Realistic Hadronic Matrix Element Approach to Color Transparency

B. K. Jennings
TRIUMF, Vancouver, British Columbia, Canada V6T 2A3

G. A. Miller
Physics Department, FM-15, University of Washington, Seattle, Washington 98195
(Received 6 July 1992; revised manuscript received 29 October 1992)

Color transparency occurs if a small-sized wave packet, formed in a high momentum transfer process, escapes the nucleus before expanding. The time required for the expansion depends on the masses of the baryonic components of the wave packet. Measured proton diffractive dissociation and electron deep inelastic scattering cross sections are used to examine and severely constrain the relevant masses. These constraints allow significant color transparency effects to occur at experimentally accessible momentum transfers.

PACS numbers: 24.85.+p, 12.38.Qk, 25.30.Fj

Color transparency (CT) is the postulated [1,2] absence of final- (or initial-) state interactions caused by the cancellation of color fields of a system of quarks and gluons with small spatial separation. For example, suppose an electron impinges on a nucleus knocking out a proton at high momentum transfer. The consequence of color transparency is that there is no exponential loss of flux as the ejected particle propagates through the nucleus. Thus, the usually "black" nucleus becomes transparent. We examine only processes for which the fundamental reaction is elastic, or at least a two-body reaction. The nuclear excitation energy must be known well enough to ensure that no extra pions are created.

The existence of color transparency depends on (1) forming a small-sized wave packet in a high momentum transfer (Q) reaction, (2) the interaction between such a small object and nucleons being suppressed (color neutrality or screening), and (3) the wave packet escaping the nucleus while still small. That color neutrality (screening) causes the cross section of small-sized color singlet configurations with hadrons to be small seems well known [3-6]. So we take item (2) as given. The others require discussion.

Asymptotic perturbative QCD predicts the wave packet size to be $\sim 1/Q$. At nonasymptotic kinematics, including the effects of gluon radiation (Sudakov suppression) reduces effects of well separated quarks [7] and may lead to a falloff more rapid than $1/Q$ [8]. But, the minimum value of Q required to form a small wave packet is not known, and unexpected enhancements may occur if the wave packet is not small [9]. It is also true that at experimentally available energies, the small object expands in its motion through the nucleus. Thus final-state interactions are suppressed, but not zero [10-12].

Tantalizing but nondefinitive evidence has been obtained in a pioneering (p,pp) experiment at Brookhaven National Laboratory (BNL) [13]. Color transparency is the object of current searches using electron [14] and proton beams [13]. The existence of color transparency has not yet been demonstrated, and it would be useful to improve the reliability of CT predictions. Here we use apparently unrelated diffractive dissociation (DD) and deep inelastic scattering (DIS) data to probe the existence of the small-sized wave packet and to constrain the expansion process.

To be specific, consider the high Q^2 quasielastic $(e,e'p)$ reaction. A wave packet is formed when a bound proton absorbs the virtual photon. This wave packet is dubbed [4] a pointlike configuration (PLC) in an optimistic notation. Thus $|\mathrm{PLC}\rangle = T_H(Q^2)|N\rangle$, where the hard photon absorption operator is denoted as $T_H(Q^2)$. Our notation is that $|N\rangle$ represents a nucleon at rest, and $|N(\mathbf{q})\rangle$ represents one of momentum $\mathbf{q}$. Then the form factor is $F(Q^2)=\langle N(\mathbf{q})|T_H(Q^2)|N\rangle$.

We assume that for some large Q^2 the PLC has no soft interaction U with the surrounding nucleons. Then [6]

$$0=UT_H(Q^2)|N\rangle . \tag{1}$$

This is an extreme assumption, and an interesting partial transparency could occur even if the left-hand side were as large as half that predicted by taking $T_H(Q^2)|N\rangle$ to be a normal-sized object. Here we examine Eq. (1).

