Spin Dynamics of the ³He $A \rightarrow B$ Phase Transition

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The magnetic signals emitted by a moving A-B interface are calculated using the spin hydrodynamic theory, including both the Leggett-Takagi relaxation and spin diffusion. The B-phase texture is appropriately simplified. Most experimental observations are well accounted for.

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The phase transition from the A to B phase of superfluid ³He is a highly singular one. It becomes hypercooled very early [1], its damping results from the Andreev scattering [2], and, due to phase coherence across the interface, the growing B phase is colder than the receding A phase at low interface velocities [3]. These are aspects that are at least qualitatively understood. In contrast, the bizarre magnetic signals that accompany the A-B transition [4] have remained a puzzle to date: Down to temperatures around $0.75T_{AB}$, equilibrium values of the magnetization ("nominal signature") are measured on both sides of the interface; below it, deviation starts to develop in the A phase and a precursor, moving with the spin wave velocity c ahead of the slower interface, can be observed. At still lower temperatures, the erratic disturbances become so large that reproducibility is all but lost; and as c and the interface velocity u become comparable, chaos subsides, while two unidentified magnetic objects, a fast and a slow one, are observed.

These features and a number of other details can be understood within a simple model involving only two variables: the longitudinal spin component S (parallel to the field) and the dipole angle θ , the dynamic variable conjugate to S. (Other pairs of conjugate variables are, e.g., the displacement vector and momentum in elasticity theory, or the phase angle and density in superfluid hydrodynamics.) Starting from their hydrodynamic equations of motion [5,6], we have obtained the stationary solution, and found that it changes drastically as the interface velocity u increases: There is a threshold velocity u_c , where $u_c \simeq 1/H$ and $u_c \approx 30$ cm/sec for H = 3 kOe. If $u < u_c$, the solution is essentially a static twist, in both the A and B phases, of the dipole angle θ by less than π . The excess magnetization (produced by the transition to a system with a smaller susceptibility) is quickly converted to orbital angular momentum by the dipole torque within the "dipole healing length" of $c/\Omega \approx 2 \times 10^{-3}$ cm (Ω is the longitudinal resonance frequency). Since this length is so small, only the equilibrium values of magnetization are observed and $u < u_c$ is the subcritical region of nominal signature.

If *u* increases, so do the excess magnetization and the dipole torque that strive to balance it. At $u = u_c$, the dipole torque reaches its upper bound. So for $u > u_c$, the overstretched dipole torque starts to oscillate in space and convert orbital angular momentum back into spin in the

appropriate intervals. This supercritical, alternating behavior would in fact go on forever, if it were not for the spin diffusion that redistributes the spin density and manages to effect a small net amount of spin being converted irreversibly.

The situation is mathematically equivalent to a pendulum, with time and pendulum angle substituting for the spatial coordinate and dipole angle, respectively. In addition, the pendulum is required to come to a standstill in its upside-down position. The subcritical behavior is depicted by the pendulum that swings up with a finite velocity to the right angle to come to a standstill. A larger initial velocity must be compensated by a larger initial angle. If the initial velocity is too large, however, the pendulum starts rotating, with alternating conversion between the kinetic and potential energy. This is the equivalent of the supercritical behavior. Now, damping becomes necessary for the pendulum to come to rest. And if the damping is weak, the new time scale is very different from the old one. So, if u exceeds u_c , the twist texture of the A-phase dipole angle is spatially rather extended. (In the B phase, the subcritical stationary behavior prevails.) The excess magnetization, being produced at an ever greater rate, yet only able to seep away slowly, builds up in strength and spatial extent quickly. This may well be the observed "region of excess magnetization."

For $u = 10^2$ cm/sec and H = 3 kOe, the spatial extent of the excess magnetization reaches 20 cm. Obviously, to build up such an extended stationary state takes some time, and understanding the transient behavior becomes important.

