Absolute Alpha Decay Width of ^{212}Po in a Combined Shell and Cluster Model

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We give a parameter-free microscopic description of the α decay of the ground state of ^{212}Po by expanding the parent wave function over a basis containing shell-model- as well as cluster-model-type elements. The resulting decay width agrees with experiment well within the range of the uncertainty inherent in the input of the model. The amount of core+α clustering in the parent state is found much higher (30%) than former microscopic estimates (~1%), which shows the soundness of assuming the existence of preformed α particles with appreciable probability in the initial state.

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The α decay of heavy nuclei slips out of the grip of theoreticians again and again. While the barrier penetration problem involved has been solved once and for all, and even the relative probability of α-particle preformation seems by now well accounted for [1], a reliable theory for the absolute probability of α formation is yet to be seen [2]. And until we can predict the absolute decay constants, we cannot feel we really understand α decay.

The numerous attempts so far have either brought partial success [3–8] or the approximations used need further justification [9–11]. The revival of interest in decay fragmentation has recently been stimulated by the discovery of heavy-cluster decay, which was predicted by fission theory [12] and poses an even tougher challenge to conventional decay theory than α decay [13].

As α-particle formation involves valence nucleons, its theoretical formulation calls for the shell model. The shell model, however, tends to underestimate the decay width substantially [3]. The problem is that conventional formalisms require that the parent wave function be correct at a channel radius that is large enough for interfragment nucleon-nucleon interaction as well as Pauli exchanges to be negligible, and that is very difficult to achieve. A partial remedy was attained by Fliessbach and Mang [4], whose formalism allows us to use radii within the range of Pauli exchanges, and by Tonozuka and Arima [5], who, in addition, included high-lying admixtures brought about by proton-proton and neutron-neutron interaction. Later it was shown by Dodig-Crnković *et al.* [8] that the proton-neutron interaction also plays a significant role. The use of the microscopic cluster model to produce four-nucleon correlation on the nuclear surface was introduced by Wildermuth and co-workers [7, 14], and the concept of an "α giant resonance" to achieve the same phenomenology was proposed by Okabe [11].

In this Letter we report on a parameter-free solution of the problem of α formation and absolute decay width in the ground state (g.s.) of ^{212}Po. We thereby get a reliable estimate for the amount of α clustering in ^{212}Po. The nucleus of ^{212}Po is the simplest case of physical interest. Our approach is essentially a shell model complemented by cluster-model-type basis states. It can be derived along the following lines.

The exponential decay is described by a Gamow wave function, in which, for large core-α distances, the relative-motion function is an outgoing Coulomb wave. Under the Coulomb barrier the outgoing wave has a long neck reminiscent of the tail of a bound state, which qualifies the state for a bound-state-type variational approximation [14, 15].

The state space is fully spanned by the shell-model basis alone, but to represent the decaying state in this way requires prohibitively large dimensions. We combine the shell model with the cluster model,

$$\Psi = \Psi^{\rm sh} + \Psi^{\rm cl},$$

to cover a large enough region of the configuration space in the α channel with manageable dimensions. The cluster-model term should have the form

$$\Psi^{\rm cl} = \mathcal{A}\{\Phi_c(\xi_c)\Phi_\alpha(\xi_\alpha)\phi(\mathbf{R})\},$$

where Φ_c and Φ_α are the fragment intrinsic wave functions, $\mathbf{R}$ connects the centers of mass, and $\mathcal{A}$ is the interfragment antisymmetrizer. It is good enough to describe the α particle by a single configuration of $0s$ harmonic-oscillator (h.o.) states $\varphi^{(\alpha)}(\mathbf{x}) = (\alpha/\pi)^{3/4}\exp(-\frac{1}{2}\alpha x^2)$. The intrinsic state Φ_α is related to the 4×4 Slater determinant $\Psi_\alpha(\{\mathbf{x}_i\})$ by $\Psi_\alpha = \varphi^{(4\alpha)}(\frac{1}{4}\sum_{i=1}^{4}\mathbf{x}_i)\Phi_\alpha$. It is now convenient to expand $\phi(\mathbf{R})$ in terms of shifted Gaussians projected to orbital momentum LM:

$$\phi(\mathbf{R}) = \sum_k f_k \int d\hat{\mathbf{s}}_k\, Y_{LM}(\hat{\mathbf{s}}_k)\varphi^{(b)}(\mathbf{R}-\mathbf{s}_k).$$

