Collapse of Minibands in Far-Infrared Irradiated Superlattices

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Quasienergy minibands in superlattices which interact with intense far-infrared laser radiation collapse under conditions that are experimentally accessible. In particular, it is shown that the miniband width becomes close to, but not identical to, zero if the ratio of the Bloch frequency and the laser frequency approaches a zero of the Bessel function J_0.

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A periodic "superlattice" results when the alloy composition in a compound semiconductor such as $Al_xGa_{1-x}As$ is varied periodically along one dimension. The technique of computer-controlled molecular-beam epitaxy allows one to fabricate superlattices of high quality with a period d that is typically of the order of 100 Å. Such a larger period in space implies that the allowed energy "minibands" are very narrow; they can have a width of only a few meV.

An essential feature of narrow-band transport is nonlinear response to an applied electric field. Indeed, the seminal paper by Esaki and Tsu [1] on superlattice transport was motivated by the possibility of nonlinear dc transport due to Bloch oscillations. Some of the qualitative predictions are now beginning to emerge [2–4]. Furthermore, models of nonlinear optical response have been developed [5]; particularly important is the response to strong electric ac fields in the nonperturbative regime.

To date, treatments of strong ac field response have been carried out in the quasiclassical limit [6,7]. But in physically realizable systems the driving frequency and/or the harmonics created by the strong fields have frequencies which exceed the miniband widths. Therefore, a realistic model of a strongly driven superlattice must be solved quantum mechanically. The purpose of this Letter is to present a quantum-mechanical discussion of the nonlinear response of superlattices employing a method that is valid for wide ranges of frequencies, field strengths, and periods; the only assumption that will be made is that excitations to higher minibands can be neglected.

The approach that is used here to investigate the quantum mechanics of laser-driven superlattices is particularly well suited for the analysis of nonperturbative effects of strong ac fields. It emphasizes primarily the temporal periodicity of the driving laser field, rather than the spatial periodicity of the superlattice. Periodicity in time leads to a formulation in terms of quasienergy eigenvalues, and due to the periodicity in space the quasienergies for the allowed quantum states group together in minibands.

A simple one-dimensional model potential $V_{SL}(x)$ allows the numerical investigation of the behavior of quasienergy minibands. It describes an array of N identical square quantum wells of width w_w that are separated by rectangular barriers of width w_b and height v. The superlattice period is $d=w_w+w_b$, but the potential extends only over a finite range: At $x_{\min}=-Nd/2$ and $x_{\max}=+Nd/2$ there are infinitely high walls which force the wave functions to vanish at these boundaries. For the parameters $w_w=100$ Å, $w_b=40$ Å, $v=0.3$ eV, a mass $m=0.066m_e$ (the effective electron mass in bulk GaAs), and $N=50$ wells, the lowest-energy miniband extends from 32.82 to 36.49 meV, and the first excited miniband from 125.4 to 144.4 meV.

If a superlattice is exposed to far-infrared laser radiation, the laser wavelength is much longer than the whole sample and, therefore, does not introduce a new length scale into the problem. Such a situation can be modeled by the Hamiltonian ($\hbar=1$)

$$H(x,t)=-\frac{1}{2m}\frac{\partial^2}{\partial x^2}+V_{SL}(x)-eFx\sin\omega t\,; \qquad (1)$$

F denotes the strength and ω the frequency of the laser field. The Hamiltonian (1) is periodic in time: $H(x,t)=H(x,t+T)$, where $T=2\pi/\omega$ is the length of an optical cycle. From this property it follows that there is a complete set of Floquet wave functions [8–11] as solutions of the Schrödinger equation:

$$\psi_\varepsilon(x,t)=\exp(-i\varepsilon t)u_\varepsilon(x,t) \qquad (2)$$

with "quasienergies" ε and T-periodic functions $u_\varepsilon(x,t)=u_\varepsilon(x,t+T)$. Exactly as a quasimomentum is defined only up to an integer multiple of the reciprocal-lattice vector $2\pi/d$, a quasienergy can only be determined up to an integer multiple of the photon energy $2\pi/T=\omega$. There is also a Brillouin-zone scheme for quasienergies, the width of one zone being ω. For vanishing field strength, the quasienergies are identical to the energies of the undriven system, modulo ω. Generally speaking, the Floquet states play the role of stationary states in periodically time-dependent quantum systems.