In the optical approximation $U=-4\pi i\,\mathrm{Im}\hat{f}\rho$, in which $\hat{f}$ represents the PLC-nucleon interaction as a sum of quark-nucleon scattering operators and ρ is the density of target nucleons. Only the dominant imaginary part of $\hat{f}$ is kept, and the nucleonic matrix element $\langle N|4\pi \times \mathrm{Im}\hat{f}|N\rangle=\sigma_p$, the proton-nucleon total cross section. Taking the nucleon matrix element of Eq. (1) and using completeness yields

$$0=\sigma_p+\sum_\alpha\int_{(M+m_\pi)^2} dM_X^2\langle N(\mathbf{q})|4\pi\,\mathrm{Im}\hat{f}|\alpha,M_X^2\rangle\frac{\langle \alpha,M_X^2|T_H(Q^2)|N\rangle}{F(Q^2)} , \tag{2a}$$

in which an intermediate state of mass M_X^2 has a set of quantum numbers (including multiplicity) α. It is useful to define the integral term of Eq. (2a) as $I(Q^2)$. Then

$$\sigma_p = -I(Q^2)\,. \tag{2b}$$

The propagating wave packet is described by the Green's operator acting on $|\mathrm{PLC}\rangle$. Expanding in a complete set of baryon states X, allows one to describe the propagating wave packet as a sum of terms of the form $G_X T_H(Q^2)|N\rangle$, where G_X is the eikonal propagator. Thus as the PLC propagates through a length l, each baryonic component X acquires a phase factor $e^{ip_X l}$ with $p_X^2 = p^2 + M_N^2 - M_X^2$. Here p, p_X, and l are magnitudes of three-vectors. The different phases upset the cancellation inherent in Eq. (1) so interactions do occur. To include these, note that the first-order scattering term is the sum (integral) of nucleonic matrix elements of $UG_X T_H$. The resulting scattering term is similar to the corresponding standard Glauber result (here Glauber always refers to calculations of nuclear distortions) except that a new quantity, defined as σ_{eff}, appears instead of σ_p [11,15]:

$$\sigma_{\mathrm{eff}}(l) \equiv \sigma_p + \sum_\alpha \int_{(M+m_\pi)^2} dM_X^2 \langle N(\mathbf{q})|4\pi \mathrm{Im} f|\alpha, M_X^2\rangle e^{i(p_X - p)l} \frac{\langle \alpha, M_X^2|T_H(Q^2)|N\rangle}{F(Q^2)}\,. \tag{3}$$

Equation (2a) follows from the assumption that the PLC does not interact, but its form as a sum rule involving hadronic matrix elements may seem surprising. It is therefore encouraging that the vanishing left-hand side does occur in the model of Jennings and Miller (JM) [11]. The consequence of their model is that Eq. (3) appears as

$$\sigma_{\mathrm{eff}}^{\mathrm{JM}}(l) = \sigma_p (1 - e^{i(p_1 - p)l})\,, \tag{4}$$

where the subscript 1 refers to the single excited state contributing to the integral term in Eq. (3). The quantity $(p - p_1) \approx (M_1^2 - M_N^2)/2p$ for large p, so that $(M_1 - M_N)^{-1} \equiv \tau_0$ plays the role of a time scale for PLC expansion. If $\tau_0 \ll l$ (a nuclear radius), the two terms in Eq. (4) cancel and transparency occurs; otherwise, final-state interactions occur.

The previous two-state model has some desirable features, but it is not realistic because a continuum of nucleon resonances and multipion states are excited in $pp \to pX$ reactions. We therefore use experimental observations of the matrix elements appearing in Eqs. (2) and (3). Thus

$$|\langle N(\mathbf{q})|4\pi f|\alpha, M_X^2\rangle| = \left[\frac{d^2\sigma^{\mathrm{DD}}(\alpha)}{dt\, dM_X^2}\right]^{1/2}, \quad |\langle \alpha, M_X^2|T_H(Q^2)|N\rangle| = \left[\frac{1}{\sigma_M}\frac{d^2\sigma^{\mathrm{DIS}}(\alpha)}{d\Omega\, dM_X^2}\right]^{1/2}, \tag{5}$$

where DD (DIS) stands for diffractive dissociation (deep inelastic scattering). In DD a fast proton breaks into the state α, M_X^2 without exciting the bound target nucleon. The matrix element of T_H is obtained by dividing the DIS cross section by the Mott cross section, σ_M.

