Self-Organized Pinning and Interface Growth in a Random Medium

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A new class of interface growth models is proposed, where global equilibration of the driving force is achieved between each local deposition. Two such models are studied numerically, and it is seen that roughness can occur with higher exponents than in situations where global equilibration of the driving force is not established. In particular, we have found a new universality class of growth models which in one dimension gives self-affine interfaces with roughness exponent $\chi = 0.63 \pm 0.02$.

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There has recently been much interest in nonequilibrium growth models and their dynamic universality classes. These studies may have a number of practical applications, e.g., in chemical vapor deposition [1], electrochemical deposition [2], molecular beam epitaxy [3], growth of bacteria colonies [4], and in fluid invasion of porous media [5–7]. Theoretical studies have fallen into two classes. The first one treats nonlocal growth, appearing, e.g., in Laplacian growth phenomena, as diffusion limited aggregation. In these the growth at a point is influenced by the overall shape of the interface through screening of the driving force, and the evolving surfaces typically become self-similar. In nonlocal growth models there is also the invasion percolation where the segments of the interface with overall minimum resistance always propagate [8]. The other class of models treats growth that is governed completely by local conditions that prevent the developing interface from developing overhangs.

As a scholastic example of the local class of models Kim and Kosterlitz [9] introduced a simple discrete model of a growing interface. In this model the growth process is simulated by randomly choosing a site and allowing the interface to grow one unit if all slopes remain small (e.g., ≤ 1). If this condition is not fulfilled a new site is chosen randomly. In the long-wavelength limit this process can be described by the Kardar-Parisi-Zhang (KPZ) equation [10]

$$\frac{dh}{dt} = \nu \Delta h + \frac{\lambda}{2}\left(\frac{dh}{dx}\right)^2 + \eta(x,t)\,, \tag{1}$$

where $\langle \eta(x,t)\eta(x',t')\rangle = \Gamma\delta(x-x')\delta(t-t')$. The results of this algorithm are the well-known scalings [11] for the ensemble averaged width w of the interface, $w^2 = \langle (h - \langle h\rangle)^2\rangle \propto L^{2\chi} f(t^{1/z}/L)$ with $\chi = 1/2$ as the roughness exponent of the saturated interface and with $\beta = \chi/z = 1/3$ describing the transient roughening. Experimentally determined χ for one-dimensional interfaces ranges from $\chi = 0.55 \pm 0.06$ in electrochemical deposition [2], $\chi = 0.78 \pm 0.07$ in growth of bacterial colonies [4], to a χ of 0.63 ± 0.04 ([7]), 0.73 ± 0.03 ([5]), and ≈ 0.81 ([6]) measured for fluid invasion of porous media. In all experiments the measured χ are above the one predicted by the KPZ universality class. And although $\chi = 0.55 \pm 0.06$ in electrochemical deposition is within the range of the KPZ prediction, the measured dynamics of the surface growth in Ref. [2] are much faster than that predicted by KPZ.

It is known that the type of noise plays a big role in the growth of an interface. If the noise is changed either to a spatially or a temporally power-law-correlated noise [12] or to an uncorrelated but non-Gaussian (i.e., power law) distribution [13] one observes different exponents. However, there is no reason to assume such power laws in the medium where the interface propagates. Thus it would be much more satisfactory if we could find dynamical reasons for obtaining the measured nontrivial exponents.

The fact that the growth behavior is influenced by the type of noise η has been further investigated by Parisi [14], who considers the equation

$$\frac{dh}{dt} = \nu \Delta h + \frac{\lambda}{2}\left(\frac{dh}{dx}\right)^2 + \eta(x,h)\,, \tag{2}$$

where $\langle \eta(x,h)\eta(x,h)\rangle = \Gamma\delta(x-x')\delta(h-h')$. This form of noise is physically appealing because, e.g., liquid penetrating a porous medium is influenced by a local resistance which does not change explicitly with time, but only with location (x,h). The result from simulations [14] with the above equation is that roughening occurs much faster, with dynamical exponent $\beta = 0.7 \rightarrow 0.8$, reflecting that part of the interface gets stuck in regions with large resistance.

We present here another approach, in which the pinning force on a given site is local (as in Parisi's model), but where the growth at a given time occurs at the site where the pinning force is minimal (in analogy with invasion percolation [8]). However, at each time step, we only consider sites where growth is indeed allowed by the constraint of small local slopes, as in the Kosterlitz-Kim model.