In the case under consideration, the restriction to far-infrared frequencies also means that the photon energy ω is much smaller than the gap between the allowed energy minibands; interband transitions require multiphoton processes of very high order. Therefore, only the dynamics in the lowest miniband will be studied in the following.

Before turning to the numerical results, an intuitive argument will be useful. Assuming the energy-quasimomentum relation $E(k)=\varepsilon_\infty-\Delta\cos(kd)/2$ for the undriven superlattice (ε_∞ is the center of the unperturbed energy band and Δ its width) and a homogeneous electric field $\mathcal{E}(t)=F\sin\omega t$, the group velocity of a wave packet which is centered around $k=k_0$ at $t=T/4$ is given by

$$v(t)=\frac{\Delta d}{2}\sin\left[k_0d+\frac{eFd}{\omega}\cos\omega t\right]. \tag{3}$$

Thus, the velocity averaged over one laser period is

$$\bar{v}=\frac{\Delta d}{2}\sin(k_0d)J_0\left[\frac{eFd}{\omega}\right], \tag{4}$$

where J_0 is the zeroth-order Bessel function. Hence, if the ratio $y=eFd/\omega$ of the Bloch frequency $\Omega=eFd$ and the laser frequency ω is equal to a zero of J_0, the average electron velocity vanishes for every initial quasimomentum k_0 and the wave packet becomes "localized." It is remarkable that J_0 simply appears as a multiplicative factor of the group velocity $\Delta d\sin(k_0d)/2$ in the unperturbed superlattice. For this reason, the Bessel function J_0 also dominates the quasiclassical results [6,7].

The average velocity $\bar{v}$ is a measure of the extent to which the individual wells "communicate"; another measure is the band width. It is, therefore, natural to assume that the width of the quasienergy minibands is affected by the laser field in the same way as $\bar{v}$, and a detailed analysis [12] shows that this is indeed the case. If the original energy eigenvalues for the lowest miniband of the undriven superlattice are given by

$$E_n=\varepsilon_\infty-\frac{\Delta}{2}\cos\left[\frac{n\pi}{N+1}\right],\quad n=1,\ldots,N, \tag{5}$$

a quantum-mechanical calculation which neglects all finite-size effects yields an approximate expression for the quasienergies ε_n that originate from them [12]:

$$\varepsilon_n=\varepsilon_\infty-\frac{\Delta}{2}J_0\left[\frac{eFd}{\omega}\right]\cos\left[\frac{n\pi}{N+1}\right]\operatorname{mod}(\omega). \tag{6}$$

As with $\bar{v}$, the width of the quasienergy miniband should become zero when $y=eFd/\omega$ is equal to a zero of J_0.

The exact Floquet states and quasienergies have been calculated numerically for the model system (1). Figure 1 shows the lowest miniband of quasienergies for $\omega=5.0$ MeV, plotted versus the laser field strength F. The frequency ω is larger than the original band width $\Delta=3.67$ meV of the unperturbed superlattice, and therefore the miniband fits completely into the first quasienergy Brillouin zone. Because of finite-size effects, a pair of almost degenerate edge states splits off from the top of the quasienergy miniband, but apart from these two edge states the agreement with the approximate formula (6) is almost complete. For the superlattice period $d=140$ Å, the parameter $y=eFd/\omega$ becomes equal to the first zero of J_0 at $F=8589$ V/cm, precisely where the miniband is seen to collapse.