In keeping with the standard shell model of heavy nuclei,

the core can be taken to be infinitely heavy. This is a very good approximation in the cluster model as well [16]. Furthermore, it is useful to choose $b = 4\alpha$. Then, because $\varphi^{(4\alpha)}(\mathbf{R}-\mathbf{s})\Phi_\alpha = \Psi_\alpha(\{\mathbf{x}_i - \mathbf{s}\})$, we have

$$\Psi^{\rm cl} = \sum_k f_k \int d\hat{\mathbf{s}}_k\, Y_{LM}(\hat{\mathbf{s}}_k)\Psi_k,$$

where

$$\begin{aligned}\Psi_k &= \mathcal{A}\{\Phi_c\varphi_k(\mathbf{x}_1)\cdots\varphi_k(\mathbf{x}_4)\chi\} \\ &= \mathcal{A}\{\Phi_c\psi_k(\mathbf{x}_1)\cdots\psi_k(\mathbf{x}_4)\chi\},\end{aligned}$$

$\varphi_k(\mathbf{x}) = \varphi^{(\alpha)}(\mathbf{x}-\mathbf{s}_k)$, χ is the antisymmetrized product of spin-isospin states, and $\psi_k = P\varphi_k$, with P projecting onto the states unoccupied by the core nucleons.

In describing ^{212}Po we assume the ^{208}Pb core to be inert, with the nucleons occupying h.o. orbits of parameter β. Since ψ_k are orthogonal to the core orbits, we can get rid of the explicit treatment of the core as is usual in the shell model. The problem is still tough technically, due mainly to the reappearance of the core orbits in the projector P. Our wave function is written as

$$\begin{aligned}\Phi &= \Phi^{\rm sh} + \Phi^{\rm cl} \\ &= \sum_n C_n\Phi_n^{(\beta)} + \sum_k f_k \int d\hat{\mathbf{s}}_k\, Y_{LM}(\hat{\mathbf{s}}_k)\psi_k(\mathbf{x}_1)\cdots\psi_k(\mathbf{x}_4)\chi,\end{aligned}$$

where the shell-model basis states $\Phi_n^{(\beta)}$ are four-nucleon h.o. states of parameter β carrying angular momentum L. We treat the coefficients C_n and f_k as linear variational parameters. The Hamiltonian we diagonalize is

$$H = \sum_{i=1}^{4}(T_i + U_i) + \sum_{i<j}^{4} V(i,j),$$

where T_i is the nucleon kinetic energy, the average potential U_i is a realistic single-particle (s.p.) potential of Woods-Saxon form, with a Coulomb term generated by a homogeneous sphere, and $V(i,j)$ is a residual interaction.

The essential novelties in our approach are as follows. Contrary to Ref. [7], we avoid a bias for α formation by using a large realistic shell-model basis. In contrast with Ref. [11], we do not presuppose the existence of a model α-cluster state near the decaying physical state. Unlike in Ref. [7], all matrix elements are calculated with exact analytical techniques, and we have no convergence problems. At variance with Ref. [11], we fix all parameters to independent experimental data, and include the interaction between protons and neutrons as well.

Last but not least, the treatment of the core-α interaction deserves some discussion. In the shell model this interaction is hidden in the s.p. energies, which are insensitive to the shape of the s.p. potential on the nuclear surface. In the conventional cluster approach [7] this interaction is involved in the potential kernel, and that is bound to be in gross error because of the compelling use of h.o. functions in Φ_c. This implies errors in the inner wall of the Coulomb barrier in both of these approaches. In our model the s.p. states are still h.o. states, but the core-α interaction, if disentangled from the formalism, emerges as a folding of the s.p. potential U with the α density, which are both chosen to be realistic.

We used the s.p. potentials labeled "POP" and "NOP" by Rost [17] for protons and neutrons, respectively, approximated them by a combination of Gaussians, and adjusted the overall strengths to the experimental energies of the first proton and neutron s.p. states above the Fermi levels. To facilitate the analytical calculations, we approximated the s.p. Coulomb potential also by a combination of Gaussians. The h.o. constants were set to α=0.5, β=0.16 fm^{-2}. The s.p. states included are the following: $0h_{9/2}$, $1f_{7/2}$, $0i_{13/2}$, $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$ for protons and $1g_{9/2}$, $0i_{11/2}$, $0j_{15/2}$, $2d_{5/2}$, $3s_{1/2}$, $1g_{7/2}$, $2d_{3/2}$ for neutrons. Bases for ^{210}Pb and ^{210}Po were then set up by forming all possible positive-parity combinations of s.p. states up to summed angular momentum 8. The shell-model basis for ^{212}Po was obtained as a direct product of the two-nucleon bases. For L=0 it has 538 elements. The cluster-model basis was chosen to have 40 elements, with $\{s_k\}$ values from 1 to 20 fm, with a distribution peaking on the surface. For the model to be physically sensible, the energies of not only ^{210}Pb, ^{210}Po, ^{210}Bi, and ^{212}Po, but also of the α particle should be reproduced, which excludes the usual interactions devised for valence nucleons. The results we show were obtained with the Volkov 1 force [18] with Majorana parameter 0.58, which is often used in the cluster model. We neglected the Coulomb interaction between the valence protons.