In the (one-dimensional) version of this algorithm we have an interface $h(x) = h(x,t)$ defined on a discrete chain $x = 1, 2, 3, \ldots, L$ as well as a string of random Gaussian distributed uncorrelated local pinning forces $\eta(x,h)$. We use periodic boundary conditions. The

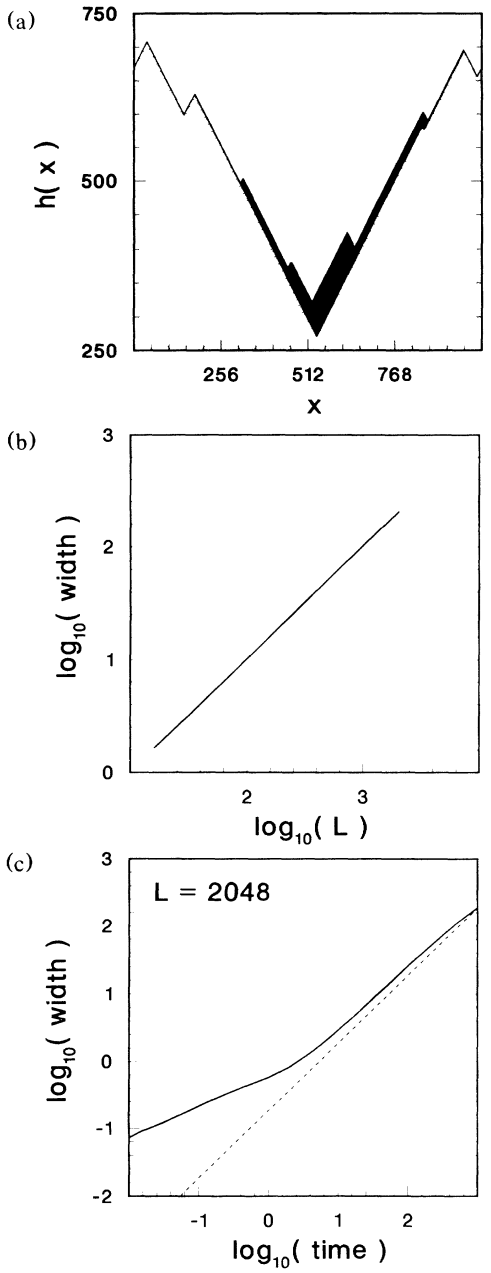

FIG. 1. Model A. (a) Typical snapshots of interfaces at two configurations at saturation. The dark areas show the activity between the respective stages. (b) Saturated width as a function of system size L for periodic boundary conditions, sampled at saturation. (c) Width as a function of time t. Time is measured from flat state, in units of increased heights per site. The dashed line has slope 1.

chain is updated by finding the site with the smallest pinning force $\eta(x,h)$ among the sites that are allowed to grow without breaking the Kosterlitz-Kim conditions $[|h(x)+1-h(x-1)| \leq 1$ and $|h(x)+1-h(x+1)| \leq 1]$. On this site one unit is added to h and a new random noise η between 0 and 1 is associated to the site. In the following we refer to this as model A.

Figure 1(a) displays a typical configuration of the string at the saturated state. The darkest areas show the latest activity of the moving interface. One observes a nonhomogeneous distribution of activity. From calculations of the saturated width $w(L)$ for various system sizes L [see Fig. 1(b)] we find a static roughness exponent $\chi = 1.00 \pm 0.01$. Figure 1(c) shows how the width w grows with time. We see two regimes, one at short times before global comparisons of pinning becomes important, and one where "self-organized pinning" dominates. The latter regime has width $w \propto t^{\beta}$ with $\beta = 0.95 \pm 0.05$. This value is within the error bars from what is expected from Galilean invariance (i.e., $\chi + \chi/\beta = 2$). Also we find that in the time interval where $\beta \approx 0.95 \pm 0.05$ the skewnes $s = \langle (h - \langle h \rangle)^3 \rangle / \langle (h - \langle h \rangle)^2 \rangle^{3/2}$ takes a constant value of about 0.6. For comparison the KPZ predicts a skewness of 0.29 in the transient regime [15]. After about 1000 time steps per site the string saturates and the skewness vanishes for the finite system of $L = 2048$. It may also be noted that an investigation of higher moments of the Fourier spectra $[f_q(k) = \langle (h_k h_{-k})^{q/2} \rangle^{1/q}$, where h_k is the Fourier transform of $h(x)$ and $\langle\,\rangle$ denotes ensemble average] revealed that for all $q_1, q_2 = 1, 2, \ldots, 10$, $f_{q_1}(k)/f_{q_2}(k)$ is independent on k at all times, both in the transient and at saturation. Thus there is no multiscaling in model A.