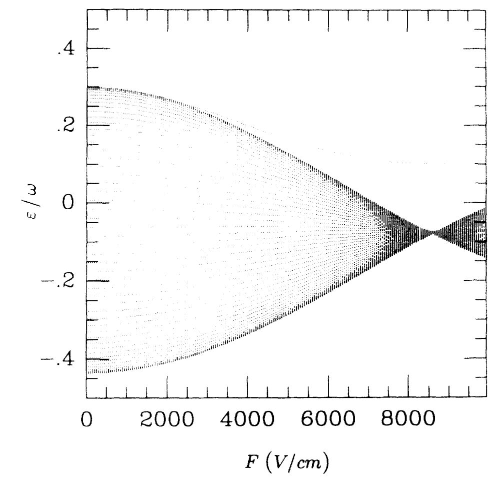

FIG. 1. The lowest miniband of quasienergies for the model potential $V_{SL}(x)$ with $N=50$ wells and a superlattice period of $d=140$ Å, plotted vs the strength F of the laser field. The vertical axis is the first quasienergy Brillouin zone, which extends from $\varepsilon/\omega=-1/2$ to $\varepsilon/\omega=+1/2$. The laser frequency is $\omega=5.0$ meV. The predicted field strength for the band collapse is $F=8589$ V/cm.

A more involved example is shown in Fig. 2: Now the laser frequency $\omega=1.0$ meV is more than 3 times smaller than Δ, which means that for small field strength the miniband has to overlap more than 3 times with itself in the Brillouin zone. Again, there are edge states which behave differently from the rest of the miniband states, but the overall agreement with Eq. (6) is quite good and, as the next figure illustrates, the band collapses at the predicted parameters.

Figure 3 shows a plot of quasienergies calculated from the approximate equation (6) for the parameters of the

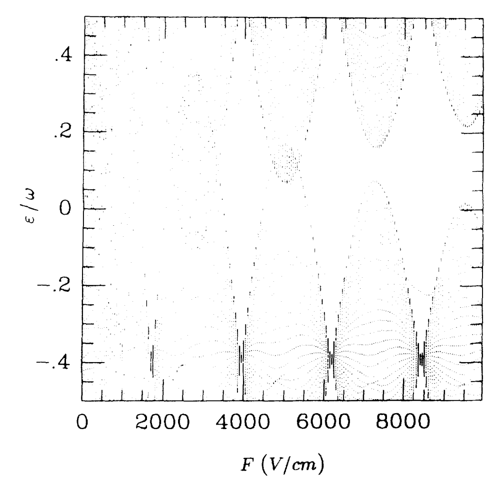

FIG. 2. The quasienergy miniband for $\omega=1.0$ MeV; all other parameters are the same as in Fig. 1.

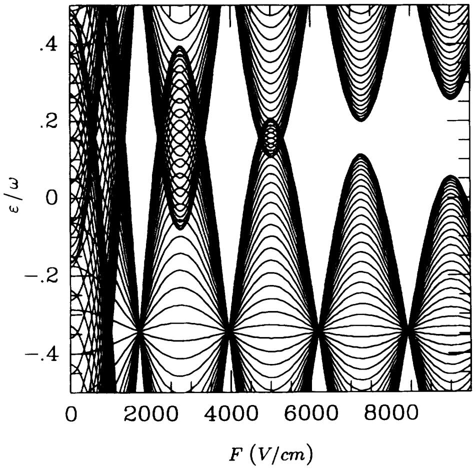

FIG. 3. Approximate quasienergies according to Eq. (6) for $\omega = 1.0$ meV.

