Ponderomotive Force of a Uniform Electromagnetic Wave in a Time Varying Dielectric Medium

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A ponderomotive force associated with a *uniform* electromagnetic wave propagating in a medium with time varying dielectric properties [e.g., $\epsilon=\epsilon(x-v_0t)$] is identified. In particular, when a laser ionizes a gas through which it propagates, a force is exerted on the medium at the ionization front that is proportional to $(\nabla\epsilon)E^2$ rather than the usual $(\epsilon-1)\nabla E^2$. This force excites a wake in the plasma medium behind the ionization front. The ponderomotive force and wake amplitude are derived and tested with 1D particle-in-cell simulations.

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The ponderomotive force of an electromagnetic wave is responsible for such diverse phenomena as radiation pressure, parametric instabilities in plasmas [1,2], "optical tweezers" for confining and manipulating particles, self-focusing of electromagnetic waves in media and in plasmas [3], profile steepening in plasmas [4], and laser accelerator schemes [5]. An expression for the ponderomotive force can be derived either by computing the divergence of the Maxwell stress tensor for a dielectric medium or by considering the motion of single particles in the medium. By either method, the ponderomotive force is generally found to be proportional to the gradient in electromagnetic wave intensity. In this paper we identify the possibility of a ponderomotive force associated with a *uniform* electromagnetic wave in a medium with time varying dielectric properties. In particular, in a stationary medium such that the dielectric function is of the form $\epsilon=\epsilon(x-v_0t)$, we show that a large ponderomotive force is exerted on the medium even when there is no appreciable gradient in electromagnetic wave intensity. One way to realize this situation is with an intense laser that ionizes a gas through which it propagates.

Besides being of fundamental interest, the ponderomotive force associated with a uniform electromagnetic wave could have important consequences for current short pulse laser experiments. These include experiments to study ionization and harmonic generation in gases [6,7], recombination x-ray lasers [8], and plasma accelerators [5]. We show that the new ponderomotive force can produce a wake field in the plasma behind the ionization front created by the laser. For plasma accelerator applications this wake excitation is an attractive means of overcoming technological barriers to demonstrating a near term laser wake-field accelerator. In particular, the need for ultrashort laser pulses (to provide a gradient in intensity and hence a ponderomotive force) is eliminated.

In this Letter we first review the ponderomotive force resulting from a laser pulse propagating (in the x direction) in a static medium with a constant dielectric function and from a laser pulse moving in a dielectric with a spatial gradient. We calculate the force from both the macroscopic and single-particle perspectives to show that it can always be expressed as a gradient in laser intensity. We then use a single-particle picture to calculate a ponderomotive force in a medium where the density depends on the variable $x-v_0t$ and v_0 is near c; we show that in this case the force depends on the gradient in ϵ, not the gradient in laser intensity. The theoretical predictions for the wake amplitude are then compared to particle-in-cell simulations.

We begin with the conservation of momentum equation for fields and particles,

$$\frac{\partial}{\partial t}\frac{\mathbf{E}\times\mathbf{B}}{4\pi c}+\nabla\cdot\left[\frac{E^2+B^2}{8\pi}\mathbf{I}-\frac{\mathbf{EE}+\mathbf{BB}}{4\pi}\right]+n\mathbf{F}=0\,, \quad (1)$$

where n is the density of free charges, $\mathbf{F}$ is the force on a free charge, and $\mathbf{I}$ is the unit dyadic. For simplicity we consider a medium in which there are no bound charges, i.e., a plasma. So $\mathbf{E}$ and $\mathbf{B}$ rather than $\mathbf{D}$ and $\mathbf{H}$ appear in Eq. (1). However, the free plasma particles affect the relationship between $\mathbf{B}$ and $\mathbf{E}$. For plane waves their relation follows simply from Faraday's law, giving

$$B=(ck/\omega)E=\sqrt{\epsilon}E\,, \quad (2)$$

where ϵ is the dielectric function for the free particles. Substituting into Eq. (1) gives

$$\frac{\partial}{\partial t}\left[\frac{\sqrt{\epsilon}E^2}{4\pi c}\right]+\frac{\partial}{\partial x}\left[(\epsilon+1)\frac{E^2}{8\pi}\right]+nF_x=0\,. \quad (3)$$