We conclude that a model-A interface is rough with the "trivial" exponents $\beta = 1$ and $\chi = 1$, indicating that it in fact shows self-similarity (like invasion percolation without any constraint). We also noticed that the breaking of the $h \rightarrow -h$ symmetry in the dynamics shows up in the transient regime through a significant and nearly constant skewness.

We now consider a different modification of the invasion percolation approach, where the constraint on the slopes acts after the least pinned site is moved. Physically this may be realized in situations where the local pinning force is reduced by a local slope. In the one-dimensional version of such an algorithm we consider a discrete interface $h(x)$ defined on a discrete chain $x = 1, 2, 3, \ldots, L$, and a string of Gaussian distributed random uncorrelated local pinning forces $\eta(x,h)$. We use periodic boundary conditions. The chain is updated by finding the site with the smallest pinning force $\eta(x,h)$ among all sites. On this site one unit is added to h. Then neighboring sites are adjusted upwards $(h \rightarrow h+1)$ until all slopes $|h(y) - h(y-1)| \leq 1$. New random noise $\eta \in [0,1]$ is assigned to all adjusted sites. We will refer to this approach as model B.

Figure 2(a) displays typical configurations at saturation. The darkest areas show the latest activity of the moving interface. One observes a highly nonhomogeneous distribution of activity. Notice that according to the rule B there will be a variable amount of sites that are adjusted for each finding of a new minimum of $\eta(x,h)$ to propagate. Thus one might consider the distribution

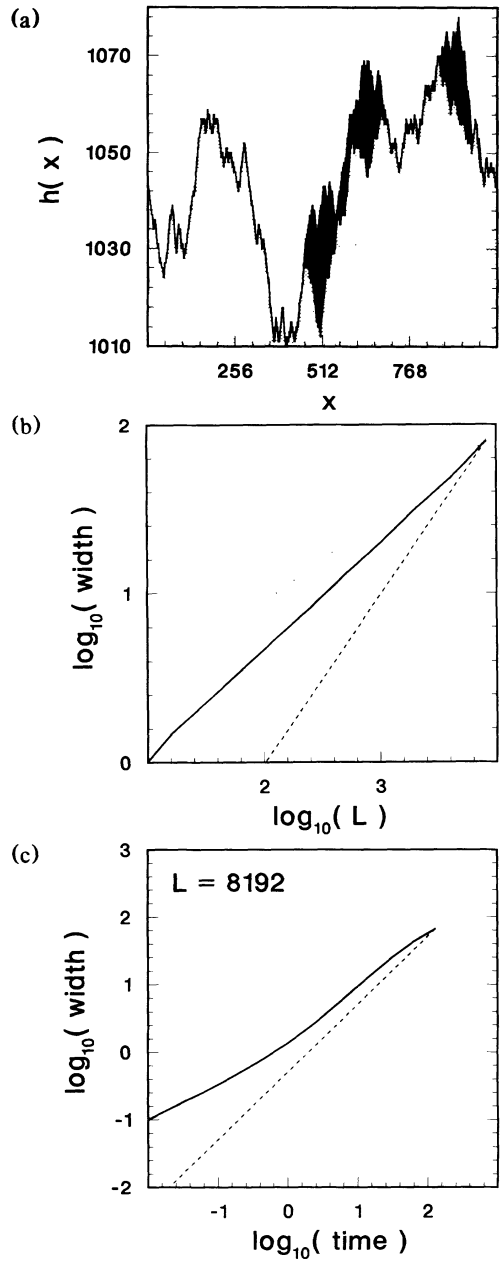

FIG. 2. Model B. (a) Typical snapshots of interfaces for two configurations at saturation. The dark areas show the activity between the respective stages. (b) Saturated width as function of system size L. (c) Width as a function of time. Time is measured from flat state, in units of increased heights per site. The dashed line has slope 1.

of these "avalanches" sampled at saturation. It turns out that these avalanches have a characteristic size (average of about 4), and that there is an exponentially small probability for bigger avalanches. Figure 2(b) presents the saturated width $w(L)$ versus system size L. We observe a static roughness exponent $\chi = 0.63 \pm 0.02$. From a plot of the width of the string versus time [see Fig. 2(c)] we see a regime before saturation where the width $w \propto t^{\beta}$ with $\beta = 0.9 \pm 0.1$. Thus in this case the Galilean invariance is not fulfilled. Investigation of the skewness shows a steady decrease during the full transient, approaching zero at saturation. Calculations of moments of Fourier spectra during both transient and at saturation show that there is no multiscaling.