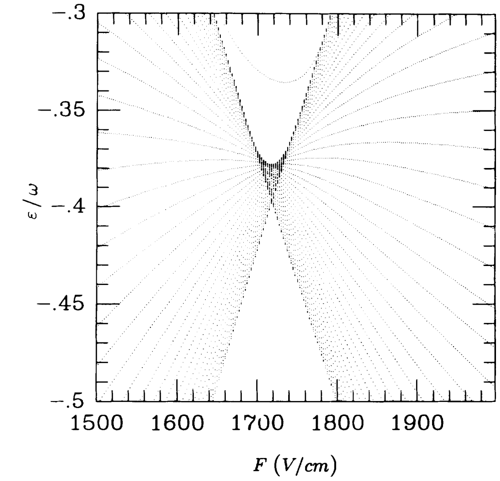

FIG. 4. Quasienergies in the vicinity of the first miniband collapse seen in Fig. 2.

previous example. There are several features of the exact quasienergies which are not accounted for by the approximation. First, the edge states on top of the miniband push the rest of the band slightly down so that the collapse value ε_∞ in a finite superlattice is lower than the center of the original band, modulo ω [12]. More important is the fact that the approximate quasienergies cross in the zone of self-overlap, whereas there are avoided crossings of the exact quasienergies. The model potential $V_{\mathrm{SL}}(x)$ is symmetric, $V_{\mathrm{SL}}(x) = V_{\mathrm{SL}}(-x)$, which implies that the Hamiltonian (1) remains invariant under the combined operation S_P: $x \rightarrow -x$ and $t \rightarrow t + T/2$. The Floquet wave functions $u_\varepsilon(x,t)$ have odd or even parity under S_P, and according to the von Neumann–Wigner noncrossing rule [13] eigenvalues belonging to functions of the same symmetry class in general do not cross each other if only one parameter is varied, as is the field strength F in the numerical calculations. The resulting avoided quasienergy crossings can, however, hardly be discerned in Fig. 2. The multiple self-overlap in the low-field region leads to a very intricate level pattern that can only be resolved on a much finer scale. Needless to say, the appearance of avoided crossings reflects the possibility of strong laser-induced intraband transitions [8].

Another consequence of the noncrossing rule is that even for an ideal finite superlattice the collapse is imperfect. The approximate equation (6) predicts a total degeneracy of all miniband quasienergies at the zeros of J_0, but the exact quasienergies within one class of the extended parity S_P will repel each other also at these parameters. This fact is illustrated in Fig. 4, which shows a magnification of the quasienergy spectrum in the vicinity of the first collapse seen in Fig. 2. Equation (6) predicts a collapse field strength of 1718 V/cm, in striking agreement with the numerical result, but it is clearly visible that the miniband maintains a finite width.

Finally, Fig. 5 shows the probability density of a Floquet state in the model potential $V_{\mathrm{SL}}(x)$ for $\omega = 1.0$ meV and $F = 1718$ V/cm, the parameters for the first collapse in Fig. 2. The displayed interval between $x = -1000$ and $+1000$ Å contains fourteen wells. During one laser cycle there is a sloshing motion right through the barriers that couples several wells, in agreement with what follows from Eq. (3). Thus, it can be seen that a vanishing quasienergy miniband width does not imply that the current between the individual wells also vanishes; it is only the time-averaged current that goes to zero.

Obviously, the main deficiency of the ideal model discussed so far is that it does not contain any scattering. A conservative estimate for the scattering time τ in a superlattice is $\tau \approx 5 \times 10^{-13}$ sec. Even for the laser frequency

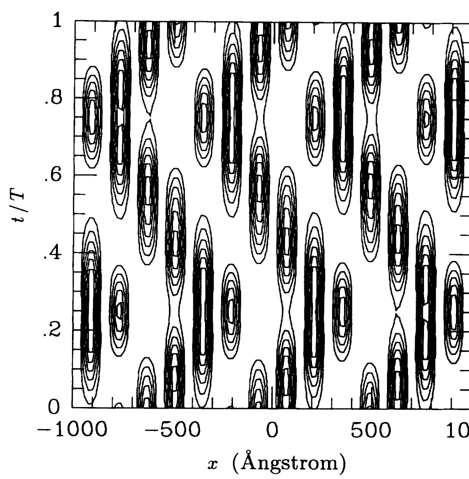

FIG. 5. A Floquet state in the model potential $V_{\mathrm{SL}}(x)$ at $\omega = 1.0$ meV and $F = 1718$ V/cm, corresponding to the first collapse in Fig. 2. Lines connect points of equal probability density $|u(x,t)|^2$. There are fourteen wells between $x = -1000$ and $+1000$ Å.

