# Anomalous Gauge Boson Couplings and Loop Calculations

C. P. Burgess[(1),(a)] and David London[(2),(b)]

[(1)]*Physics Department, McGill University, 3600 University Street, Montréal, Québec, Canada H3A 2T8*
[(2)]*Laboratoire de Physique Nucléaire, Université de Montréal, CP 6128 Montréal, Québec, Canada H3C 3J7*
(Received 20 March 1992)

Calculations with anomalous trilinear gauge boson vertices have been criticized as overestimates due to a failure to incorporate gauge invariance. We show here that the criticized calculations *are* gauge invariant, although this symmetry is realized nonlinearly. We instead trace the overestimates to an incorrect treatment of cutoffs in loop diagrams.

PACS numbers: 12.15.Cc, 11.10.Gh

Probably the least satisfactory aspect of the standard model is the *ad hoc* manner in which symmetry breaking and the concomitant generation of particle masses are implemented. We can expect to learn a great deal more about the symmetry-breaking mechanism at higher energies through the study of the Higgs boson (should it be found) and/or the scattering of gauge bosons. In the meantime, however, the low-energy effects of this sector can be parametrized in terms of an effective Lagrangian in which any new physics appears as nonrenormalizable, effective interactions. The dependence on $M$, the new-physics scale, of a dimension-$n$ operator in this Lagrangian is generically $M^{4-n}$.

One subgroup of such effective operators which has undergone particular theoretical scrutiny during the past ten years is that of anomalous trilinear gauge boson vertices (TGV's). These couplings are of special interest because they have virtually the lowest dimensions possible for nonstandard interactions, and so should be the least suppressed by inverse heavy masses, and yet they are as yet poorly probed by experiment. The self-couplings of gauge bosons also directly test the gauge structure, and hence the underlying symmetry breaking, of the standard model. Much work has gone into constraining such terms via their loop-induced processes at low energies, as well as examining their effects at future colliders (see Ref. [1] for references to the literature).

Recently, de Rújula and co-workers have claimed that many of these analyses are wrong because the Lagrangians used are not gauge invariant [2]. We argue here that this claim rests upon unstated assumptions that are not sufficiently general. It is true that if the Higgs boson should be light enough to appear in the low-energy theory then the electroweak gauge symmetry should be linearly realized, as de Rújula *et al.* take it to be. In this case, some previous calculations do overlook the Higgs contribution to certain processes. However, the Higgs boson need not be light; indeed it may not exist at all. We argue instead that any Lagrangian containing $W$'s and $Z$'s which satisfies Lorentz invariance and $U_{em}(1)$ gauge invariance is *automatically* $SU_L(2)\times U_Y(1)$ gauge invariant—this gauge invariance is simply nonlinearly realized.

We do, however, agree that most of the bounds on anomalous TGV's are considerably overestimated [3]. The real culprit is not gauge invariance at all—rather, it is the improper use of cutoffs to estimate the effect of such operators in loop diagrams. Since TGV's cannot yet be measured directly, bounds on such operators come solely from their contributions to loop-induced processes, which are commonly regularized using a cutoff $\Lambda$. Typical results vary as $\log\Lambda$, $\Lambda^2$, or $\Lambda^4$, where $\Lambda$ is taken to be of the order of the scale of new physics, leading to very stringent bounds on anomalous TGV's.

This procedure errs in attributing a physical significance to the cutoff, and often violates the decoupling theorem. The real bounds to be deduced from such loop calculations are, in general, much weaker than those found in the literature because the cutoff dependence of an amplitude in the low-energy theory rarely gives an accurate indication of the true dependence on the high-energy physics scale. We argue that at most a logarithmic dependence on the heavy-mass scale can be inferred purely from the low-energy effective Lagrangian.

Although well known in some circles, given the confusion in the literature it is clear that these ideas have not made their way out into the wider community which is now finding applications for effective-Lagrangian techniques. For this reason we feel it is appropriate to reexamine them here.