Model B demonstrates that systems with global equilibration of pinning forces can give interfaces that are self-affine but not self-similar. It is important to understand that neither of the presented models needs any fine tuning. They automatically develop into a "critical" state, for model B with the new and interesting roughness exponent $\chi = 0.63 \pm 0.02$. It is in fact interesting that the exponent $\chi = 0.63 \pm 0.04$ was measured in Ref. [7] for ink propagation in a 20 cm strip of paper.

In order to understand how the exponent $\chi = 0.63 \pm 0.02$ can appear, consider Ref. [7] where wetting invasion in paper is modeled by wetting invasion in a percolating network. When the density p of inert sites in the network is fine tuned to the critical density p_c of directed percolation, the propagating interface stops along a directed percolating string and becomes self-affine with a roughness exponent $\chi = 0.633 \pm 0.001$. The global comparison of pinning forces in model B gives a dynamical reason for such fine tuning. Thus the argument in Ref. [7] suggests that the $\chi = 0.63$ exponent of model B can be understood from the scaling of the transverse correlation length $\xi_{\perp} \propto | p - p_c |^{-1.097}$ with the parallel correlation length $\xi_{\parallel} \propto | p - p_c |^{-1.733}$ for a directed percolating string on a close to critical network: $\xi_{\perp} \propto \xi_{\parallel}^{1.097/1.733} = \xi_{\parallel}^{0.633}$. For exponents of directed percolation see Ref. [16].

It is stressed, however, that the particular model presented in Ref. [7] has a dynamical exponent $\beta = 0.70 \pm 0.05$ which is different from β of model B. Thus the difference between the propagation rules of model B and of that of Ref. [7] leads to fundamentally different large-scale dynamical behaviors.

Finally we would like to stress the following.

(1) Both the presented models show exponents which are bigger than that of the corresponding local rules. For model A this is seen by comparing with the Kosterlitz-Kim deposition rule (which has KPZ behavior). For model B we compare with a model where a site $x = 1, 2, \ldots, L$ is chosen randomly (equiprobable) for a forced move $h(x) \rightarrow h(x) + 1$. Then neighbors are adjusted upwards until all slopes ≤ 1. Simulations with this model give $\chi = 0.51 \pm 0.01$ and $\beta = 0.33 \pm 0.01$, thereby indicating that this local rule also is in the KPZ universality class, as is expected from its symmetries. Modifications of this local rule by choosing the site $x = 1, 2, \ldots, L$ with a chance proportional to a local $\eta(x, h) \in [0, 1]$ does not alter the KPZ behavior.

(2) The behavior of the interface roughening can be very sensitive to whether the constraint of the slopes acts before (model A) or after (model B) the motion takes place. Nearly all experiments with self-affine structures

have χ exponents in between the ones predicted by these two rules.

(3) There are severe limitations on the applicability of the presented growth models. First of all, they simplify the global equilibrium by assuming that the chance to advance only depends on pinning forces, but is completely independent of the overall shape of the interface. In this way they will not apply to situations where screening can occur (e.g., where the Mullins-Serkerka instability [17] is important). Second, as they assume that pinning forces are compared globally before movement takes place, the models are valid only for experimental situations where the time to decide which site advances next is much smaller than the time of the actual movement. If, in reverse, one can only compare over a small restricted region of neighbors, then on large scales the behavior will be governed by local rules. Experiments might turn out to be in the crossover regime between the conditions giving global equilibrium and the conditions of completely local dynamics.

In conclusion, we have presented two especially simple members of a new class of growth models, which exhibits self-affine scalings in the no-man's land between the scale invariance observed in locally driven models and the understanding of scale invariance as a signal of self-organized criticality.

