Vacuum Polarization and the Electric Charge of the Positron

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We show that higher-order vacuum polarization would contribute a measurable net charge to atoms, if the charges of electrons and positrons do not balance precisely. We obtain the limit $|Q_e + Q_{\bar{e}}| < 10^{-18}e$ for the sum of the charges of electron and positron. This also constitutes a new bound on certain violations of PCT invariance.

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In a recent Letter [1] Hughes and Deutch discussed the possibility that the charges of positrons and antiprotons may not be exactly opposite to those of electrons and protons. Whereas the equality in magnitude of the charges of electrons and protons is known to the extreme accuracy [2, 3]

$$|Q_e + Q_p| < 10^{-21}e, \tag{1}$$

the equality in magnitude of the charges of electrons and positrons is much more difficult to study directly. After reviewing the available body of evidence, Hughes and Deutch conclude that the present limit on the net neutrality of an electron-positron pair is

$$|Q_e + Q_{\bar{e}}| < 4 \times 10^{-8}e. \tag{2}$$

Here we would like to point out that there exist far more stringent bounds on this quantity from indirect sources. Our argument is based on the fact that the vacuum polarization in heavy atoms contains an equal number of electrons and positrons and hence would contribute to the overall charge of an atom, if the charges of electrons and positrons do not balance each other precisely. This reasoning is closely related to the observation first made by Morrison [4] and Schiff [5], that the equality of the gravitational masses of electrons and positrons is probed to about 1% accuracy by the fact that the contribution of vacuum polarization to the mass of an atom does not lead to a violation of the equivalence principle.

As we will show below, this argument is much more powerful in the case of the electric charge. In fact, our bound would be even more precise were it not for the necessity of charge renormalization. Since the amount of charge contained in the lowest order (in $Z\alpha$, where Z is the nuclear charge) vacuum polarization is directly proportional to the source charge of the Coulomb field, the net vacuum polarization charge to this order can be absorbed in the *renormalized* charge of the source, rendering it effectively unobservable. This reasoning does not apply to higher orders in $Z\alpha$ of the atomic vacuum polarization, which do not contribute to charge renormalization.

If the charges of electrons and positrons are not opposite and equal, the first nonvanishing contribution to the overall charge of an atom by the vacuum polarization would come in order $(Z\alpha)^2$. According to Furry's theorem [6], this order normally vanishes identically due to the invariance of quantum electrodynamics (QED) against charge conjugation (C invariance). However, if Q_e and $Q_{\bar{e}}$ do not balance each other, this would imply a violation of C invariance and hence invalidate Furry's theorem.

It is not clear that a completely consistent quantum field theory of QED without C invariance can be constructed, but for our purposes it is sufficient to consider an effective theory that is consistent at the one-loop level. This is provided by the Lagrangian

$$L = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + [Q_e\bar{\psi}_e\gamma^\mu\psi_e + Q_{\bar{e}}\bar{\psi}_{\bar{e}}\gamma^\mu\psi_{\bar{e}} + \tilde{Q}(\bar{\psi}_e\gamma^\mu\psi_{\bar{e}} + \bar{\psi}_{\bar{e}}\gamma^\mu\psi_e)]A_\mu, \tag{3}$$

where $\psi_{e/\bar{e}} = P_\pm\psi$ denotes the Dirac field projected on positive and negative energies, respectively, Q_e and $Q_{\bar{e}}$ are the charges of electron and positron, and $\tilde{Q}$ denotes the coupling constant associated with pair creation. From the success of QED precision measurements we know that $Q_e \equiv -e \approx -Q_{\bar{e}}$ at least to within 10^{-8} [1] and $\tilde{Q} \approx -e$ to within 10^{-3} [7].