*Gauge invariance.*—Suppose we wish to specify a low-energy effective Lagrangian which describes the interactions of the spin-one $W^{\pm}$ and $Z^0$ bosons among themselves and with other matter fields. There are several choices one can make. One approach is to require only that the Lagrangian respect the electromagnetic gauge group. (We refer to this as the "non-gauge-invariant" Lagrangian.) Another possibility (the "nonlinearly realized gauge-invariant" Lagrangian) is to demand invariance with respect to the full $SU_L(2)\times U_Y(1)$ gauge symmetry, with all but the unbroken $U_{em}(1)$ being nonlinearly realized [4]. In this case the only light particles in the unknown symmetry-breaking sector are assumed to be the three Nambu-Goldstone bosons which give mass to the $W^{\pm}$ and $Z^0$.

In fact, these two Lagrangians are equivalent, as was first shown by Chanowitz, Golden, and Georgi (CGG) [5], and has been more recently discussed in Ref. [1]. The non-gauge-invariant Lagrangian may be obtained from the other by working in a specific gauge—unitary gauge. The proof of this is briefly sketched below. The argument is presented in more detail, including explicit calculations, elsewhere [1,5].

In the formulation with nonlinearly realized gauge invariance, one introduces the three Nambu-Goldstone fields $\varphi_a(x)$ nonlinearly via the matrix-valued scalar field $\xi(x)=\exp[iX_a\varphi^a(x)/f]$, where the $X_a$ are the broken generators of $SU_L(2)\times U_Y(1)$ (we take $X_3=T_3-Y$ and normalize unconventionally: $\mathrm{tr}[T_aT_b]=\frac{1}{2}\delta_{ab}$ for all generators including the hypercharge $Y$). The Nambu-Goldstone boson decay constant $f$ is of the order of the symmetry-breaking scale. It is related to the $W$ mass by $M_W=g_2f$, in which $g_2$ is the $SU_L(2)$ gauge coupling. By assigning the same decay constant to all three Nambu-Goldstone bosons, we implicitly assume a custodial SU(2) symmetry (broken only by hypercharge), although this is irrelevant for the argument we wish to make (see, e.g., Ref. [5]).

The nonlinearly realized gauge-invariant Lagrangian is constructed from $\xi$ and its covariant derivative: $\mathcal{D}_\mu(\xi)\equiv\xi^\dagger\partial_\mu\xi-i\xi^\dagger\mathbf{W}_\mu\xi$, where $\mathbf{W}_\mu=g_2W^a_\mu T_a+g_1B_\mu Y$ represents the electroweak gauge potentials. It is convenient for these purposes to define the following fields:

$$g_2\mathcal{W}_\mu\equiv i\sqrt{2}\,\mathrm{tr}[T_+\mathcal{D}_\mu(\xi)]\,, \tag{1}$$

$$(g_1^2+g_2^2)^{1/2}\mathcal{Z}_\mu\equiv 2i\,\mathrm{tr}[(T_3-Y)\mathcal{D}_\mu(\xi)]\,, \tag{2}$$

in which $T_+=T_1+iT_2$. These fields are constructed in such a way as to transform purely electromagnetically under arbitrary $SU_L(2)\times U_Y(1)$ transformations. It follows that an arbitrary $U_{\mathrm{em}}(1)$-invariant Lagrangian that is constructed from these fields becomes *automatically* invariant under the full nonlinearly realized electroweak gauge group.

This shows that the non-gauge-invariant Lagrangian and the Lagrangian with nonlinearly realized gauge invariance are equivalent, term by term. One simply makes the correspondence $\mathcal{Z}_\mu\leftrightarrow Z_\mu$ and $\mathcal{W}^\pm_\mu\leftrightarrow W^\pm_\mu$, where $Z_\mu$ and $W^\pm_\mu$ are the fields used in the non-gauge-invariant Lagrangian. This connection is explicit in unitary gauge, which is defined by the condition $\xi(x)\equiv 1$ throughout spacetime. In this gauge the fields $\mathcal{W}_\mu,\mathcal{Z}_\mu$ and $W_\mu,Z_\mu$ become identical, reducing the nonlinearly realized gauge-invariant Lagrangian to the non-gauge-invariant Lagrangian. Inspection of the corresponding ghost determinants [1] does not modify this conclusion. Of course, this equivalence only holds for energy scales at which the effective Lagrangians themselves make sense. For $W$'s and $Z$'s, the maximum applicable scale is roughly $4\pi M_W/g_2$, where $g_2$ is the $SU_L(2)$ coupling. (The conclusions further address the conditions for the validity of the low-energy expansion.)

