Induced QCD and Hidden Local Z_N Symmetry

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(Received 29 June 1992)

We show that a lattice model for induced lattice QCD which was recently proposed by Kazakov and Migdal has a Z_N gauge symmetry which, in the strong coupling phase, results in a local confinement where only color singlets are allowed to propagate along links and all Wilson loops for nonsinglets average to zero. We argue that if this model is to give QCD in its continuum limit, it must have a phase transition. We give arguments to support the presence of such a phase transition.

PACS numbers: 12.38.Gc

The large N expansion is one of the few analytical tools available for gauge theories in the strong coupling regime. However, in greater than two dimensions the solution of even the leading, $N=\infty$, order of the expansion has not been found. Recently, an intriguing new approach to this problem has been proposed by Kazakov and Migdal [1, 2]. They have suggested using a lattice gauge model in which the Yang-Mills interaction is induced by a heavy scalar field in the adjoint representation of SU(N). The action is

$$S=\sum_x N\,\mathrm{Tr}\left(m_0^2\Phi^2(x)-\sum_\mu \Phi(x)U_\mu(x)\Phi(x+\mu)U_\mu^\dagger(x)\right), \tag{1}$$

where the scalar $\Phi(x)$ lives on lattice sites and the link operator $U_\mu(x)$ is the usual SU(N)-group-element-valued lattice gauge field (in the fundamental representation). Integrating over the scalar field Φ,

$$\int DU\,D\Phi\,\exp(-S)\sim\int DU\exp(-S_{\mathrm{ind}}[U]), \tag{2}$$

results in the induced gauge action

$$S_{\mathrm{ind}}[U]=-\frac{1}{2}\sum_\Gamma \frac{|\mathrm{Tr}U[\Gamma]|^2}{l[\Gamma]m_0^{2l[\Gamma]}}, \tag{3}$$

where $l[\Gamma]$ is the length of the loop Γ, $U[\Gamma]$ is the ordered product of link operators along Γ, and the summation is over all closed loops. It was argued in [1] that in the continuum limit this action is equivalent to the ordinary Yang-Mills action with the coupling constant depending on the scalar mass and ultraviolet cutoff. Indeed, for an elementary plaquette, $\Box$, $\mathrm{Tr}U[\Box]\sim N-\mathrm{Tr}F^2/2+\cdots$, where F is the continuum field strength and $S[U]\sim$ const $+\,\mathrm{Tr}F^2/4+\cdots$ in the naive continuum limit. Furthermore, Kazakov and Migdal [1] showed that, when the gauge field is integrated first, one obtains an effective matrix model for the field $\Phi(x)$ which can be analyzed at large N and behaves like a master field in that its fluctuations are frozen in the large N limit. They conjectured that $\Phi(x)$ is the correct master field for QCD.

In this Letter we shall point out that the lattice gauge theory described by (1) has unusual properties which are not shared by the usual Wilson formulation of lattice QCD. This is a result of the fact that the induced action (3) for the gauge fields depends only on the modulus of the terms $\mathrm{Tr}U(\Gamma)$ and, unlike the Wilson theory, is insensitive to their phases. This is of course connected with the fact that, since the scalar field transforms in the adjoint representation of SU(N), the original action (1) has both a U and a $U^\dagger$ on each link. The action has two local gauge symmetries: The first one is usual local SU(N) which acts on the both the scalar and gauge field as left and right group multiplication,

$$\begin{aligned}U_\mu(x)&\to\Omega(x)U_\mu(x)\Omega^{-1}(x+\mu),\\ \Phi(x)&\to\Omega(x)\Phi(x)\Omega^{-1}(x),\ \ \Omega(x)\in\mathrm{SU}(N),\end{aligned} \tag{4}$$

and whose group elements are defined on the sites x. The second local symmetry is a hidden Z_N symmetry which is not seen in the weak coupling continuum limit and which is also absent in the standard Wilson formulation of lattice QCD:

$$U_\mu(x)\to Z_\mu(x)U_\mu(x)Z_\mu(x)\in Z_N, \tag{5}$$

where Z_N is the center of SU(N) and the local gauge group elements are defined on links. We stress that this symmetry will appear for any induced QCD if the original matter fields are invariant under the action of the center of the gauge group (as is the adjoint representation which we use here).