The model energies of the yrast states of ^{210}Pb, ^{210}Po, and ^{210}Bi all agree with experiment within 0.2 MeV (mostly much better), which is very good for this unconventional interaction. In the shell model the g.s. energy of ^{212}Po, with respect to the four-nucleon threshold, is -18.61 MeV, and the inclusion of the cluster state lowers this to -18.88 MeV. The α-particle energy was found to be -27.79 MeV, thus the α-decay energy obtained is 8.91 MeV, which is in reasonable agreement with experiment (8.95 MeV). Through the cluster-model terms, the ^{212}Po energy is very sensitive to the value of β, and the correct energy found verifies the consistency between β and U.

It is worth noting that the pure cluster model puts the g.s. at -16.47 MeV, which is not much higher than the shell-model energy. This state can be identified with the model cluster state postulated by Okabe [11], but its "excitation energy," $-16.47 + 18.88$ MeV=2.41 MeV, is substantially smaller than Okabe's assumption, 5 MeV.

The α formation is characterized by the components of the state vector Ψ in the standard cluster basis. The elements of this nonorthogonal basis $\{\Psi_r\}$ are

$$\Psi_r = \mathcal{A}\{\Phi_c\Phi_r\}, \quad \Phi_r = r\int d\hat{\mathbf{r}}\, Y^*_{LM}(\hat{\mathbf{r}})\Phi_\alpha\delta(\mathbf{r}-\mathbf{R}).$$

The overlap $\langle\Psi_r|\Psi_{r'}\rangle$, as a function of r and r', gives the integral kernel of the norm operator $\hat{A}$, which comprises the core-α Pauli effects. The α-formation amplitude or covariant component in the cluster-model subspace [19] is

$$g(r) = \langle\Psi_r|\Psi\rangle = \langle\Phi_r|\Phi^{\rm sh}\rangle + \hat{A}\langle\Phi_r|\Phi^{\rm cl}\rangle.$$

The amplitude of the amount of clustering [4, 19] is

$$G(r) = \hat{A}^{-1/2}g(r).$$

To calculate the decay width Γ, we adopt the R-matrix formula

$$\Gamma = 2P_L(\hbar^2/2M_\alpha a_L)g_L^2(a_L),$$

where M_α is the (reduced) mass, P_L is the Coulomb penetration factor, and a_L is the channel radius. This is valid [20] for a narrow isolated level deep below the Coulomb barrier, provided a_L is in a region where the interfragment potential is $\sim 1/r$, and the Pauli exchanges are negligible. [Since in this region $g(r) = G(r)$, the amplitude $g(r)$ could as well be replaced by $G(r)$.]

The (conventional) spectroscopic factor S, which enters into direct reaction theories [21], and the amount of clustering, $\mathcal{S}$, which is the weight of the cluster-core component in Ψ [4, 19], are given by

$$S = \int dr\,|g(r)|^2, \quad \mathcal{S} = \int dr\,|G(r)|^2.$$

The formation amplitudes produced by the shell model, the cluster model, and the hybrid model are shown in Fig. 1. (Note the magnification of the shell-model curve.) The curves show the enormous importance of the inclusion of the cluster states. The amplitudes of the amount of α clustering of the three models are compared in Fig. 2. In the pure cluster model $G(r)$ has the shape of a radial wave function and, as is assumed in Ref. [1], has 12 nodes. But the shape of $G(r)$ of the shell model is as irregular as those in Refs. [4,5], and, in agreement with Ref. [11], its contribution distorts $G(r)$ of the full model as well.

The width, calculated with a_0=10–12 fm, has an uncertainty of $\pm$10% due to the graininess of the cluster basis. The result, $\Gamma = 1.45\times10^{-15}$ MeV, agrees with the experimental value ($\Gamma = 1.5\times10^{-15}$ MeV) very well.

The spectroscopic factor is S=0.025, which is 20 times higher than the best shell-model estimate [5, 22]. The amount of clustering, $\mathcal{S}$=0.302, is also substantially larger than previous microscopic estimates ($\sim$0.01 [23]). At the same time the ratio $\mathcal{S}/S$=12 is much smaller than ever assessed ($\sim$300 [23]), obviously because $g(r)$ and $G(r)$ are dominated by the large peaks out of the nucleus more than formerly believed, and there the Pauli effects are smaller.