Starting at around $u = 10^2$ cm/sec, one can neglect the dipole torque, just as one ignores the effect of gravitation on the angular velocity of a quickly rotating pendulum. Then the dynamics of the system reduces to two spin wave step functions, where the steps propagate with $\pm c$ in opposite directions. (The situation is in complete analogy to the two second sound steps emitted by the interface [3].) Moving ahead of the interface, the step in the *A* phase is probably the observed precursor. The two steps and the interface are three discontinuities in the magnetization, moving with c, u, and -c, respectively. If u is large enough, they are of comparable sizes. With five pickup coils providing simultaneous input and the possibility of the spin wave pulses reflected off both ends of the

vessel and traversing the interface, it is easy to construct jumbled signals, whose exact shapes depend sensitively (and therefore irreproducibly) on times of flight. Publishing the magnetic signals of different runs (with presumably slightly modified initial or ambient conditions) the authors therefore had to resort to plots of varying colors. If u exceeds c, the interface rushes ahead, leaving both spin wave steps behind. More experimental data are needed to decide whether this scenario is in fact connected to the observed fast and slow objects.

Now to our model: A planar interface in the x-y plane, moving with u and subject to a field H, both along \hat{z} . In the A phase on the right, we have $\hat{l}, \hat{d} \perp \hat{z}$, and the dynamics is given by two variables S and θ : S is the spin component along H, and θ the angle between \hat{l} and \hat{d} , \hat{l} being fixed. In the B phase on the left, if we assume $\hat{n} \parallel \hat{z}$, \hat{n} being the rotation axis, the same pair of variables is relevant during phase transition. Now, θ is the rotation angle around \hat{n} , defined as the deviation from the dipole minimum θ_L . We shall justify this simplifying assumption later, when the physics of the present model is clarified. The equations of motion, for both phases and in the frame of the interface, are [5,6]

$$(\partial_t - u \,\partial_z)\omega - D \,\partial_z^2 \omega = C^2 \,\partial_z^2 \theta - \Omega^2 f \,, \tag{1}$$

$$(\partial_t - u \,\partial_z)\theta = \omega + \tau \left(c^2 \partial_z^2 \theta - \Omega^2 f\right). \tag{2}$$

The notations are D = spin diffusion coefficient, $\tau = \text{the}$ spin relaxation time, $\chi = \text{the}$ (perpendicular) magnetic susceptibility, $\gamma = \text{the}$ gyromagnetic ratio, $\omega = \gamma^2 S/\chi$ $-\gamma H$ is the spin rotational velocity and proportional to the excess magnetization $\chi \omega/\gamma$, and $f = dF/d\theta$ where $\varepsilon_D = \chi \Omega^2 F/\gamma^2$ is the dipole energy $(F_A = \frac{1}{2} \sin^2 \theta)$ and $F_B = \frac{1}{2} [\cos \theta_L - \cos(\theta_L + \theta)]^2/\sin^2 \theta_L$, with θ_L the Leggett angle).

We first look for solutions that are stationary in the frame of the interface. Setting $\partial_t \omega = \partial_t \theta = 0$ and eliminating ω , we find the dimensionless equation

$$(1 - \zeta_1 \nabla - \zeta_2 \nabla^2) (\nabla^2 \theta - f) = (\eta_1 + \eta_2 \nabla) \nabla f.$$
(3)

