path of the incoming electron, and flow back into the reservoir.

In our analysis we assume that the scattering in the semiconductor is "classical" [10], which implies that there is a well-defined relation between the direction of the incoming and reflected waves. This is valid when the size W of the scatterers is large compared to the Fermi wavelength λ_F, in which case the scatterers act as mirrors. For point scatterers [11] ($W\ll\lambda_F$) our analysis cannot be applied without modification.

Depending on the configuration of the scatterers, the reflected electron can return directly to the reservoir, or (as shown in Fig. 1) proceed along the loop formed by path 2, and be reflected at the S-Sm interface once more. The final result is that the incoming electron wave partially returns to the reservoir as an electron wave (along path 3) and as a hole wave (along path 1).

For the calculation of the amplitudes of these waves, and the charge currents carried by them, it is crucial to take into account the interference due to electrons traveling along path 2 in one direction, and holes traveling in the opposite direction. The reflection amplitudes for trajectories involving two reflections are

$$r_{eh}(2)=i\left(\frac{1-|r|^2}{1+|r|^2}\right)\left(\frac{1+\exp(i\Delta\phi)}{1+[(1-|r|^2)^2/(1+|r|^2)^2]\exp(i\Delta\phi)}\right), \tag{3a}$$

$$r_{ee}(2)=\frac{4|r|^2\exp[i(2\beta+\phi_e)]}{(1+|r|^2)^2+(1-|r|^2)^2\exp(i\Delta\phi)}, \tag{3b}$$

with $\Delta\phi=\phi_e+\phi_h$. The phase shift acquired by electrons is given by $\phi_e=(k_F+E/\hbar v_F)L+2\pi\Phi/\Phi_0$, with L the length of path 2, and Φ the magnetic flux enclosed between path 2 and the superconductor ($\Phi_0\equiv h/e$). The phase shift of the holes is $\phi_h=-(k_F-E/\hbar v_F)L+2\pi\Phi/\Phi_0$. The total phase shift is therefore [12]:

$$\Delta\phi=\frac{2EL}{\hbar v_F}+4\pi\frac{BA}{\Phi_0} \tag{4}$$

with A the area enclosed between the superconductor and the loop formed by path 2, and B the applied magnetic field.

In the absence of a magnetic field $\Delta\phi=0$ for particles at E_F, *irrespective of the length and shape of path 2.* This is a consequence of the phase conjugation between electrons and holes. Equations (3a) and (3b) show that for $\Delta\phi=0$ the probability $|r_{eh}(2)|^2$ for Andreev reflection has a *maximum*, whereas the probability $|r_{ee}(2)|^2$ for normal reflection has a *minimum*. Using recursion relations one obtains the amplitudes $r_{eh}(N)$ and $r_{ee}(N)$ for trajectories having been reflected N times at the S-Sm interface.

For $E\neq0$, or $B\neq0$, the phases $\Delta\phi_N$ associated with different loops between the Nth and the $(N+1)$th reflection will generally be different. Simulations of a continuous random walker in a three-dimensional slab with a width d have shown that the average loop length $\langle L\rangle$ scales linearly with d. The relation between $\langle L\rangle$ and the transmission probability $T_n\simeq l_e/d$ of the slab is given by $\langle L\rangle\simeq0.35l_e/T_n$. The distribution $P(L)$ of L exhibits a power-law behavior for not too large lengths. For large L, $P(L)$ decays exponentially.

In the following calculations we have chosen L randomly according to the approximated distribution

$$P(L)=\begin{cases}1/3l_e, & \text{for } 0<L<l_e,\\ (1/3l_e)(L/l_e)^{-1.5}, & \text{for } l_e<L<L_c,\end{cases} \tag{5}$$

where L_c is a cutoff length chosen such that the average length $\langle L\rangle$ is equal to the value given above.

Figure 2 shows the N dependence of $I_N=1+|r_{eh}(N)|^2-|r_{ee}(N)|^2$, calculated for a typical barrier with $T=0.2$ [in Ref. [9] the transparency of the barrier is described by a parameter Z, with $T=1/(1+Z^2)$]. The quantity I_N indicates the average contribution to the charge current of trajectories which have been reflected N times. Figure 2 shows that for $V=0$ (and $B=0$) I_N rapidly approaches 2, indicating complete Andreev reflection. This shows that coherent multiple reflections drastically enhance the probability of Andreev reflection.