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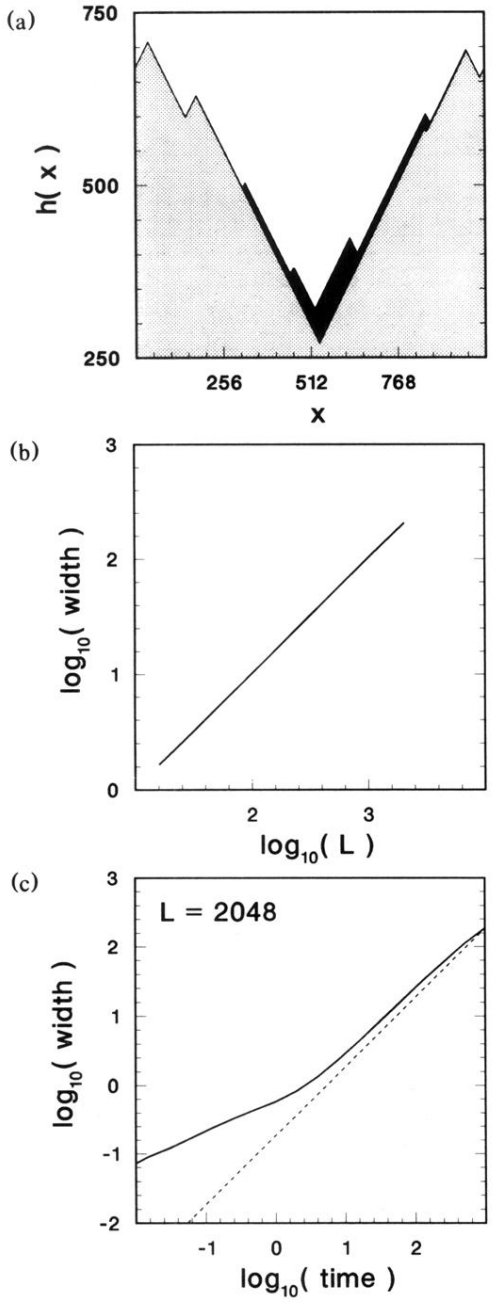

FIG. 1. Model A. (a) Typical snapshots of interfaces at two configurations at saturation. The dark areas show the activity between the respective stages. (b) Saturated width as a function of system size L for periodic boundary conditions, sampled at saturation. (c) Width as a function of time t. Time is measured from flat state, in units of increased heights per site. The dashed line has slope 1.

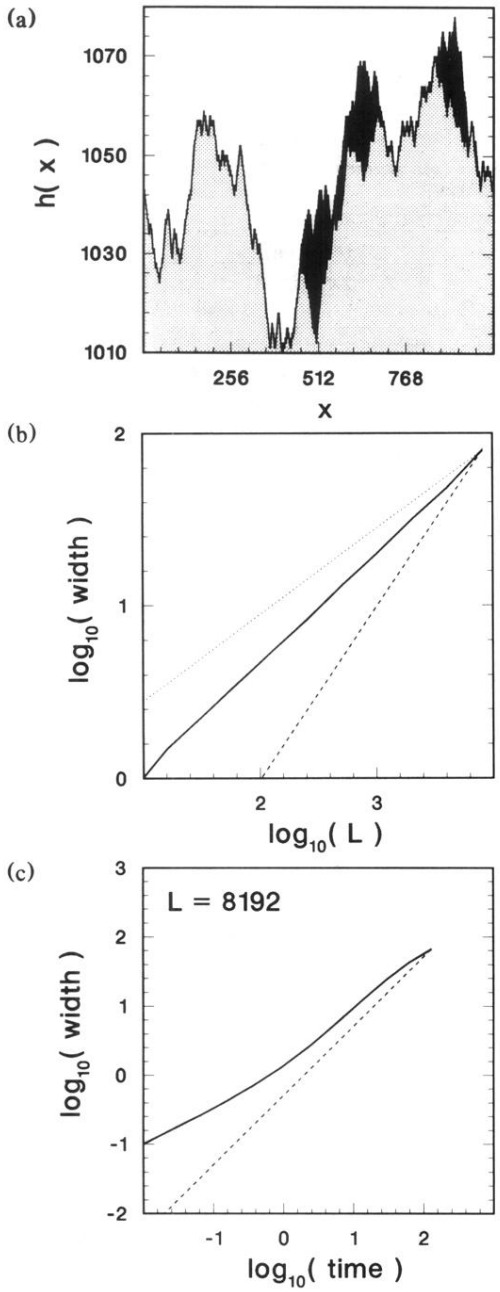

FIG. 2. Model B. (a) Typical snapshots of interfaces for two configurations at saturation. The dark areas show the activity between the respective stages. (b) Saturated width as function of system size L. (c) Width as a function of time. Time is measured from flat state, in units of increased heights per site. The dashed line has slope 1.