The properties of the strong coupling (confining) phase in a theory with this additional local symmetry differ from those of ordinary Wilson lattice QCD—in some sense confinement is stronger and we call the strong coupling phase *local confinement.* Let us consider the vacuum average of the Wilson line operator in the fundamental representation

$$W(C)=\left\langle \mathrm{Tr}P\exp\left(i\oint_C A_\mu dx^\mu\right)\right\rangle=\left\langle \mathrm{Tr}\prod_{\gamma\in C}U(\gamma)\right\rangle. \tag{6}$$

Using the local Z_N symmetry $U(\gamma)\to e^{2\pi i n_\gamma/N}U(\gamma)$

and assuming Z_N invariance of the ground state we get $W(C) = 0$ for any contour C with nonzero area. A nonzero result can only be obtained when in a Wilson loop or array of Wilson loops, either there is an equal number of U and $U^\dagger$ factors γ $[\cdots U(\gamma)\cdots U^\dagger(\gamma)\cdots]$ or there are "baryon" factors $\prod_1^N U(\gamma)$ for every link. Thus we see that local Z_N symmetry prohibits the propagation of unscreened color along the links—the only possible type of excitation in this theory is "locally" white objects—contrary to Wilson QCD where confinement was not so restrictive and the Wilson line behaves as $\exp[-k\mathcal{A}(C)]$ with string tension k. One can say that local Z_N symmetry makes the string tension undefined.

We also note that the Z_N fluxons [3] whose contributions are relevant to the Wilson theory are gauge artifacts in the present model with local Z_N symmetry. Indeed, the flux through a plaquette is defined modulo elements of the center of the group—due to the local Z_N symmetry one can change the product $\prod_{\gamma\in C} U(\gamma)$ by the phase factor $\exp(2\pi i n/N) \in Z_N$.

The strong coupling phase of this theory exhibits local confinement. An interesting question is whether there can be a phase transition to a conventional confining phase or even a confinement-deconfinement phase transition in this theory. In usual lattice gauge theory the latter transition [4] is thought to occur at some value of the gauge coupling when the number of links N_t in one of the Euclidean directions is finite. It is connected with spontaneous breaking of a *global* Z_N symmetry and the appearance of a nonzero vacuum expectation value of the relevant order parameter—the Polyakov line $\langle L\rangle = \langle \mathrm{Tr}\prod_{l=1}^{N_t} U(l)\rangle$. In the confining phase $\langle L\rangle = 0$; in the deconfined phase there are N degenerate states (global Z_N is broken) and $\langle L\rangle \in Z_N$.

However, in our case besides the usual global Z_N symmetry we also have a *local* Z_N which will guarantee that $\langle L\rangle = 0$. A local gauge symmetry cannot be spontaneously broken [5] and whether we can obtain a Wilson QCD-like phase of this theory and a confinement-deconfinement transition within that phase is a subtle question. This question is an important one for the Kazakov-Migdal model and for all induced QCD models where one starts with the action without any explicit symmetry breaking terms.

To better understand this issue, consider the following model of lattice QCD with the gauge group SU(N) and with an action [6]

$$S[U] = -\sum_{\Box}\left(\beta\,\mathrm{Re}\,\mathrm{Tr}U(\Box) + \frac{1}{2\lambda}|\mathrm{Tr}U(\Box)|^2\right), \quad (7)$$

where $\Box$ are fundamental plaquettes. The first term is the conventional Wilson action and the second term is the local Z_N symmetric term for elementary plaquettes which appears in (3). The Wilson term breaks the local Z_N symmetry explicitly. This theory was considered in [7] in the limit $N\to\infty$ where the possibility of phase transitions between the confining phase and local confining phase (it was called the "absence of quarks" phase) was discussed. Such a phase transition in a somewhat different theory which also has an additional local Z_N symmetry was discussed in [8]. In fact in the early 1980s there were several papers discussing models with a local Z_N symmetry [9].

The $\beta\to 0$ limit of (7) has local Z_N symmetry and can be regarded as a toy version of the Migdal-Kazakov model with the identification $\lambda = 4m_0^8$. In the strong coupling (large λ) limit it is a locally confining theory. In its weak coupling limit it resembles continuum QCD in the sense that it produces the correct naive continuum limit. At least for N=2 this model is known to have a phase transition at a critical value of λ when β vanishes [7–9]. An interesting question is the connection of this phase transition to confinement. One may speculate that for $\lambda < \lambda_c$ the Z_N symmetry is realized in a Higgs phase and the physical properties of that phase resemble those of Wilson's lattice QCD. For the purpose of studying such a Higgs phase we temporarily ignore the fact that this local symmetry cannot be broken at finite N. We shall later define a generalization of the fundamental Wilson loop which avoids the problem that the expectation value of the ordinary Wilson loops all vanish.

