Identical Bands at Normal Deformation: Criteria and Challenges

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Identical rotational bands (i.e., those with surprisingly similar moments of inertia) have been proposed through several analyses of known bands in normally deformed nuclei. In this paper, we analyze bands in odd-proton nuclei ($A=150$–190) with criteria more stringent than those of previous approaches. The identical bands are found to be associated with up-sloping particle states, suggesting that the cause may be a cancellation between pairing and deformation decreases.

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It has been known since 1958 [1,2] that pairing correlations play a substantial role in the collective motion of nuclei. In odd-A nuclei, the pairing correlation is reduced because of the blocking effect of the odd nucleon, which in turn increases the moment of inertia of the rotating nucleus. In general, the effective moments of inertia of odd-A nuclei exceed those of adjacent even-even nuclei by roughly 20%, although there are large variations, as summarized by Bohr and Mottelson [3]. This "effective" moment of inertia is simply the ratio of angular momentum (I) to rotational frequency (ω), which is now generally called the kinematic moment of inertia, $\mathcal{J}^{(1)}$ [4,5], and is hereafter called "moment of inertia" (unless specified otherwise) for simplicity in this paper. However, it has recently been found that there exist in nuclei of normal deformations "identical bands" which exhibit differences in moments of inertia more than an order of magnitude less than the typical value mentioned above, either between nearby even-even nuclei [6–8] or, especially, between odd-A nuclei and their even-even neighbors [9–12].

While the observation of superdeformed rotational bands was met by a significant degree of theoretical understanding, the discovery of identical bands, at both superdeformed and normal shapes, has been a real surprise. That is, the occurrence of rotational bands based on different configurations in different nuclei but exhibiting such identical moments of inertia presents real challenges to contemporary nuclear models. And it is especially difficult to explain the fact that identical bands are found at normal values of deformation, where pairing effects are known to be strong. It is the intent of this paper to refine the definition of identical bands and demonstrate that this phenomenon occurs for very special single-particle orbitals.

Several ideas have been proposed to help understand the identical-band phenomenon at normal deformation, such as a need [9,11] for modification of the traditional theory of the pairing-blocking effect [3] in odd-A nuclei, the N_p-N_n scheme of effective proton-neutron interaction [7], and the cancellation of the various terms associated with the contribution of the odd nucleon to the moment of inertia [12]. However, there is no generally accepted explanation yet, especially since there is no theory which can calculate the moment of inertia to an accuracy as high as that observed in the identical-band phenomenon.

Different groups [6–12] have used different criteria for defining the existence of an identical band. It is very important to properly define crucial criteria, so that the least ambiguity is included in the identification of the identical bands. The requirements for such criteria are the following. (a) The quantities compared should have either the same angular momentum (when the γ-ray energies E_γ are compared) or the same E_γ values (when the angular momenta are compared). (b) The extracted quantities should be model independent. (c) There should be as little as possible interpolation in constructing the quantities to be compared, because of the high accuracy required in the comparison. (d) Averaging of experimental quantities should be avoided as much as possible, since this may dilute or even eliminate the difference. (e) There should be no extra factors involved in the construction of the criteria.

The judgment of the identical nature of rotational bands depends somewhat on whether the kinematic ($\mathcal{J}^{(1)}$) or dynamic ($\mathcal{J}^{(2)}$) moments of inertia of adjacent nuclei are compared. The former is defined to be the ratio of angular momentum to rotational frequency, the latter the ratio of the variations of these two quantities. A rotational band with an aligned angular momentum exhibits differing $\mathcal{J}^{(1)}$ and $\mathcal{J}^{(2)}$ values, which could give a different conclusion concerning the identical nature of this band relative to that in a neighbor. We present here an analysis based on both moments, with emphasis on $\mathcal{J}^{(1)}$, because the influence of pairing and deformation variations on $\mathcal{J}^{(1)}$ are clearer. The possibility of a cancellation between these factors giving rise to the identical condition is easier to assess in the $\mathcal{J}^{(1)}$ analysis.