With $z_0 = z \Omega/c_R$ fixing the length scale $(c_R^2 = c^2 - u^2, c_R/\Omega \approx 2 \times 10^{-3} \text{ cm})$, we have $\nabla = (c_R/\Omega)\partial_z$ and three small dimensionless parameters: $D_0 = \Omega D/c_R^2 \approx 3 \times 10^{-2}$, $\tau_0 = \Omega \tau$, and $u_0 = u/c_R$ ($\tau_0 \approx 6 \times 10^{-3}$ in the *A* phase and 0.3 in the *B* phase; u_0 varies widely, but we shall confine it to be smaller than $\frac{1}{3}$). Then the four explicit parameters $\zeta_1 = u_0[D_0 + \tau_0(1 + u_0^2)]$, $\zeta_2 = D_0\tau_0(1 + u_0^2)$, $\eta_1 = (D_0 + \tau_0 u_0^2)u_0$, and $\eta_2 = D_0 \tau_0 u_0^2$ are also small compared to 1. Neglecting all of them at first, we see that the static texture $\nabla^2 \theta = f = dF/d\theta$ solves Eq. (3) approximately. This texture can be visualized as a pendulum in potential -F, oscillating or rotating along z_0 , rather than *t*. The potential is that of the gravitational field in the *A* phase and has two dips in the *B* phase. The conserved quantity is (twice) the energy $E = (\nabla \theta)^2 - 2F$.

Including ζ_1 and ζ_2 but still neglecting η_1 and η_2 , we can integrate Eq. (3) to obtain $\nabla^2 \theta - f = \sum \beta_i \exp(q_i z_0)$,

where $q \pm \approx \pm \zeta_2^{-1/2}$ since $\zeta_1^2 \ll 4\zeta_2$. To avoid infinite textural disruptions, we set $\beta_+ = 0$ for the *A* phase on the right and $\beta_- = 0$ for the *B* phase on the left. This leaves us with a thin boundary layer, of order $\zeta_2^{1/2}$, outside of which the static texture is unperturbed. Including now also η_1 and η_2 , but staying clear of the boundary layer, the energy develops a spatial dependence

$$\nabla E = 2\nabla \theta (\eta_1 + \eta_2 \nabla) \nabla f = 2\eta_1 (E + 2F) df/d\theta.$$
(4)

With $\eta_1, \eta_2 \ll 1$, this implies a weak perturbance, effective over many periods, of the static texture $\nabla^2 \theta = f$. Independent of the form of F, η_2 does not contribute in Eq. (4). This is easy to understand: Approximating $\nabla^2 f$ by $\nabla^4 \theta$. we see that η_2 is reactive and only renormalizes the pendulum's mass. More importantly, one can also show $\nabla E < 0$, i.e., the energy (leaving the interface) increases into the B phase and decreases into the A phase. This reduces the number of possible solutions: For $z_0 \rightarrow \pm \infty$, we require homogeneity $(\nabla \theta \rightarrow 0)$ and dipole minimum $(\varepsilon_D \simeq F \rightarrow 0 \text{ or } \theta \rightarrow 0)$. Consequently, $E = (\nabla \theta)^2 - 2F$ vanishes as well. What is more, the dipole minimum is where the maximum of the pendulum potential -F is. The pendulum comes to a standstill ($\nabla \theta = 0$), and, therefore, is in its upside-down position ($\theta = 0$). As a result, there are two solutions for each phase. The singular one with E = 0 exists in both: The pendulum swings up at the interface with a finite kinetic energy and spends it all to reach the upside-down position. The rotating one, with E > 0 at the interface, exists only in the A phase. It loses its energy over many rotations, gradually coming to its "awkward" position with E = 0. The oscillating pendulum, on the other hand, exists only in the B phase. It starts with E < 0 at the interface, gains energy while oscillating, and eventually comes to a stop, at E = 0, some distance away.

As we shall see later when studying the boundary conditions, in the A phase the singular solution is realized for low, and the rotating one for high, interface velocities, while the B phase always entertains the singular solution. So we only need to know the rate of damping in the A phase. Since F and $df/d\theta$ vary on a different scale from *E*, we can calculate the change in energy $\Delta E = \int d\theta \nabla E / d\theta \nabla E$ $\nabla \theta$ and distance $\Delta z_0 = \int d\theta / \nabla \theta$, both per period $\Delta \theta = 2\pi$, to obtain $\nabla E = \Delta E / \Delta z_0$. They are $\Delta E = \eta_1 \pi / 2 \sqrt{E}$ and $8\eta_1/3$, $\Delta z_0 = 2\pi/\sqrt{E}$ and $2[2 - \ln(E/4)]$ for $E \gg 1$ and $E \ll 1$, respectively. Note that $\Delta z_0 \rightarrow \infty$ for $E \rightarrow 0$, i.e., the texture is formally always infinitely extended. The region of excess magnetization is $R_0 = \int dE \,\Delta z_0 / \Delta E$, integrated from E_i at the interface to E_f at a distant spot with an excess magnetization that is still large enough to be measurable. (The choice of the second spot is obviously to a large extent discretionary.) The excess magnetization density $\chi \omega / \gamma$ is obtained via Eq. (2):