To identify the Higgs field, we use a Gaussian transformation to write the partition function as

$$\begin{aligned} Z &= \int DU\,\exp\left(\sum_{\Box}\beta\,\mathrm{Re}\,\mathrm{Tr}U(\Box) + \frac{1}{2\lambda}|\mathrm{Tr}U(\Box)|^2\right) \\ &= \int DU\,D\phi\,\exp\sum_{\Box}[-2\lambda|\phi(\Box)|^2 + \phi(\Box)\mathrm{Tr}U(\Box) + \phi^\dagger(\Box)\mathrm{Tr}U^\dagger(\Box) + 2\beta\lambda\,\mathrm{Re}\,\phi(\Box)]\,. \end{aligned} \quad (8)$$

Here $\phi(\Box)$ is a scalar field which lives on plaquettes. It is a singlet under the gauge transformation in (4). The local Z_N transformations (5) act on links. $\phi(\Box)$ transforms as

$$\phi(\Box)\to\phi(\Box)\prod_{\mathrm{links}\in\delta\Box}(Z_N)^{\pm 1}\,, \quad (9)$$

where $\pm$ depends on the orientation of the link in the boundary $\delta\Box$ of $\Box$. The Wilson term in (7) results in a constant external field for $\phi(\Box)$ in (8) which breaks local Z_N explicitly.

We wish to investigate the possibility that the local Z_N symmetry is realized in a Higgs phase. To this end we consider the effective action for the Higgs field which is obtained by integrating the gauge fields in (8),

$$V_{\text{eff}}^{\beta} = \sum_{\Box}[2\lambda|\phi(\Box)|^2 - 2\beta\lambda\,\text{Re}\phi(\Box)] - \ln\int DU\,\exp\left(\sum_{\Box}[\phi(\Box)\text{Tr}U(\Box) + \phi^{\dagger}(\Box)\text{Tr}U^{\dagger}(\Box)]\right). \qquad (10)$$

Note that the integral is just the partition function of conventional QCD with a position-dependent coupling constant given by $1/g^2 \sim \phi(\Box)$.

For small ϕ we can use the conventional strong coupling expansion to evaluate S_{eff}:

$$\begin{aligned} S_{\text{eff}}(\Box) &= -\ln\int DU\,\exp\left(\sum_{\Box} 2\,\text{Re}\phi(\Box)\text{Tr}U(\Box)\right) \\ &= \sum_{\Box,\Box'}\phi(\Box)\phi(\Box')\,\frac{\partial}{\partial\phi(\Box)}\frac{\partial}{\partial\phi^*(\Box')}S_{\text{eff}}\bigg|_{\phi=0} + \cdots \\ &= -\sum_{\Box\Box'}[\langle\text{Tr}U(\Box)\text{Tr}U^{\dagger}(\Box')\rangle - \langle\text{Tr}U(\Box)\rangle\langle\text{Tr}U(\Box')\rangle]\phi(\Box)\phi(\Box') + \cdots \\ &= -\sum_{\Box}|\phi(\Box)|^2 + \cdots, \end{aligned} \qquad (11)$$

where the averaging $\langle\cdots\rangle$ is done at $\phi = 0$; i.e., it is simple integration $\int DU$ with zero action. To obtain the leading term above note that $\langle\text{Tr}U(\Box)\rangle = 0$ and that $\langle\text{Tr}U(\Box)\text{Tr}U^{\dagger}(\Box')\rangle = \delta_{\Box\Box'}$. Higher-order terms depend on either higher powers of $|\phi|$ or on ϕ^N and on $\phi^{\dagger N}$. Terms such as ϕ^N appear since the effective action is invariant only under Z_N but not under U(1). "Gradient" terms appear at order ϕ^6 due to a term in the effective action which is the product of ϕ over the six plaquettes on the face of any cube. Such a product is invariant under local Z_N. Note that higher-order terms in $|\phi|$ will have positive signs and the potential appears not to be bounded from below. Therefore, this expansion is good only for small $|\phi|$. If we assume that the expansion has some nonzero radius of convergence the effective potential for ϕ in the small ϕ approximation is (for $\beta = 0$)

$$V_{\text{eff}}[\phi] = 2\lambda|\phi|^2 - |\phi|^2 + \cdots \qquad (12)$$

which exhibits the typical behavior of a phase transition to a Higgs phase at $\lambda = 1/2$. This suggests that for large λ this model exists in a locally confining phase and that at some critical value λ_c of λ there is a phase transition which we conjecture is a conventional confining phase resembling the Wilson lattice QCD.