The above are cross sections for final states, α, M_X^2. These can be related [16,17] to cross sections obtained by summing over α by defining probabilities $P^{\mathrm{DD,DIS}}(\alpha)$:

$$d\sigma^{\mathrm{DD,DIS}}(\alpha) = P^{\mathrm{DD,DIS}}(\alpha, M_X^2)\, d\sigma^{\mathrm{DD,DIS}}\,,$$

where $\sum_\alpha P^{\mathrm{DD,DIS}}(\alpha, M_X^2) = 1$. Measurements [16,17] show that $P^{\mathrm{DD,DIS}}(\alpha)$ is a peaked but broad function of multiplicities.

Evaluating the integrals of Eqs. (2),(3) using only data requires knowledge of the measurable relative phases of the matrix elements; these are presently unknown. Nevertheless, we can see if existing data rule out Eq. (2). This is because the integral term has a lower (negative) limit, obtained by taking each product of matrix elements to be negative. If this limit was much less (in magnitude) than σ_p color transparency would be ruled out. The integral $-I(Q^2)$ of Eq. (2b) can be written as

$$-I(Q^2) \le \int_{(M+M_\pi)^2} dM_X^2 \left[\frac{d^2\sigma^{\mathrm{DD}}}{dt\, dM_X^2}\frac{W_2(x,Q^2)}{2M}\right]^{1/2} \frac{\sum_\alpha [P^{\mathrm{DD}}(\alpha, M_X^2) P^{\mathrm{DIS}}(\alpha, M_X^2)]^{1/2}}{F(Q^2)} \equiv I_{\max}(Q^2)\,, \tag{6}$$

where M and M_π are nucleon and pion masses. The factor $W_2(x,Q^2)/2M$ arises from relating the matrix element of T_H to measured DIS cross sections for relevant experimental kinematics where W_2 is more important than W_1; $F(Q^2)$ is a linear combination of electric and magnetic form factors evaluated at the same kinematics.

We now evaluate $I_{\max}(Q^2)$ and compare it to measured values of σ_p [18]. We use Atwood's [19] parametrization of $W_2(x,Q^2)$ and Goulianos's [20] tabulation of $d^2\sigma^{\mathrm{DD}}/dt\, dM_X^2$ at $t = -0.047$ GeV^2. The factor

$$\sum_\alpha [P^{\mathrm{DD}}(\alpha, M_X^2) P^{\mathrm{DIS}}(\alpha, M_X^2)]^{1/2}$$

is replaced by the function $g(M_X^2)$:

$$g(M_X^2) = \sum_\alpha [P^{\mathrm{DD}}(\alpha, M_X^2) P^{\mathrm{DIS}}(\alpha, M_X^2)]^{1/2} \times \mathrm{Phase}(\alpha)\,, \tag{7}$$

where Phase(α) is the relative phase of the matrix elements. This includes the effects of the currently unknown phases. Taking the probability functions $P^{\mathrm{DD}}, P^{\mathrm{DIS}}$ from Ref. [16] for DIS and Ref. [17] for diffractive dissociation we estimate the sum over α to be approximately 0.6 for low values of M_X^2. Assuming $g(M_X^2)$ has a sharp cutoff at $M_X^2 = M_c^2$, we evaluate $I_{\max}$ by integrating over M_X^2 up to this cutoff.

With these inputs $I_{\max}(Q^2)$ is equal to σ_p for values of M_c^2 between 2.4 and 2.6 GeV^2, depending slightly on s for $Q^2 \gtrsim 1$ GeV^2. These values of M_c^2 do not exceed the bound required for diffractive dissociation to occur nor lead to highly virtual states. For partial transparency a lower value of M_c^2 would be obtained. However, even the extreme condition of Eq. (1) can be satisfied with a reasonably small value of M_c^2. Thus existing DD and DIS data allow color transparency to occur.

The above treatment of the integrand is now used to evaluate σ_{eff} of Eq. (4). This could be unrealistic: Not all of the products of matrix elements are negative and a sharp cutoff of the DD cross section is not expected. It is reasonable to try a form $g(M_X^2)=(M/M_X)^\beta$ (power law) instead of the previously used $g(M_X^2)=\theta(M_c^2-M_X^2)0.6$ (sharp cutoff). Values of β ranging from 2.4 to 4.0 allow the sum-rule relation (2) to be satisfied at each value of Q^2. The use of the power-law falloff allows high-mass M_X^2 states ($M_X^2 \approx Q^2$) to participate in the integral without emphasizing the importance of highly virtual states.