$$\omega = -u \,\partial_z \theta = -\,\Omega u_0 \nabla \theta \,. \tag{5}$$

(The term $\simeq \tau$ is smaller by a factor of $\tau_0 u_0$ and can be neglected.) Now, $\nabla \theta \approx -\sqrt{E}$ if $E \gg 1$ and, when aver-

aged, $\nabla \theta = -2\pi/\Delta z_0$ if $E \ll 1$. Hence, the excess magnetization is large only if E is large. For the singular solution or the last rotation we can take $\Delta z_0 = 200$ (length of the pickup coil) and $u_0 < 10^{-1}$ to conclude that $\omega < 5 \times 10^{-3} \Omega$ indeed yields a vanishingly small excess magnetization.

From the consideration of boundary conditions below, we shall learn that $E_i = 10$ for $u_0 = 0.1$ (or $u = 10^2$ cm/sec). So even if we take the cutoff energy at $E_f \gg 1$, the region of excess magnetization $R_0 = (E_i - E_f)4/\eta_1$ is still huge: For $E_f = 5$, it is $R_0 = 10^4$ or R = 20 cm. This is also the length of the experimental cell, and one would expect the vessel wall not to exert disruptive influence on the stationary solution only if it absorbs the spin current. Moreover, the approach to such an extended stationary solution will certainly take a macroscopic, measurable time interval and therefore warrants attention. Fortunately, for $E \gg 1$, or $(\nabla \theta)^2 \gg F$, one can neglect the dipole torque; then the spin dynamics becomes strictly linear (if the ω dependence of χ and c can be neglected) and is given by the spin wave behavior. In close analogy to the second-sound pulses [3] sent out by the interface at the instant it starts moving, Eqs. (1) and (2) yield two spin wave step functions. In the laboratory frame they are $\omega \theta_H (\mp z + ct)$, where $\omega = \mp c \partial_z \theta$ and θ_H is the Heaviside step function. These formulas remain valid for u > c, except that both steps are then in the B phase.

We now consider the boundary, or better, connecting, conditions across the interface, first for the spin waves. They are

$$\Delta(u\gamma^2 S + \chi c^2 \partial_z \theta) = 0 = u\Delta(\partial_z \theta) + \Delta\omega, \qquad (6)$$

where $\Delta x \equiv x_B - x_A$ and $\langle x \rangle \equiv \frac{1}{2} (x_B + x_A)$ below. The first of Eqs. (6) expresses the continuity of the spin current across the interface and accounts for the fact that the dipole torque is too weak to appreciably alter it on a length scale of $\zeta_2^{1/2} \ll 1$. The second of Eqs. (6) comes from $\Delta \theta = 0$ (phase coherence of the dipole angle across the interface, for the same reason as for the phase [3,7]) and is obtained by setting $\Delta(\partial_t \theta) = 0$ in Eq. (2). Equations (6) are equivalent to the corresponding boundary conditions for heat transfer [3], $\Delta Q = 0$ and $\Delta(\mu + v_n v_s)$ =0 or, in fact, for second sound shock waves [8]. As in Ref. [3], dissipative terms $\simeq \tau_{,D}$ are neglected. They only lead to boundary conditions for the squashing modes [9]. As a result of the smallness of the magnetic susceptibility, the feedback of the spin dissipation to the interface motion is usually feeble; hence we can take u and the temperatures T_A and T_B , calculated at zero field [2,3], as an input to Eqs. (6) to determine the two amplitudes of the spin wave step function. For u < c, we have

$$[\omega(1-u/c)]_{A} = [\omega(1+u/c)]_{B} = -\frac{1}{2} u\gamma H \Delta \chi / \langle \chi c \rangle.$$
(7)