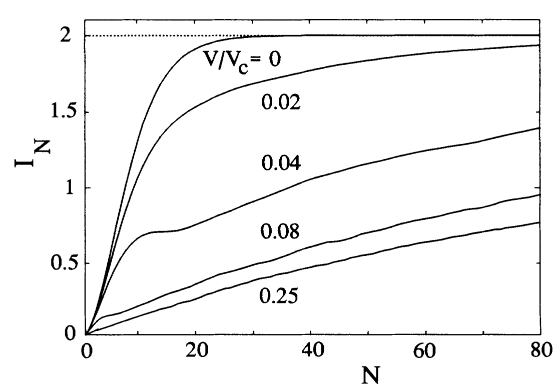

FIG. 2. Averaged contribution I_N to the charge current of trajectories involving N reflections at the interface, calculated as a function of N for several values of the voltage. The corresponding energy is $E=eV$.

Figure 2 also shows I_N calculated for $V \neq 0$ (the corresponding energy $E = eV$). As a reference we define a critical voltage $V_c = \frac{1}{2} hv_F/el_e$. Each curve represents the average over 2000 different sets of lengths L chosen according to distribution (5). This averaging removes the fluctuations in I_N associated with different choices of L. As a result of the random path lengths, the constructive interference is broken down with increasing V.

The evaluation of the differential conductance $dI(V,B)/dV$ requires the calculation of the sum over all trajectories with different N. The fraction $F(N)$ of trajectories which return to the reservoir after N reflections can be written as

$$F(N) = \begin{cases} 1 - T_n \quad (N=0), & \text{(6a)} \\ T_n^2(1-T_n)^{N-1} \quad (N \neq 0), & \text{(6b)} \end{cases}$$

where T_n is the transmission probability through the middle region.

The differential conductance can now be expressed as

$$G(V,B) = \frac{dI(V,B)}{dV} = G_s \sum_{N=1}^{\infty} F(N) I_N(eV,B) \,, \tag{7}$$

where G_s is the (Sharvin) conductance of the interface. Figure 3 shows the voltage dependence of $G(V,B=0)$, at zero temperature, for several values of T_n. For $T_n = 0$ (not shown), $G(V)$ is independent of V. Note that the voltage V_c^{eff} at which the excess conductance is suppressed is substantially smaller than V_c. This is due to the fact that the average length of an interference loop is much larger than l_e for the values of T_n shown. The effective temperature T^{eff} for suppression of the excess conductance can be estimated by $kT^{\mathrm{eff}} \approx eV_c^{\mathrm{eff}}$.

From the random-walk simulations we find that for $T_n = 0.1$, the rms average of the area is approximately given by $\langle A^2 \rangle^{1/2} \approx 12 l_e^2$. This yields an effective critical magnetic field $B_c^{\mathrm{eff}} \approx \Phi_0/24 l_e^2$ for the suppression of the excess conductance [13].

Figure 4 shows the normalized excess differential conductance $g = [G(V=0) - G(V \gg V_c)]/G(V \gg V_c)$ at zero temperature as a function of T_n for three different values of T. For a given value of T, the excess conductance first increases when T_n is reduced below unity. This means that the *addition* of scatterers *increases* the conductance. Note also that g increases with decreasing T.

We now compare our results with the experiments of Kastalsky *et al.* [4], who observed excess conductance in Nb/$In_{0.53}Ga_{0.47}As$ contacts. The authors explained their results in terms of a proximity-effect-induced pair current [14,15]. In our description the semiconductor is in the normal state (which implies $\Delta = 0$), and the superconductor affects the transport exclusively by imposing boundary conditions (corresponding to phase-coherent Andreev and normal reflection) on the wave functions.

The mean free path in their experiment is estimated to be [16] $l_e \simeq 50$ nm, and the Fermi velocity $v_F \simeq 2 \times 10^6$ m/s. For our choice for the transmission of the interface barrier ($T = 0.2$) this corresponds to $V_c^{\mathrm{eff}} \simeq 4$ mV, and $B_c^{\mathrm{eff}} \simeq 0.06$ T. In the experiments the excess conductance is suppressed for $V \gtrsim 0.5$ mV, and for $B \gtrsim 0.04$ T. Given the restrictions of our model, the latter result is consistent with the calculated B_c^{eff}. However, a discrepancy exists between the calculated and experimentally observed V_c^{eff}.

It may be that phase-coherent transport breaks down for large voltages, due to the reduction of the inelastic scattering length. Also our assumption of classical scattering may not be justified. The typical size of a scattering center may well be smaller than the Fermi wavelength ($\lambda_F \approx 5$ nm in Ref. [4]).

Finally, we note that because of our assumption of classical scattering in the semiconductor, the excess conductance vanishes in the absence of a barrier at the interface. As already noted, however, our model is expected to fail for point scatterers with dimensions $W < \lambda_F$. In this case the scattering in the semiconductor should be treated in a fully quantum-mechanical way. An excess conductance might then also occur for an ideal interface.