The results for σ_{eff} at $s=13$ GeV^2 are shown in Fig. 1 (for electron scattering $s=Q^2+4M^2$). If $g(M_X^2)$ is given by the power falloff, $\sigma_{\text{eff}}(l)\sim l$ for small l as in Ref. [10]. If the sharp cutoff is used, $\sigma_{\text{eff}}(l)\sim l^2$ for small values of l as in Ref. [11]. σ_{eff} is generally smaller with the sharp cutoff because with $M_c^2 \sim 2.2$ GeV^2 large values of M_X do not appear. Thus p_X-p is prevented from becoming large, and the cancellation between the two terms of Eq. (4) is not disturbed much by the phase factor $(p_X-p)l$.

We now turn to predicting nuclear color transparency. We use σ_{eff} to compute (e,e',p) cross sections to be measured at SLAC [13]. The ratios of cross sections (or transparency) $\mathcal{T}=\sigma/\sigma^{\text{BORN}}$ are shown in Fig. 2. By σ^{BORN} we mean just Z times the free cross section. The quantities σ are $(e,e'p)$ differential cross sections integrated over the scattering angles of the outgoing proton. (See Ref. [11] for details.) Full color transparency corresponds to a ratio of unity. We want to know the energies for which $\mathcal{T}$ approaches unity and for which it is substantially greater than that obtained with the standard Glauber treatment. Both choices of $g(M_X^2)$ show that observable increases are obtained for values of $q=|\mathbf{q}|$ as low as 5 GeV/c, or $Q^2=9$ GeV/c^2. The results of using the sharp cutoff are very similar to those of using the model of Ref. [10], with $M_1=1.44$ GeV. This follows from the small value of M_c^2.

The single published experiment aimed at observing the effects of color transparency is the BNL (p,pp) work [13] at beam momenta p_L ranging from 6 to 12 GeV/c. The kinematics of the BNL experiment are such that the basic pp elastic scattering occurs at a center-of-mass angle of 90° if the target proton is at rest. Figure 3 shows that the experimentally determined transparency $\mathcal{T}=d\sigma/d\sigma^{\text{BORN}}$ (ratio of nuclear to hydrogen cross section per nucleon after removing the effects of nucleon motion) has

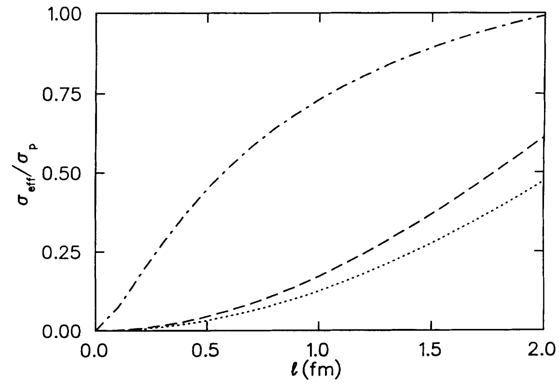

FIG. 1. The real part of $\sigma_{\text{eff}}(l)/\sigma$. Dashed line: sharp cutoff $g(M_X^2)$; dotted line: Eq. (5) with $M_1=1.44$ GeV; dash-dotted line: power law $g(M_X^2)$.

unexpected oscillations with energy. Also shown is the energy-independent expectation of standard Glauber theory. This independence survived the examinations of Refs. [21] and [22].

One possibility, suggested by Ralston and Pire [23], is that the energy dependence is caused by an interference between a hard amplitude, which produces a small object, and a soft one (the Landshoff process), which does not. Kopeliovich and Zakharov [12] and Jennings and Miller [24] extended this idea by including effects of the expansion of the small object. Another mechanism is that of Brodsky and de Teramond [25] in which the two-baryon system couples to charmed quarks [there is a small ($6q$) and a large ($6q,c\bar{c}$) object]. These two well-motivated ideas, when combined with the expansion technique of Ref. [24], do not reproduce the data satisfactorily.