The A, B-phase ratios $\omega/\gamma H$ of the excess to the equilibrium magnetization are, respectively, $-\frac{1}{2}u(1 \mp u/c)^{-1}$

 $\times \Delta \chi \langle \chi c \rangle^{-1}$. Since $\Delta \chi < 0$, both ω_A and ω_B are positive, and especially the former suggests itself naturally as the observed precursor. The three discontinuities in magnetization are, in laboratory frame, respectively, at *ct*, *ut*, -ct, and of the heights $(\chi \omega / \gamma)_A$, $\Delta (\chi H + \chi \omega / \gamma)$, $(\chi \omega / \gamma)_B$. For *u* large yet not too close to *c*, they are of comparable size. The total signal of four or five pickup coils depends critically on times of flights, and must therefore appear jumbled and irreproducible.

The divergence of $\omega_A \simeq (1 - u/c)^{-1}$ can be understood as a compensation for the diminishing velocity c - u, with which the spin wave carries the excess magnetization out of the interface region. This divergence may be diverted by (a) the pair-breaking critical velocity, (b) the ω dependence of χ and c, and most intriguingly (c) the greater thermodynamic stability of the A phase at large values of ω . If the divergence proceeds this far, there will definitely be a strong feedback to the interface velocity, possibly already observed.

For u > c, causality forces $\omega_A \equiv 0$. The three discontinuities are then at ut, ct, and -ct, with the heights $H\Delta\chi + \chi\omega_B/\gamma$, $\chi\omega_+/\gamma$, and $\chi\omega_-/\gamma$, respectively, where $\omega_B = \omega_+ + \omega_-$ and $\omega_{\pm} = -\frac{1}{2}\gamma H(\Delta\chi/\chi)u/(u \mp c)$.

Now to the boundary condition for the stationary case. Defining $u_* = \chi_A \Omega_A / \gamma H |\Delta \chi|$ ($u_* \approx 3 \times 10^{-2}$ for H = 3 kOe) and $\beta = (\chi \Omega c_R)_B / (\chi \Omega c_R)_A$, we can write the first of Eqs. (6) as

$$\nabla \theta_A - \beta \nabla \theta_B = -u_0 / u_* \,. \tag{8}$$

The second of Eqs. (6), though certainly correct, does not contain any useful information. In contrast to the spin wave case, $\partial_t \theta_A = \partial_t \theta_B = 0$ is an integral part of the quasistatic, stationary solutions and $\Delta(\partial_t \theta) = 0$ is trivially satisfied. Instead, the proper boundary condition is back to $\Delta \theta = 0$. Surprisingly at first, perhaps, these two are not enough and we need two additional boundary conditions: In the spin wave case, two boundary conditions determine the amplitudes of the two outgoing spin wave pulses. And the implicit and causal assumption is that the incoming pulses have zero amplitude. For the quasistatic solutions, since there is no time or causality, we need two explicit boundary conditions for each side, say E and θ , making four altogether. Given the strong static character, $E_{A,B}$ and $\theta_{A,B}$ at the interface have to be determined by minimizing the total textural energy, subject to the constraint of $\Delta \theta = 0$, and of Eq. (8), which drives the system out of uniformity. In the following, the minimization procedure is sketched.

Minimal textural energy is achieved by the smallest possible value of |E|, since it determines the extent of the texture. So, for small u_0 , $E_A = E_B = 0$ and $\nabla \theta_A < 0$, $\nabla \theta_B > 0$ minimize the textural energy and yield two singular solutions. [The chosen signs yield, for a given u_0 , the smallest spatial extent within four singular solutions, cf. Eq. (8).] With $\Delta \theta = 0$, the two remaining parameters are equal, $\theta_A = \theta_B = \theta$, and determined by Eq.