This work is part of the research program of the

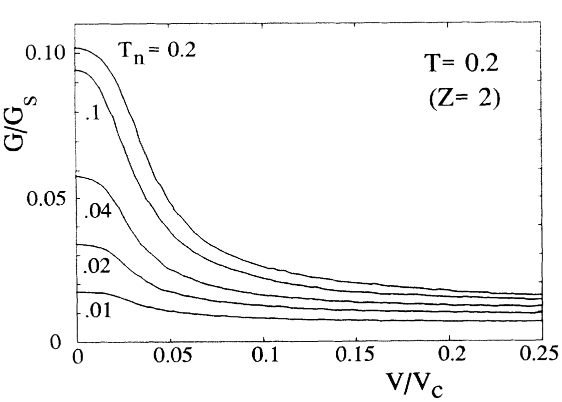

FIG. 3. Voltage dependence of the differential conductance at zero temperature, calculated for several values of T_n.

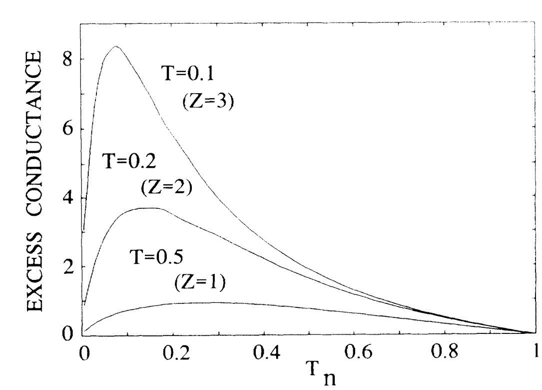

FIG. 4. Normalized excess conductance g (see text) as a function of T_n, for three values of interface barrier transmission.

Stichting voor Fundamenteel Onderzoek der Materie (FOM), which is financially supported by the Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO).

[1] A. F. Andreev, Zh. Eksp. Teor. Fiz. **46**, 1823 (1964) [Sov. Phys. JETP **19**, 1228 (1964)].

[2] P. C. van Son, H. van Kempen, and P. Wyder, Phys. Rev. Lett. **59**, 2226 (1987).

[3] The analogy with optical phase conjugation is discussed by H. van Houten and C. W. J. Beenakker, in *Analogies in Optics and Microelectronics*, edited by W. van Haeringen and D. Lenstra (North-Holland, Amsterdam, 1991).

[4] A. Kastalsky, A. W. Kleinsasser, L. H. Greene, R. Bhat, F. P. Milliken, and J. P. Harbison, Phys. Rev. Lett. **67**, 3026 (1991).

[5] Van der Post *et al.* (unpublished) have observed excess conductance at low voltage bias in interfaces between niobium and (degenerately doped) silicon.

[6] When a current flows, the reservoir is not in true equilibrium. However, the voltage drop inside the reservoir region can be described by an Ohmic series resistance.

[7] This barrier can be due to a Schottky barrier, or it may result from the Fermi wave number mismatch between the superconductor and the semiconductor.

[8] The barrier is extended along the y direction, and k_y is conserved. The S matrix describes the scattering of the x-dependent part of the wave function.

[9] G. E. Blonder, M. Tinkham, and T. M. Klapwijk, Phys. Rev. B **25**, 4515 (1983), calculated the normal and Andreev reflection probabilities for a delta-function potential barrier.

[10] C. W. J. Beenakker and H. van Houten, Phys. Rev. B **43**, 12066 (1991).

[11] J. Herath and R. Rainer, Physica (Amsterdam) **161C**, 209 (1989), have studied the effect of point scatterers near a normal-metal–superconductor interface.

[12] In Eqs. (2) and (3) the energy dependence of the phase shift associated with Andreev reflection is not taken into account. This is allowed since we restrict ourselves to energies $E \ll \Delta$.

[13] The effects of a finite E or the presence of B are not exactly equivalent. A finite $E > 0$ results in a positive $\Delta\phi$. However, a magnetic field can produce both a positive as well as a negative $\Delta\phi$, depending on whether the electrons move around the enclosed flux in a clockwise or counterclockwise direction.

[14] V. B. Geshkenbein and A. V. Sokol, Zh. Eksp. Teor. Fiz. **94**, 259 (1988) [Sov. Phys. JETP **67**, 362 (1988)].

[15] An alternative explanation has been formulated by A. V. Zaitsev, Pis'ma Zh. Eksp. Teor. Fiz. **51**, 35 (1990) [JETP Lett. **51**, 41 (1990)]; Physica (Amsterdam) **185-189C**, 2539 (1991); and A. F. Volkov and T. M. Klapwijk (unpublished).

[16] A. W. Kleinsasser, T. N. Jackson, D. McInturff, F. Rammo, G. D. Pettit, and J. M. Woodall, Appl. Phys. Lett. **55**, 1910 (1989).