The dependence of these results on the input parameters has been subjected to due scrutiny. They were found to be rather insensitive to the choice of the nucleon-nucleon force, to extensions or minor truncations of the basis, etc., provided three plausible conditions are fulfilled: (i) The basis should span the spatial region required by the formula of Γ. (ii) The decay energy should be correct. (iii) The s.p. potentials should be consistent with the β values and should not differ too much from our realistic choice [17]. Thus, for instance, with $\beta^{-1/2}$

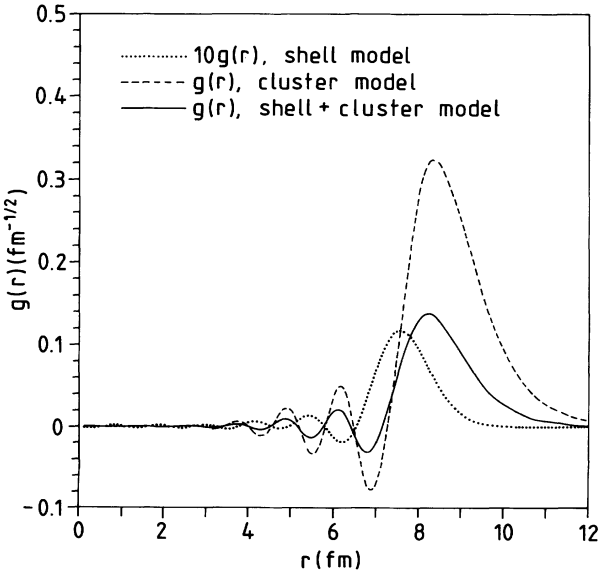

FIG. 1. Formation amplitudes in the pure shell model (multiplied by 10, dotted line), in the pure cluster model (dashed line), and in the combined model (solid line).

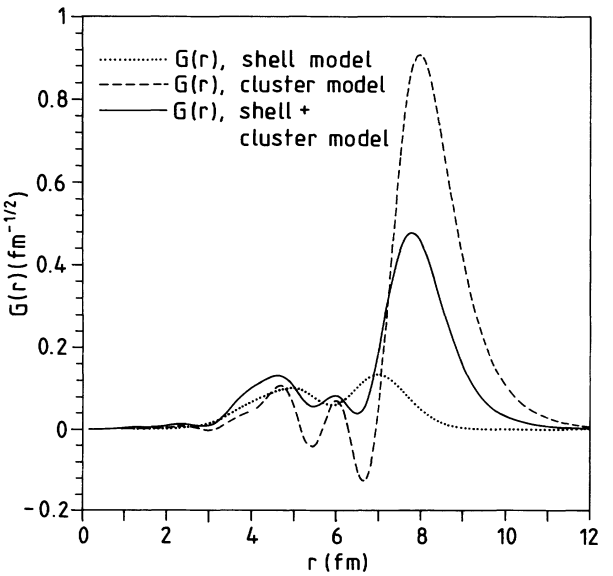

FIG. 2. The amplitudes of the amount of core+α clustering in the pure shell model (dotted line), in the pure cluster model (dashed line), and in the combined model (solid line).

reduced by 1%, Γ is enhanced by a factor of 2.5, but with proportionately reduced potential radii, the original Γ (as well as S and $\mathcal{S}$) is restored, provided that the s.p. potential depths are readjusted in each step to produce the same energy. Since, however, neither the s.p. potentials nor their correspondence to the h.o. parameter are sharply defined, the prediction for Γ might well have been a factor of 2 or 3 larger or smaller. Such an uncertainty must be inherent in any other models of α decay as well.

To sum up, we have reproduced the absolute α-decay width of the g.s. of ^{212}Po in a microscopic model containing no free parameters. Since the best shell-model calculations [5, 6, 8], which underrate Γ by an order of magnitude, were only able to allow for correlations between like nucleons properly, the present improvement can be interpreted as a result of a treatment of all correlations on the same footing. In view of the agreement of the width with experiment, the high value (0.302) obtained for the amount of α clustering can be considered realistic. Microscopic calculations have as yet revealed such high degrees of clustering only in light nuclei.

This result is, however, consistent with the good performance of the model of Buck, Merchant, and Perez [1, 13], which assumes $\mathcal{S}$=1 and treats $G(r)$ as a wave function. This model describes a whole range of cases of cluster decay, including the α decay of ^{212}Po, and its success helps to view our results over a more general background. One can thus infer that the high amount of clustering must be a general property of the decaying states of heavy nuclei.

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