Here we see that using σ_{eff} of Eq. (3) leads to a more accurate description of the data. To approximate $T_H(Q^2)$ by $W_2^{1/2}$ is to assume that the proton-proton high-Q^2 data vary in a manner similar to W_2. This is reasonable because in each case the reaction starts with a quark absorbing high momentum. The Ralston-Pire mechanism is evaluated using a recent fit by Carlson,

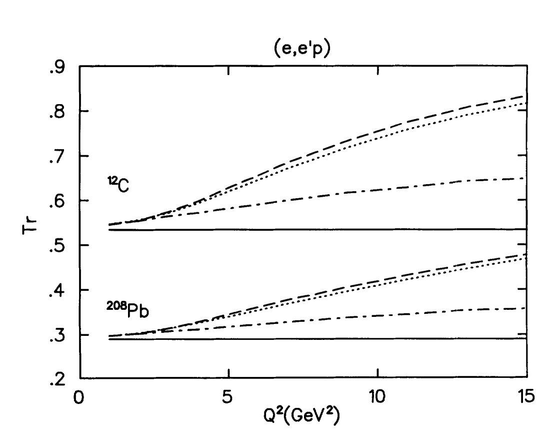

FIG. 2. The transparency $\mathcal{T}$ for the $(e,e'p)$ reaction. The solid line represents the standard Glauber calculation ($\sigma_{\text{eff}}=\sigma_p$). The other curves are defined in Fig. 1.

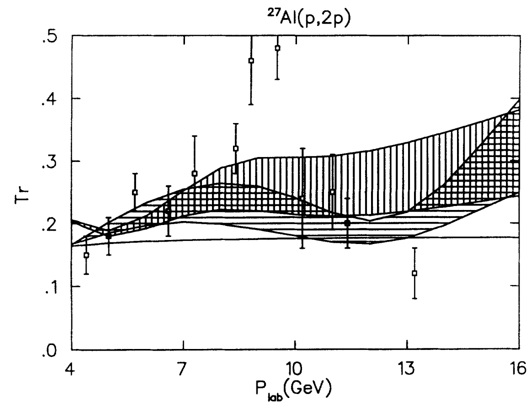

FIG. 3. Energy dependence of the transparency $\mathcal{T}$. The data points are from Carroll *et al.* [13]. The area shaded vertically is obtained from the mechanism of Ref. [23] and amplitude of Ref. [26]. The area shaded horizontally is obtained from the mechanism of Ref. [25]. In each case the upper bound uses the sharp cutoff for $g(M_X^2)$ and the lower bound a power law. The solid curve assumes no color transparency.

Chachkhunashvili, and Myhrer [26] of the hard pp scattering data. Both the usual quark-counting and Landshoff amplitudes are included in their description of A_{nn} and the differential cross sections. The results for the mechanisms of Refs. [23] and [25] are shown in Fig. 3. Both the power-law and sharp cutoff versions of $g(M_X^2)$ are used. These represent lower and upper limits to the predictions, and obtain a range of variation by shading the area between these curves. The enhancement at about 4 GeV is a new consequence of the amplitude of Ref. [26]. The Brodsky-de Teramond model along with the sharp cutoff $g(M_X^2)$ seems closest to the data, but no calculation achieves good agreement. One can say that the general trend is reproduced. The strong dependence on $g(M_X^2)$ shows that at least one measurement of color transparency is needed to determine this function. The new experiment [14] designed for higher energies and greater accuracy will certainly help.

Measured diffractive dissociation and deep inelastic scattering data lend support to the idea that color transparency occurs. In fact we have shown that there exists a set of hadronic matrix elements that reproduce the DD and DIS data and also give color transparency. This is our strongest conclusion. The formation of a PLC is allowed, and its expansion is not too rapid. We eagerly await the new experimental results [13,14].

The authors acknowledge financial support from NSERC and U.S. DOE, and thank L. Frankfurt, W. R. Greenberg, J. P. Ralston, and M. Strikman for useful discussions.

[1] A. H. Mueller, in *Proceedings of the Seventeenth Rencontre de Moriond, Moriond, 1982,* edited by J. Tran Thanh Van (Editions Frontieres, Gif-sur-Yvette, France, 1982), p. 13.