According to the requirements discussed above, a direct and simple derivation of the relative deviation in the moment of inertia, $\delta\mathcal{J}^{(1)}$, between two adjacent nuclei follows:

$$\mathcal{J}^{(1)}=I/\omega\,, \tag{1}$$

$$\delta\mathcal{J}^{(1)}=\frac{\mathcal{J}^{(1)}(A_1)-\mathcal{J}^{(1)}(A_2)}{\mathcal{J}^{(1)}(A_1)} =\frac{I(A_1)-I(A_2)}{\omega(A_1)}\bigg/\frac{I(A_1)}{\omega(A_1)} =\frac{I(A_1)-I(A_2)}{I(A_1)}=\frac{i(A_1)}{I(A_1)}\,, \tag{2}$$

where A_1 and A_2 are the mass numbers for the odd-A and $A-1$ (even-even) nuclei, respectively. In order to compare the $I(A_2)$ with $I(A_1)$ at the same frequency $\omega(A_1)$ as required in the above list of guidelines, the value of $I(A_2)$ is obtained through a normal interpolation procedure using a Lagrange polynomial [13]. The numerical accuracy of the interpolation is guaranteed in this case by the fact that the states analyzed are all before the first band crossing, and therefore the frequency dependence of the spin for those states is monotonic and smooth. The relative variation in the moment of inertia between two adjacent nuclei is thus the ratio of the angular momentum difference, i, to the angular momentum of the odd-A nucleus. It is necessary to average this angular momentum difference over N different states in a band to give a total measure of the relative variation in the moment of inertia:

$$\delta\mathcal{J}^{(1)}_{\rm avg}(A_1)=\frac{1}{N}\sum_{n=1}^{N}\frac{|i_n(A_1)|}{I_n(A_1)}\,, \qquad (3)$$

where the index n is the label of the state. The relative difference in the moment of inertia due to the variation in mass number, $\delta\mathcal{J}_m$, is used as a unit to evaluate the relative deviation of $\mathcal{J}^{(1)}$ between two adjacent nuclei:

$$\delta\mathcal{J}_m(A_1)=\frac{A_1^{5/3}-A_2^{5/3}}{A_1^{5/3}}\,; \qquad (4)$$

$\delta\mathcal{J}_m$ is around 1% in the mass region discussed. One possible criterion is then

$$\Delta\mathcal{J}^{(1)}=\frac{\delta\mathcal{J}^{(1)}_{\rm avg}(A_1)}{\delta\mathcal{J}_m(A_1)}<1.0\,, \qquad (5)$$

which means that *we are defining an identical band as one where the average relative difference of the moment of inertia in two nuclei is less than* $\delta\mathcal{J}_m$ *[defined by Eqs. (2)-(4)].*

Using this criterion, we have compared approximately 160 rotational bands (all bands with not less than three rotational transitions before the first backbending or sharp upbending) previously known for odd-proton nuclei in the region $A=150$–190 and $Z=67$–77 with yrast bands of their corresponding $A-1$ even-even nuclei [14,15]. Ten identical bands are found to satisfy our definition and are listed in Table I, which shows the nucleus, the particular configuration (signature is labeled by $+$ and $-$), the number of transitions used in the odd-A nucleus ($N_{\rm odd\,tr}$), and the $\Delta\mathcal{J}^{(1)}$ value for each case. *It is clear that all of the identical bands found by our criterion correspond to the particle states with the occupation of up-sloping (with respect to quadrupole deformation) orbitals,* $[404]\frac{7}{2}$ *and* $[402]\frac{5}{2}$, *and occur mostly in the lutetium isotopes.*

Factors which influence the moment of inertia include pairing correlations, deformation (quadrupole, hexadecapole, triaxial, etc.), proton-neutron interaction, and quasiparticle alignment. It has long been known that in the traditional pairing theory [2,3] an odd nucleon always reduces the monopole pairing correlation via the blocking effect, consequently increasing the moment of inertia. It is reasonable to assume that to compensate or cancel this increase in the moment of inertia, there must be other factors which act to reduce $\mathcal{J}^{(1)}$ in these particular odd-A nuclei. One possibility is that the quadrupole deformation provides this compensating reduction of the moment of inertia. Of course, such a scenario is a necessary but not sufficient condition for observing identical bands, since the occurrence of identical bands requires a *precise* cancellation of the factors affecting the moment of inertia.

TABLE I. Identical bands in the $A=150$–190 region.