We now apply Eq. (3) to two specific cases: an electromagnetic pulse in a uniform plasma and an infinite plane wave propagating along a density gradient. Consider an electromagnetic pulse propagating through plasma. The pulse moves at the group velocity of light, $v_g=c^2k/\omega=c\sqrt{\epsilon}$. Therefore, the envelope depends on the variable $x-v_gt=x-c\sqrt{\epsilon}t$ from which it follows that $\partial/\partial t=-c\sqrt{\epsilon}\,\partial/\partial x$. Upon making this substitution Eq. (3) reduces to

$$-c\sqrt{\epsilon}\frac{\partial}{\partial x}\frac{\sqrt{\epsilon}\langle E^2\rangle}{4\pi c}+\frac{\partial}{\partial x}\frac{(\epsilon+1)\langle E^2\rangle}{8\pi}+nF_x=0\,, \quad (4)$$

where $\langle\ \rangle$ refers to averaging over the high-frequency oscillations. If $\partial\epsilon/\partial x=0$ then we obtain

$$(\epsilon-1)\frac{\partial}{\partial x}\frac{\langle E^2\rangle}{8\pi}=nF_x\,, \tag{5}$$

which is the usual expression for the ponderomotive force [9].

We next consider the steady-state situation of radiation propagating up (or down) an underdense density gradient. In this case $\partial/\partial t\rightarrow 0$ and the spatial envelope for E is given by the WKB solution

$$E=E(x)\cos\left[\int dx\,k(x)-\omega t\right], \tag{6}$$

where $k(x)=\sqrt{\epsilon}\omega_0/c$, $E(x,t)=E_0/\epsilon^{1/4}$, and E_0 is the incident electric field. Substituting Eq. (6) into Eq. (3) and letting $\partial/\partial t\rightarrow 0$ gives

$$\frac{E_0^2}{16\pi}\frac{1}{2}\left\{\frac{\epsilon-1}{\epsilon^{3/2}}\right\}\frac{\partial\epsilon}{\partial x}+nF_x=0\,. \tag{7}$$

This can be rewritten as

$$(\epsilon-1)\frac{\partial}{\partial x}\frac{\langle E^2\rangle}{8\pi}=nF_x\,, \tag{8}$$

where we have substituted $\langle E^2\rangle/8\pi$ for $(E_0^2/16\pi)/\epsilon^{1/2}$. We once again recover the usual formula for the ponderomotive force, expressed as a gradient in laser intensity. The spatial gradient in ϵ affects the force implicitly by causing a gradient in E^2.

These points can be seen clearly if we recall the derivation of the force from a single-particle picture. The only force in the x direction on a particle with charge q and mass m is the $q\mathbf{v}\times\mathbf{B}/c$ force. We use the expression in Eq. (6) where the envelope $E(x)$ and the wave number $k(x)$ can be arbitrary. Solving the linearized equation of motion for $\mathbf{v}$ and using Faraday's law to calculate $\mathbf{B}$, the $q\mathbf{v}\times\mathbf{B}/c$ force is

$$F_q=-\frac{q^2}{m\omega_0^2}\frac{1}{2}E(x)\frac{\partial}{\partial x}E(x)\,.$$

For electrons of density N, this can be rewritten as

$$nF_e=-\frac{\omega_p^2}{\omega_0^2}\frac{\partial}{\partial x}\frac{\langle E^2\rangle}{8\pi}=(\epsilon-1)\frac{\partial}{\partial x}\frac{\langle E^2\rangle}{8\pi}\,, \tag{9}$$

where $\omega_p^2=4\pi ne^2/m$. Therefore, we find explicitly that the usual ponderomotive force exists only when there is a gradient in electric field intensity. This gradient could be the result of a pulse envelope, a gradient in dielectric properties, or simple reflection (as in radiation pressure on a mirror).

Next we consider the ponderomotive force for the special case of a light wave propagating along with a comoving ionization front. In this case E^2 is approximately constant, but ϵ is not. The ionization may or may not be created by the light wave itself. To determine the force exerted at the ionization front we return to the single-particle picture. The force follows by considering the well-known constants of motion for particles in a plane electromagnetic plane wave [10]:

$$p_\perp-\frac{eA}{c}=\frac{-eA(0)}{c}\,, \tag{10}$$

$$p_\parallel-\gamma mc=-mc\,, \tag{11}$$

where $A(0)$ is the value of the vector potential at the time when the electron is born. The first equation follows from conservation of the transverse canonical momentum and the second follows from subtracting the energy equation from the parallel equation of motion. The constants on the right-hand side follow from the assumption that the electron's momentum is zero at the instant it is first ionized. Combining (10) and (11) yields

$$p_\parallel=\frac{e^2}{2mc^3}[A-A(0)]^2=\frac{e^2A_0^2}{2mc^3}[\sin^2(\omega t-kx+\phi)+\sin^2\phi-2\sin(\omega t-kx+\phi)\sin\phi]\,, \tag{12}$$

where we have assumed the electromagnetic wave-vector potential to be of the form $A=A_0\sin(\omega t-kx+\phi)$ and that ionization occurs when the phase of A is ϕ.