There is another way to demonstrate that for small λ one can get the nonzero vacuum expectation value for the master field $\phi(\Box)$. Asymptotically, when ϕ is large, the logarithm of the integral in (10) is bounded using the triangle inequality,

$$\begin{aligned} \int dU\,\exp\sum_{\Box}(2\,\text{Re}\phi\,\text{Tr}U) &\le \exp\left(\sum_{\Box}2|\phi|\right)\int dU \\ &= \exp\left(\sum_{\Box}2|\phi|\right), \end{aligned} \qquad (13)$$

so that for large $|\phi|$,

$$S_{\text{eff}} = \sum_{\Box}[2\lambda|\phi(\Box)|^2 - 2|\phi(\Box)| - 2\beta\lambda\,\text{Re}\phi(\Box)]. \qquad (14)$$

The effective potential is extremized by the configuration $\phi = \beta/2 + \text{sgn}(\phi)/2\lambda$. When $\beta < 1/\lambda$ there are two solutions which are degenerate at $\beta = 0$. This nontrivial solution is reliable for small λ since the estimate of the effective potential is valid for large ϕ. The nature of the solution changes when the external field $\beta > 1/\lambda$. Note that our results are in qualitative agreement with the analysis [7].

It is unclear how to generalize these arguments to the case of the induced action (3) $S_{\text{ind}}(U)$ where we have the sum of all possible closed paths, not only the minimal path as in our toy model. One option is to consider the sum of different master fields ϕ_Γ (analogous to a string field) and if even one of them has a nonzero expectation value, the local Z_N symmetry may be realized in the Higgs mode. It is easy to see that for each master field ϕ_Γ the effective coupling $\lambda_\Gamma = l[\Gamma]m_0^{2l[\Gamma]}$ and the leading quadratic term in the effective action is like $\sum_\Gamma[2\lambda(\Gamma) - 1]|\phi(\Gamma)|^2$. If one decreases m_0, it will be the coupling constant corresponding to the smallest loop, i.e., $\lambda_\Box$, which becomes smaller than 1/2 first. One can imagine that if all of the other coupling constants are larger than the corresponding critical values the effective theory will be the Wilson theory with the fundamental plaquette action.

We believe that one of the principal questions about induced QCD with local Z_N symmetry is whether there is a weak coupling phase with ordinary confinement and an area law for Wilson loops. In this paper we have speculated about such a possibility based on the assumption that the Higgs phase for our auxiliary model is equivalent to a confinement phase. To better understand how in the Higgs phase for ϕ one can get an area law for Wilson loops let us consider the generalization of the usual Wilson loop,

$$W_\phi(C) = \left\langle W(C)\prod_{\Box\in A(C)}\phi(\Box)\right\rangle. \qquad (15)$$

We have introduced this "filled" Wilson line in which we preserve the local Z_N symmetry by introducing the product of factors $\phi(\Box)$ for each plaquette in the interior of the contour C. Order parameters of this kind have been discussed in somewhat different contexts in Refs. [8] and [10].

The filled Wilson loop operator has a nonzero expectation value. If in fact there is a phase transition from a locally confining to a confining phase, we expect a discontinuity in the behavior of the filled Wilson loop at the critical point. Now one can see that if the field $\phi(\Box)$ fluctuates about a nonzero value $\bar{\phi}$ (due to Elitzur's theorem this is only possible with appropriate gauge fixing) the filled Wilson loop is approximately given by

$$W_\phi(C) \sim \exp[A(C)\ln\bar{\phi}]\langle W(C)\rangle_{\phi=\bar{\phi}} \,. \qquad (16)$$

In this phase, the behavior of the Wilson loop operator is approximately like that of the usual Wilson formulation of lattice QCD.

In this paper we have pointed out an important problem with the scenario of induced QCD which has been proposed by Kazakov and Migdal, namely, the presence of an extra Z_N gauge symmetry which forces the expectation values of ordinary Wilson loops to vanish. We have presented two possible solutions to this problem. The first is the possibility of a phase transition to a phase in which the Z_N symmetry is realized in a Higgs mode. The second is the use of an order parameter alternative to the Wilson loop which we call the filled Wilson loop. The latter has a nonvanishing expectation value and reduces to the ordinary Wilson loop in the naive continuum limit. Such filled Wilson loops form a natural class of observables for models with Z_N gauge symmetry.

It is interesting to consider whether the phase transition found by Kazakov and Migdal which occurs at $m_0^2 = 2D$ is related to the phase transition between the local confinement and confinement phases so that the strong coupling phase corresponds to local confinement with unbroken Z_N and the weak coupling phase corresponds to restoration of the area law and (some kind of) spontaneous breaking of Z_N symmetry [Higgs phase for $\phi(\Box)$ field]. It is very natural to conjecture that the gap in the eigenvalue distribution function $\rho(\mu)$ is proportional to some inverse power of the string tension α'. Then in a weak coupling phase the gap, as well as α', is nonzero and one recovers the area law. However, in the strong coupling case the gap disappears and string tension diverges $\alpha' \to \infty$, which means that we are in a local confinement phase. In a future publication we shall present the results of more detailed investigations.

This work was supported in part by the Natural Sciences and Engineering Council of Canada. We thank A. Morozov for discussions.

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