$\omega = 5.0$ meV this gives $\omega\tau \approx 4$; of course, for higher frequencies the product $\omega\tau$ becomes still larger. This means that scattering will not completely blur the miniband collapse. In an actual experiment, a compromise must be made between high frequencies, attainable field strengths, and convenient superlattice periods. It is also interesting to note that the collapse can even be found in superlattices with only a small number of wells [12].

To conclude, it has been shown that the width of a quasienergy miniband in a far-infrared irradiated superlattice can efficiently be controlled by the strength and the frequency of the driving laser field; the minibands collapse almost completely if the ratio of the Bloch frequency $\Omega = eFd$ and the laser frequency ω becomes equal to a zero of the Bessel function J_0. This collapse is the result of an interplay of the spatial periodicity of the superlattice and the temporal periodicity of the external laser field. Both the periodic structure in space and that in time are man-made and can be manipulated by the experimentalist; indications for the miniband collapse can possibly be found by measuring the electronic transport properties of a superlattice in the presence of the driving field. Experiments of this type, which might eventually open up a new line of semiconductor research, are presently being prepared [14] at the free-electron-laser facility of the University of California at Santa Barbara.

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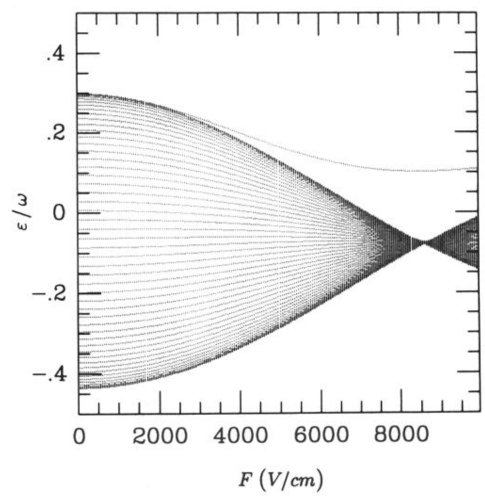

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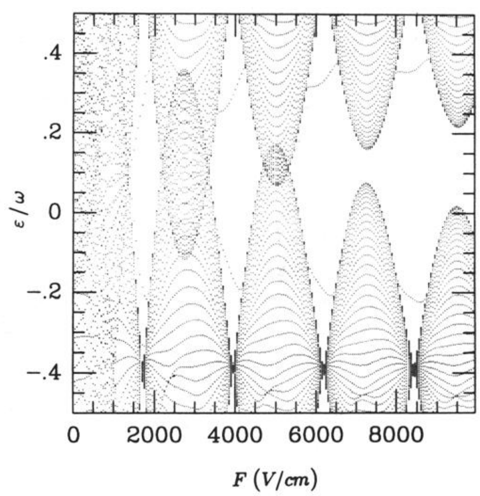

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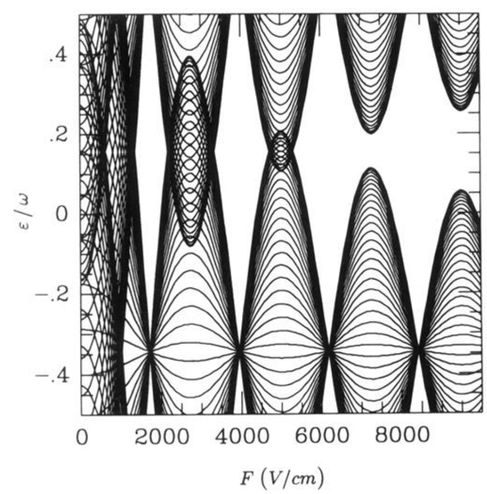

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