From (12) we see that each successive ionization will add a time-averaged nonzero momentum to the fluid. The ponderomotive force on the fluid follows from the rate of momentum increase:

$$nF=\left\langle\frac{d}{dt}np_\parallel\right\rangle\approx\frac{\partial n}{\partial t}\langle p_\parallel\rangle\,. \tag{13}$$

Here we have neglected the convection derivative, valid for $eA/mc^2<1$. Taking the time average of (12) and substituting $\epsilon-1=-4\pi ne^2/m\omega^2$ gives

$$nF=-\left[\frac{\partial}{\partial t}(\epsilon-1)\right]\frac{\langle E^2\rangle}{8\pi c}(1+2\sin^2\phi)\,. \tag{14}$$

This expression is valid whether the ionization is created by a moving front or by flash ionization everywhere in space. When the ionization is produced by a front of velocity $v_0\approx c$, we can approximate $\partial/\partial t\approx -c\,\partial/\partial x$ to obtain

$$nF=\left[\frac{\partial}{\partial x}(\epsilon-1)\right]\frac{\langle E^2\rangle}{8\pi}(1+2\sin^2\phi)\,. \tag{15}$$

This is the ponderomotive force on the plasma arising from the ionization front. In this case the force results from the gradient in ϵ rather than E^2. Equation (15) is analogous to Eqs. (5) and (8) except for the term $1+2\sin^2\phi$, which depends on the phase ϕ of the electric field at the time that an electron is born. If electrons are born only at the peak of the electromagnetic wave field, then $\phi=0$ and the ponderomotive force is simply $[(\partial/\partial x)(\epsilon-1)]\langle E^2\rangle/8\pi$ as one might have guessed from heuristic arguments. However, if electrons are born at other phases of **E**, then the ponderomotive force is increased by the factor $1+2\sin^2\phi$. We emphasize that this force is different from the usual ponderomotive force in that free electrons do not continue to accelerate in response to it. Rather the force represents a local increase in fluid momentum due to the addition of new particles with identical parallel momentum.

To make connection with the usual ponderomotive force we consider an isolated electron starting from rest which is overtaken by a laser pulse. The electron will obtain a parallel drift proportional to the laser's instantaneous intensity [Eq. (12) with $\phi=0$]. Therefore, it takes the entire rise time to obtain the peak drift momentum. In a plasma the electron is prevented from drifting by the space charge of the ions unless the laser pulse is shorter than a plasma period. However, when the electron is born inside the laser pulse it acquires the drift associated with the laser's local intensity within a single laser cycle; i.e., it responds as though the laser had an instantaneous rise. This effect has important consequences for the ionization of atoms. Specifically, it will push the electron away from the atom immediately after ionization and may prevent stabilization that has been predicted for high-intensity lasers [11].

In order to prove that this ponderomotive force is real we demonstrate that it will produce a wake just as the ponderomotive force from a short laser pulse does. The force at the ionization front will displace plasma electrons relative to plasma ions. The displaced electrons experience a restoring force due to the ion space charge. The resulting electron oscillation supports a plasma wake at frequency ω_p and phase velocity $c\sqrt{\epsilon}$.

The amplitude of the wake is easily obtained from a Lagrangian description. The initial velocity of a Lagrangian fluid element is found from (12):

$$v_{\parallel 0}=\tfrac{1}{4}A^2(1+2\sin^2\phi)\,, \tag{16}$$

where $A\equiv eE_0/m\omega c$ is the normalized vector potential amplitude of the light wave, and we have assumed $\omega \gg \omega_p$ so that we could average Eq. (12) over a laser cycle. If $v_{\parallel 0}\ll c$ then the fluid element satisfies

$$\ddot{\xi}+\omega_p^2\xi=0\,, \tag{17}$$

where ξ is the displacement of the fluid element and $\dot{\xi}(t=-x/v_0)=v_{\parallel 0}$ is from Eq. (15). Solving for ξ and applying $E_w(x,t)=4\pi e n_0\xi(x,t)$ gives for the wake electric field $E_w(x,t)$:

$$eE_w/m\omega_p c=\tfrac{1}{4}A^2(1+2\sin^2\phi)\sin\omega_p(t-x/v_0)\,. \tag{18}$$

This is the plasma wake field caused by the ponderomotive force of a uniform light wave that ionizes a gas at position $x=v_0t$. Note that the wake amplitude depends on the phase ϕ of the laser when the ionization occurs.