Configuration[a]	Nucleus	$N_{\rm odd\,tr}$	$\Delta\mathcal{J}^{(1)}$
$\pi[404]\frac{7}{2}^+$	^{169}Tm	3	0.900
$\pi[404]\frac{7}{2}^-$	^{169}Tm	3	0.868
$\pi[404]\frac{7}{2}^+$	^{171}Lu	3	0.817
$\pi[404]\frac{7}{2}^-$	^{171}Lu	3	0.948
$\pi[404]\frac{7}{2}^+$	^{175}Lu	5	0.743
$\pi[404]\frac{7}{2}^-$	^{175}Lu	4	0.825
$\pi[402]\frac{5}{2}^+$	^{177}Ta	3	0.390
$\pi[402]\frac{5}{2}^-$	^{177}Ta	3	0.424
$\pi[402]\frac{5}{2}^+$	^{183}Re	5	0.763
$\pi[402]\frac{5}{2}^-$	^{183}Re	4	0.995

[a]The signature of the particular band is denoted by $+$ or $-$.

The observation of identical bands among the specific orbitals detailed in Table I further supports such a simple cancellation picture. It seems that the effect of pairing reduction is compensated by the decrease of the quadrupole deformation from the polarization caused by the occupation of an up-sloping orbital, as discussed earlier with respect to the $A=130$ region [12].

The concentration of identical bands in lutetium nuclei may come from the fact that the proton Fermi surface for ytterbium ($Z=70$) lies right above the $[411]\frac{1}{2}$ orbital (which is flat with respect to quadrupole deformation), and those immediately above this are all up-sloping orbitals, precisely the ones for which the identical-band condition is found, i.e., $[404]\frac{7}{2}$ and $[402]\frac{5}{2}$.

To test qualitatively this cancellation picture, we have performed a model calculation. The equilibrium deformation was found for different configurations in ^{171}Lu and ^{170}Yb at $\omega=0$ through a potential energy surface calculation with a normal BCS plus Strutinsky method [16]. Using these deformations, we then performed a self-consistent pairing calculation with particle number projection [17] at rotational frequencies of 0 and 0.15 MeV. The values of the minimized pairing factors (Δ_{psc} and Δ_{nsc}) and deformations (ε_2 and ε_4) are shown in Table II. Compared with the ground band in ^{170}Yb, the quadrupole deformation of ^{171}Lu decreases for the configurations $[404]\frac{7}{2}$ and $[402]\frac{5}{2}$, but increases for $[541]\frac{1}{2}$, the down-sloping orbital. However, the proton pairing decreases for all configurations in ^{171}Lu. Consequently, for these up-sloping configurations, deformation reduces and therefore compensates the effect of the reduction of pairing on the moment of inertia, as men-

TABLE II. Results of self-consistent pairing calculations for ^{171}Lu and ^{170}Yb.

Configuration[a]	Nucleus	$\hbar\omega$ (MeV)	Δ_{psc} $(\hbar\omega_{0p})$[b]	Δ_{nsc} $(\hbar\omega_{0n})$[c]	ε_2	ε_4
Ground band	^{170}Yb	0.00	0.145	0.121	0.275	0.026
Ground band	^{170}Yb	0.15	0.142	0.116	0.275	0.026
$\pi[404]\frac{7}{2}^+$	^{171}Lu	0.00	0.122	0.122	0.265	0.026
$\pi[404]\frac{7}{2}^+$	^{171}Lu	0.15	0.118	0.118	0.265	0.026
$\pi[402]\frac{5}{2}^+$	^{171}Lu	0.00	0.131	0.122	0.265	0.024
$\pi[402]\frac{5}{2}^+$	^{171}Lu	0.15	0.127	0.118	0.265	0.024
$\pi[541]\frac{1}{2}^+$	^{171}Lu	0.00	0.127	0.120	0.285	0.024
$\pi[541]\frac{1}{2}^+$	^{171}Lu	0.15	0.123	0.116	0.285	0.024

[a]The signature of the particular band is denoted by + or −.
[b]$\hbar\omega_{0p} = (41/A^{1/3})[1-(Z-N)/3A]$ MeV.
[c]$\hbar\omega_{0n} = (41/A^{1/3})[1+(Z-N)/3A]$ MeV.