In addition to C invariance, the Lagrangian (3) breaks gauge and PCT invariance. The former expresses the fact that charge conservation is violated if one assigns unequal charges in magnitude to electron and positron, but allows for pair annihilation into a neutral photon. The violation of PCT invariance is reconciled with the Pauli-Lüders theorem [8] by noting that the projection operators $P_\pm$ appearing in (3) are nonlocal. They are given by

$$(P_\pm\psi)(\mathbf{x},t) = \mp\gamma^0\frac{m}{4\pi^2}\int d^3x'\,\frac{K_2(m|\mathbf{x}-\mathbf{x}'|)}{|\mathbf{x}-\mathbf{x}'|^2}\psi(\mathbf{x}',t) + \frac{1}{2m}(\pm i\boldsymbol{\gamma}\cdot\boldsymbol{\nabla} + m)\psi(\mathbf{x},t), \tag{4}$$

which is nonlocal on the scale of the electron Compton wavelength. Although the breaking of gauge and PCT invariance may seem unattractive, it is unavoidable if one wants to construct a low-energy effective theory describing particles and antiparticles with unequal opposite charges.

We now apply the Lagrangian (3) to the calculation of order $\alpha(Z\alpha)^2$ vacuum polarization in atoms, which is the lowest order where a nonvanishing $Q_e + Q_{\bar{e}}$ would contribute. The relevant Feynman diagrams describing the contribution of vacuum polarization to Rutherford scattering on a nucleus are shown in Fig. 1. Intuitively, they correspond to scattering on the virtual positrons (a) and electrons (b) in the polarization cloud around the nucleus. There is a time ordering ($x_0 > y_0$) assumed, which is imposed by the nucleus. Therefore arrows pointing up represent electron propagators $S_e(x-y) = \theta(x_0 - y_0)\, S^+(x-y)$, whereas arrows pointing down correspond to positron propagators $S_{\bar{e}}(y-x) = -\theta(x_0-y_0)S^-(y-x)$. Here the propagators $S^\pm$ are related to the Feynman propagator by [9]

$$S_F(x-y) = \theta(x_0-y_0)\, S^+(x-y) - \theta(y_0-x_0)\, S^-(x-y). \quad (5)$$

After Fourier transformation we obtain

$$S_e(p) = \frac{\gamma_0 E_p - \boldsymbol{\gamma}\cdot\mathbf{p} + m}{2E_p}\frac{1}{p_0 - E_p + i\epsilon},$$
$$S_{\bar{e}}(-p) = \frac{\gamma_0 E_p + \boldsymbol{\gamma}\cdot\mathbf{p} - m}{2E_p}\frac{1}{p_0 + E_p - i\epsilon}, \quad (6)$$

where $E_p = \sqrt{\mathbf{p}^2 + m^2}$. These propagators together with the vertices, modified by the coupling constants Q_e, $Q_{\bar{e}}$, and $\tilde{Q}$, respectively, define the Feynman rules which have to be used in Fig. 1. The contribution of the two diagrams to the scattering matrix is strictly proportional to the charge imbalance:

$$\Delta S_{fi} = (Q_e + Q_{\bar{e}})Q\tilde{Q}^2 \int \frac{d^4q}{(2\pi)^4}\frac{d^4p}{(2\pi)^4} A_\mu(q)A_\nu(q')D(q+q')\, \bar{u}_f\gamma_\lambda u_i \,\mathrm{tr}[S_{\bar{e}}(p+q')\gamma^\nu S_e(p)\gamma^\mu S_{\bar{e}}(p-q)\gamma^\lambda], \quad (7)$$

where $A_\mu(q)$ represents the electromagnetic potential generated by the nucleus and Q denotes the charge of the scattering particle. The loop integral over p in (7) is superficially linearly divergent, but is actually finite due to cancellation of the leading orders in p. We also note that, in contrast to the fourth-order contribution to vacuum polarization, there is no need for a finite subtraction [10].