If the electrons are born at the peak of a laser oscillation ($\phi=0$), then the wake amplitude is $|eE_w/m\omega_p c| =A^2/4$. This is exactly the wake amplitude that would be excited by a square light pulse (of amplitude A) in a uniform plasma. If the electrons are born at other phases of the laser, even larger wakes result.

To verify the predictions of these simple models we have performed simulations of the wake excited by a uniform light wave and a moving front. Sample simulation results are shown in Fig. 1. A laser propagates to the right at frequency $\omega=5\omega_p$ as shown in Fig. 1(a). The laser self-consistently ionizes a gas at a rate given by the Keldysh formula producing the plasma (ion) density profile shown in Fig. 1(b). The ionization is very rapid once the laser exceeds an approximate ionization threshold ($eE/m\omega_0 c\approx 0.1$). The resulting plasma wake potentials are shown in Figs. 1(c) and 1(d) for peak laser fields $eE/m\omega_0 c=0.15$ and 0.3, respectively. The wake amplitudes agree with the scalings predicted by Eq. (18) for $\sin^2\phi\sim\frac{1}{2}$. We have also obtained similar results from simulations in which a uniform light wave is initialized and an ionization front moving at $v_0\sim v_g$ is imposed.

We comment that plasma temperatures in the range of 0 to 2.5 keV were modeled with no apparent difference in the wakes produced. In these simulations, the ionization took place very rapidly. In order for a wake to be excited, the ionization must take place on a time scale shorter than ω_p^{-1}. This is analogous to the need for a short rise time laser to excite a wake with the conventional ponderomotive force.

The wake produced by the mechanism of this Letter may be important in near term laser experiments in several ways. First, for high-ionization-potential gases such as He, wakes of order 100 MV/m can be excited and may be a convenient means of studying future plasma-based concepts for accelerating particles. Second, the

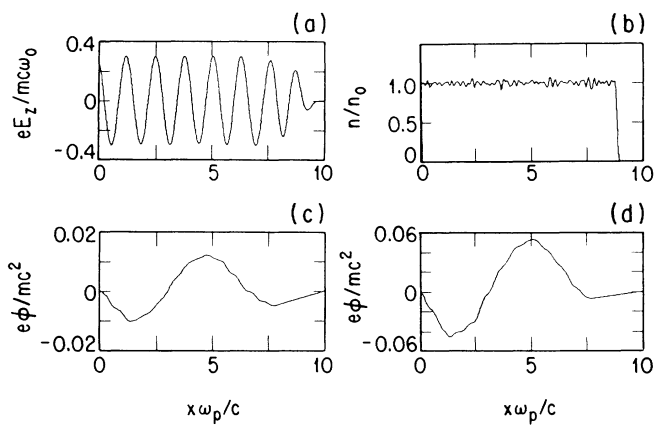

FIG. 1. 1D particle-in-cell simulations of wakes excited by the ponderomotive force at the moving interface between a gaseous and plasma medium. (a) Laser field $eE/m\omega_0 c$, (b) plasma ion density, and electrostatic wake potential $e\phi/mc^2$ for peak laser fields (c) $eE/m\omega_0 c=0.15$ and (d) 0.30.

wake created, though itself small, may seed the growth of the Raman forward scatter instability of the laser. Third, the scattering of the laser on the wake can produce Stokes and anti-Stokes sidebands of the laser at $\omega \pm n\omega_p$. The sideband structure may be quite different from that characteristically arising from Raman forward scatter in a preformed plasma. In particular, it may favor upshifts $(\omega + n\omega_p)$. We have seen this in the laser spectra of our simulations. Fourth, the density response associated with the last term in Eq. (12) beating with the transverse quiver oscillation will lead to the generation of even harmonics of the laser not usually present in preformed plasmas. Finally, we note that multiply ionized atoms can produce a new wake each time the laser amplitude exceeds the threshold for a succeeding ionization (for plasma densities low enough that collisional ionization does not occur). By the time the final electron is stripped, the laser amplitude may be quite high and the wake correspondingly large.

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