tioned above. In contrast, for the down-sloping configuration $[541]\frac{1}{2}$, both the reduction in pairing correlation and the increase in deformation result in a rise in the moment of inertia. Therefore, in general, compared to the yrast band in $A-1$ even-even nuclei, the band of an up-sloping configuration in the odd-A nucleus has less increase in moment of inertia than that of a down-sloping configuration. This general trend is further demonstrated in Table III, which shows the so-called quasi-identical bands, defined as those for which the $\Delta\mathcal{J}^{(1)}$ value is larger than 1 but less than 3. Once again, these particular bands are found among those with up-sloping configurations (with one exception, $[523]\frac{7}{2}$, a slightly down-sloping orbital). In contrast, the $\Delta\mathcal{J}^{(1)}$ values for the dominant down-sloping configuration in this region ($[541]\frac{1}{2}$) are all large (19 to 68) compared to the values for the up-sloping configurations, as expected in the cancellation picture discussed here.

Now let us consider a similar analysis based on the dynamic moment of inertia $\mathcal{J}^{(2)}$. This quantity reflects the local characteristics of the effective moment of inertia

TABLE III. Quasi-identical bands in the $A=150$–190 region.

Configuration[a]	Nucleus	$N_{\mathrm{odd\,tr}}$	$\Delta\mathcal{J}^{(1)}$
$\pi[404]\frac{7}{2}^-$	^{157}Ho	5	2.002
$\pi[404]\frac{7}{2}^-$	^{161}Tm	4	1.292
$\pi[404]\frac{7}{2}^+$	^{161}Tm	3	1.278
$\pi[523]\frac{7}{2}^-$	^{167}Tm	5	2.667
$\pi[404]\frac{7}{2}^-$	^{165}Lu	4	2.684
$\pi[404]\frac{7}{2}^+$	^{165}Lu	3	2.803
$\pi[402]\frac{5}{2}^+$	^{165}Lu	4	2.772
$\pi[402]\frac{5}{2}^+$	^{167}Lu	3	1.424
$\pi[402]\frac{5}{2}^-$	^{171}Lu	4	2.424
$\pi[402]\frac{7}{2}^-$	^{173}Lu	4	1.341
$\pi[402]\frac{5}{2}^+$	^{173}Ta	5	2.432
$\pi[514]\frac{9}{2}^+$	^{177}Ta	4	2.996
$\pi[514]\frac{9}{2}^-$	^{177}Ta	3	2.897
$\pi[402]\frac{5}{2}^+$	^{185}Re	3	1.134

[a]The signature of the particular band is denoted by + or −.

and is different from the kinematic moment ($\mathcal{J}^{(1)}$) when a band has an alignment, i. The above analysis of $\Delta\mathcal{J}^{(1)}$ picks out those bands with small alignments compared to the even-even core nucleus. The question now is whether there are bands that have constant values of i and should be judged as identical to the core nuclei based on a $\mathcal{J}^{(2)}$ analysis. The criterion is set up in a manner analogous to that in Eqs. (1)–(5):

$$\mathcal{J}^{(2)} = \frac{dI}{d\omega}, \tag{6}$$

$$\begin{aligned}\delta\mathcal{J}^{(2)} &= \frac{\mathcal{J}^{(2)}(A_1)-\mathcal{J}^{(2)}(A_2)}{\mathcal{J}^{(2)}(A_1)}\\ &= \frac{dI(A_1)-dI(A_2)}{d\omega(A_1)} \Big/ \frac{dI(A_1)}{d\omega(A_1)}\\ &= \frac{d[I(A_1)-I(A_2)]}{dI(A_1)} = \frac{di(A_1)}{2}.\end{aligned} \tag{7}$$

It is then necessary to average this relative angular momentum difference to give a total measure of the relative variation in the dynamic moment $\mathcal{J}^{(2)}$:

$$\delta\mathcal{J}^{(2)}_{\mathrm{avg}}(A_1) = \frac{1}{N}\sum_{n=1}^{N}\frac{|i_{n+1}(A_1)-i_n(A_1)|}{2}, \tag{8}$$

$$\Delta\mathcal{J}^{(2)} = \frac{\delta\mathcal{J}^{(2)}_{\mathrm{avg}}(A_1)}{\delta\mathcal{J}_m(A_1)} < 1.0. \tag{9}$$

Table IV shows the identical bands found by this $\mathcal{J}^{(2)}$ criterion. Once again all are based on up-sloping configurations, and some are the same as those identified in the $\mathcal{J}^{(1)}$ analysis of Table I. An important finding is that the down-sloping $[541]\frac{1}{2}$ bands still have $\Delta\mathcal{J}^{(2)}$ values (3 to 51) much larger than those listed in Table IV. So, even removing the possible influence of a sizable but constant alignment (which would give a large $\Delta\mathcal{J}^{(1)}$ value) results in still rather large $\Delta\mathcal{J}^{(2)}$ values in $[541]\frac{1}{2}$ bands and the conclusion that the identical bands are found only among the up-sloping configurations. Baktash *et al.* [9,11] also found that the identical bands are mostly associated with up-sloping orbitals in this same $A=150$–190 region.