(8), or $\alpha(\theta) = (2F_A)^{1/2} + \beta(2F_B)^{1/2} = u_0/u^*$. Now, α has an upper bound, $\alpha_{\max} = \alpha(\theta_0)$, where $\tan \theta_0 = -(1+1/\beta)$ $\times \tan \theta_L$ for both *B*-phase minima. Therefore, this solution is no longer possible for $u_0/u_* > \alpha_{max}$. Then E_A starts to grow, while E_B remains zero. [With $E_B \leq 0$, E_B and $\nabla \theta_B$ could only decrease, with an accordingly larger $\nabla \theta_A$ as prescribed by Eq. (8). This would result in an undesirable extension of the texture in both phases.] At still lower initial temperatures, $u_0 \gg u_*$, the A-phase energy at the interface [neglecting the B-phase contribution in Eq. (8)] becomes $E_i \approx (\nabla \theta)^2 \approx u_0^2/u_*^2$. The region of excess magnetization extends to $R_0 = 4(E_i - E_f)/\eta_1$ $\approx 4E_i/D_0u_0 \approx 10^2 u_0/u_*^2$ and its magnitude is $\omega = -\Omega u_0 \nabla \theta \approx \Omega u_0^2 / u_*$, yielding a ratio to equilibrium magnetization of $(|\Delta \chi|/\chi) u^2/(c^2 - u^2)$.

This completes the calculation of our model interface. Now the basic assumptions of our one-dimensional, longitudinal model are critically reviewed. There are three points. The first concerns the validity of the hydrodynamics, or Leggett theory of spin dynamics. Most of the predictions in this paper are arrived at with brokensymmetry concepts such as preferred direction, spin wave, and dipole energy. These ingredients should remain valid as long as the coherence length can be considered small [10]. Spin diffusion, on the other hand, is a valid concept only if the mean free path ξ is small. However, ξ should be compared to the huge length scale R_0 on which E relaxes rather than to c/Ω . Second, the assumption $\hat{\mathbf{n}} \| \mathbf{H}$ is, within the magnetic healing length of the B phase, certainly wrong [11]. However, even in a realistic model, the results concerning the transient spin wave pulses (in which the gradient energy dominates) and the A-phase stationary behavior would remain unscathed. The Bphase stationary solution will change, though probably not qualitatively, such as into an extended region of strong excess magnetization: Any magnetization that is not parallel to the magnetic field would be rotated by the Larmor precession on a scale usually much smaller than c/Ω . Averaging over it, the leftover spin is longitudinal. again with the typical length scale c/Ω . In addition, the sign of ∇E remains unchanged in a three-dimensional model [5], as we shall discuss in a future publication. Third, there is the textural critical velocity. It was proposed [4] as the underlying cause for the stalled drainage of the excess magnetization. We did not embrace this possibility because the interface rushes over the system with a high velocity, while the perpendicular motion of the vortex lines is slow in comparison. Hence it is hardly possible for the system to realize the required low textural critical velocity. The pair-breaking critical velocity, on the other hand, is too high [6] to be relevant.

In summary, we have studied a simple model in which many features of the spin dynamics accompanying the A-B transition can be understood. We do not expect it to be appropriate in every detail and we see a number of aspects that can be improved. But we believe that the present model presents an adequate framework in which the plethora of reported phenomena can be coherently ordered.

After submission of this Letter, we received a preprint by Bunkov and Timofeevskaya [12] that studies the same problem. Since they have neglected the dipole energy and terms $\sim u/c$, their results pertain to moderately high transition velocities, slow compared to c yet high enough to emit spin waves. By explicitly considering reflections and transmissions of the spin wave steps, they were able to reproduce the erratic appearing experimental curves. Their beautiful results confirm and complement ours in the relevant velocity window.

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