[2] S. J. Brodsky, in *Proceedings of the Thirteenth International Symposium on Multiparticle Dynamics,* edited by W. Kittel, W. Metzger, and A. Stergiou (World Scientific, Singapore, 1982), p. 963.

[3] S. Nussinov, Phys. Rev. Lett. **34**, 1286 (1975); F. E. Low, Phys. Rev. D **12**, 163 (1975); J. Gunion and D. Soper, *ibid.* **15**, 2617 (1977).

[4] L. Frankfurt and M. Strikman, Prog. Part. Nucl. Phys. **27**, 135 (1991); Phys. Rep. **160**, 235 (1988).

[5] B. Z. Kopeliovich, Fiz. Elem. Chastits At. Yadra **21**, 117 (1990) [Sov. J. Part. Nucl. **21**, 49 (1990)].

[6] L. Frankfurt, G. A. Miller, and M. Strikman, University of Washington Report No. 40427-26-N91 (to be published).

[7] H-n Li and G. Sterman, ITP Report No. ITP-SB-92-10 (to be published).

[8] L. Frankfurt, G. A. Miller, and M. Strikman, University of Washington Report No. 40427-16-N92 (to be published).

[9] O. Benhar, B. Z. Kopeliovich, C. Mariotti, N. N. Nikolaev, and B. G. Zakharov, Phys. Rev. Lett. **69**, 1156 (1992); S. Gardner, Indiana University Report No. IU/NTc92-27 (to be published); N. N. Nikolaev, A. Szczurek, J. Speth, J. Wambach, B. G. Zakharov, and V. Zoller, Julich Report No. KFA-IKP(TH)-1992-15 (to be published).

[10] G. R. Farrar, H. Liu, L. L. Frankfurt, and M. I. Strikman, Phys. Rev. Lett. **61**, 686 (1988).

[11] B. K. Jennings and G. A. Miller, Phys. Lett. B **236**, 209 (1990); Phys. Rev. D **44**, 692 (1991).

[12] B. Z. Kopeliovich and B. G. Zakharov, Phys. Lett. B **264**, 434 (1991).

[13] A. S. Carroll *et al.*, Phys. Rev. Lett. **61**, 1698 (1988); A. S. Carroll *et al.*, BNL Experiment No. 850.

[14] SLAC Experiment No. NE-18, R. Milner, Spokesman.

[15] Using $\sigma_{\rm eff}$ instead of σ in standard scattering wave functions incorporates CT for terms of first order in the optical potential U, and is also a very accurate approximation to the appropriate multiple-scattering series. $\sigma_{\rm eff}$ is complex because it is part of a contribution to a scattering amplitude. See W. R. Greenberg and G. A. Miller, University of Washington Report No. 40427-23-N92 [Phys. Rev. D (to be published)].

[16] See, e.g., N. Schmitz, in *Hadronic Multiparticle Production,* edited by P. Carruthers (World Scientific, Singapore, 1988), p. 3.

[17] K. Goulianos, H. Sticker, and S. N. White, Phys. Rev. Lett. **48**, 1454 (1982).

[18] Particle Data Group, J. J. Hernández *et al.*, Phys. Lett. B **239**, 1 (1990).

[19] A. Bodek *et al.*, Phys. Rev. D **20**, 1471 (1979).

[20] K. Goulianos, Phys. Rep. **101**, 169 (1983).

[21] T.-S. H. Lee and G. A. Miller, Phys. Rev. C **45**, 1863 (1992).

[22] I. Mardor, Y. Mardor, E. Piasetsky, J. Alster, and M. M. Sargasyan, Phys. Rev. C **46**, 761 (1992).

[23] J. P. Ralston and B. Pire, Phys. Rev. Lett. **61**, 1823 (1988).

[24] B. K. Jennings and G. A. Miller, Phys. Lett. B **274**, 442 (1992).

[25] S. J. Brodsky and G. F. De Teramond, Phys. Rev. Lett. **60**, 1924 (1988).

[26] C. E. Carlson, M. Chachkhunashvili, and F. Myhrer, Phys. Rev. D **46**, 2891 (1992).