From Tables I, III, and IV one can see that there is a

TABLE IV. Identical bands in the $A=150$–190 region (with the $\mathcal{J}^{(2)}$ criterion).

Configuration[a]	Nucleus	$N_{\mathrm{odd\,tr}}$	$\Delta\mathcal{J}^{(2)}$
$\pi[402]\frac{5}{2}^-$	^{165}Lu	4	0.703
$\pi[404]\frac{7}{2}^-$	^{171}Lu	3	0.392
$\pi[404]\frac{7}{2}^+$	^{171}Lu	3	0.601
$\pi[404]\frac{7}{2}^-$	^{173}Lu	4	0.957
$\pi[404]\frac{7}{2}^+$	^{173}Lu	3	0.407
$\pi[514]\frac{9}{2}^+$	^{173}Lu	3	0.253
$\pi[514]\frac{9}{2}^-$	^{173}Lu	3	0.714
$\pi[404]\frac{7}{2}^+$	^{175}Lu	4	0.719
$\pi[402]\frac{5}{2}^+$	^{177}Ta	3	0.470
$\pi[402]\frac{5}{2}^-$	^{177}Ta	3	0.980
$\pi[402]\frac{5}{2}^+$	^{185}Re	3	0.078

[a]The signature of the particular band is denoted by + or −.

continuous variation in the $\Delta\mathcal{J}^{(1)}$ and $\Delta\mathcal{J}^{(2)}$ values for the bands surveyed here. That is, there is no distinct gap between these values for identical compared to nonidentical bands [8]. Different criteria can therefore find a different number of identical bands. For example, the criterion of Baktash *et al.* [11] is similar [it includes one more factor in the definition, compared to our Eq. (8)], but results in the conclusion of roughly 3 times more identical bands among the same group as analyzed here. Our emphasis in this paper has been to make a more stringent definition of an identical band in order to suggest a possible origin of the physics involved.

To summarize, based on the crucial criteria proposed in this paper, there exist identical bands in certain configurations of odd-A nuclei compared with $A-1$ even-even neighbors. The total number of identical bands is approximately (6–7)% of the roughly 160 known bands in $A=150$–190, $Z=67$–77 nuclei. These bands and our so-called quasi-identical bands are all associated with the occupation of up-sloping orbitals ($[402]\frac{5}{2}$, $[404]\frac{7}{2}$, $[514]\frac{9}{2}$) as particle states, and are mostly concentrated in Lu isotopes. There is one exception to this strong correlation among the quasi-identical bands, and this involves the $[523]\frac{7}{2}$ orbital, which is slightly down-sloping with deformation. The simple picture of a cancellation between the effects of a pairing reduction and a quadrupole deformation decrease must be considered as a possible explanation for the identical-band phenomenon at normal deformation. A recent systematic analysis of the identical-band phenomenon in even-even nuclei has revealed [8] that identical bands exist for a pair of nuclei with close values of the ratio ε_2/Δ. This is the same cancellation effect: The decrease (increase) of the pairing correlation is compensated by a roughly proportional decrease (increase) of the quadrupole deformation. If this cancellation picture works, the extremely difficult task is then to explain the mechanism which stabilizes the effect of the odd nucleon so that the precise cancellation works for an extended transition sequence. Of course, another theoretical need is a model which can reproduce the experimental moment of inertia with such high accuracy (within a fraction of a thousandth) that the condition of identical rotational structures can be tested. That is why the cancellation picture described here can only be viewed in a qualitative, but not quantitative, light. We have emphasized in this paper that the occurrence of identical bands based on the proposed criteria is always associated with up-sloping particle states in odd-A nuclei. This correlation gives us strong hints about the underlying physics involved in the formation of identical bands at normal deformation.

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