For our purposes it is sufficient to consider the limit of forward scattering ($q + q' = 0$) in the nonrelativistic limit, where only the timelike components ($\mu = \nu = \lambda = 0$) contribute. Then it is easy to see that the effect of (7) on the scattering amplitude corresponds to the presence of an additional charge

$$\Delta Ze = (Q_e + Q_{\bar{e}})Q(Ze\tilde{Q})^2 \int \frac{d^3q}{(2\pi)^3}\frac{F(\mathbf{q}^2)^2}{\mathbf{q}^4} \int \frac{d^3p}{(2\pi)^3}\frac{E_pE_{p-q} - E_p^2 + \mathbf{p}\cdot\mathbf{q}}{E_pE_{p-q}(E_p + E_{p-q})^2} \quad (8)$$

surrounding the nucleus. Here $F(q^2)$ is the nuclear elastic form factor and $E_{p-q} = \sqrt{(\mathbf{p}-\mathbf{q})^2 + m^2}$. Since we are not interested in extreme precision, we simply cut off the q integration at the inverse nuclear radius R and evaluate the integrals in (8) to leading order in the cutoff. We also set $\tilde{Q} = -e$. The result is

$$\Delta Ze \approx (Q_e + Q_{\bar{e}})\frac{2Z^2\alpha^2}{3\pi^2}\left[\ln\left(\frac{1}{mR}\right) + c\right], \quad (9)$$

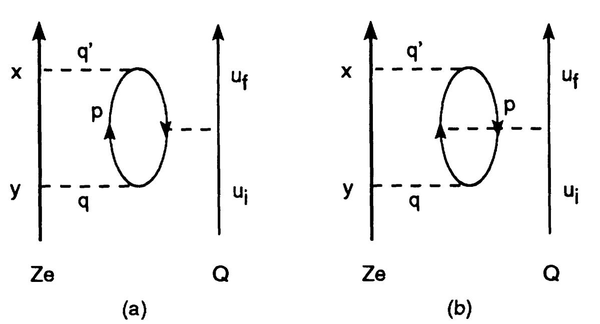

FIG. 1. Diagrams contributing to second-order (in $Z\alpha$) vacuum polarization correction to the forward Coulomb scattering cross section on an atomic nucleus. Diagram (a) is proportional to the positron charge $Q_{\bar{e}}$ and (b) to the electron charge Q_e.

where the constant c depends on the details of the nucleon form factor and can be neglected for our purpose. For a heavy atom, such as lead ($Z = 82, R = 7$ fm), we find $\Delta Ze \approx \frac{1}{10}(Q_e + Q_{\bar{e}})$. With the limit (1) on the apparent residual charge of the atom per proton, $\Delta Z/Z$, we obtain the bound

$$|Q_e + Q_{\bar{e}}| < 10^{-18}e. \quad (10)$$

Because the net vacuum polarization charge is quadratic in the nuclear charge Z, it is impossible to simultaneously adjust the electron-positron and electron-proton charge differences such that all atoms are neutral, without satisfying the bound (10). Since the momentum integrations in (8) involve only momenta up to R^{-1}, and the structure of QED has been tested to very high precision over that range, we believe that our result is essentially model independent. Because our effective Lagrangian (3) breaks *PCT* invariance, the bound (10) can also be taken as a new test of *PCT* symmetry, which is better by a factor of 4 than the limit derived from the neutral kaon system [11], but tests a different mode of *PCT* symmetry breaking.

In conclusion, we have shown that the existing limit on violations of the neutrality of atoms sets a very stringent limit on the opposite equality of electron and positron charge, if one considers the second-order vacuum polarization, which normally vanishes due to Furry's theorem. It is unlikely that direct experimental tests can improve on this bound soon.

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[10] This can be seen in two ways: (a) No third-order term in the external field arises in Pauli-Villars regularization. (b) The only possible Lorentz invariant subtraction term of third order in the external potential, $A_\mu A_\nu \partial_\mu A_\nu$, vanishes for a static Coulomb potential.
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