Being related to one another by a gauge transformation, both Lagrangians clearly express the same physical content. From a conceptual point of view, this demonstrates that there is little to choose between a description of light spin-one particles in terms of a nonlinearly realized gauge symmetry and no gauge symmetry at all. In this way it is seen that criticisms of analyses involving TGV's based on the supposed failure of gauge invariance in the Lagrangians used are simply red herrings. We next argue that the overestimates in the literature instead arise from the careless use of cutoffs in regularizing divergent loop diagrams.

*Cutoffs.*—In the absence of direct measurements, the only way to constrain anomalous TGV's is through their contributions to such loop-induced processes as the $\rho$ parameter, $(g-2)_\mu$, etc. Typically these loop diagrams diverge and are regularized through the use of a cutoff $\Lambda$. In this way many authors [3] find that their results depend quadratically or even quartically on $\Lambda$. Taking $\Lambda$ to be of the order of the scale of new physics, these authors find large contributions from anomalous TGV's, and so infer extremely stringent bounds.

There are several problems with this reasoning. First of all, one cannot take $\Lambda$ to be of the order of the new-physics scale. More importantly, all quadratic (or higher) dependence on $\Lambda$ is simply canceled by counterterms coming from the high-energy theory. At one loop at best only a logarithmic dependence on the scale of new physics can be extracted purely from the low-energy effective Lagrangian.

Consider a theory with two very different mass scales, $M\gg m$, and suppose we calculate, for example, the contributions to a low-energy mass shift $\delta\mu^2$ (such as to the $W$ or $Z$ mass):

$$\delta\mu^2(m^2,M^2)=c_0M^2+c_1m^2+c_2m^4/M^2+\cdots\,, \tag{3}$$

where the ellipses represent terms that are suppressed by more than two powers of $m/M$. We remind the reader that only logarithmic infrared divergences are possible at zero temperature in four dimensions [6], so that terms like $M^4/m^2$—which would diverge as a power when $m/M$ tends to zero—cannot arise. The dimensionless coefficients are functions of the other (renormalized) parameters of the theory and may depend at most logarithmically on the large mass ratio $M/m$.

Now consider splitting the calculation up into a "high-energy" and a "low-energy" piece. First choose a cutoff $\Lambda$ satisfying $\Lambda\ll M$. Then calculate the high-energy contribution to $\delta\mu^2$ by integrating out all frequencies above $\Lambda$. This produces an effective Lagrangian which is applicable at energies less than $\Lambda$. Finally, using this effective Lagrangian, compute the low-energy contribution to $\delta\mu^2$. Clearly this is simply a reorganization of the complete calculation and thus the sum of the high-energy and low-energy contributions must equal the full result of Eq. (3):

$$\delta\mu^2(m,M)=\delta\mu^2_{\mathrm{le}}(m,\Lambda,M)+\delta\mu^2_{\mathrm{he}}(m,\Lambda,M)\,, \tag{4}$$

where the subscript "le" ("he") refers to the low-energy (high-energy) contribution.

Note that, contrary to much common usage, the cutoff *must* to be taken much smaller than the scale of the new physics: $\Lambda \ll M$. This requirement has two roots. It is required in order to justify the neglect within loops of high-dimension interactions of the low-energy effective Lagrangian. It is also necessary if the loop expansion itself is to be trusted. This is because the nonrenormalizability of the low-energy interactions makes successive loops depend on larger and larger powers of $\Lambda/M$. Neglect of these divergent higher-loop contributions is only possible if $\Lambda \ll M$. (This argument needs some modification in dimensional regularization, as we discuss below.)

The largest contributions to the low- and high-energy parts of $\delta\mu^2$ in a particular calculation might look like

$$\delta\mu_{\rm he}^2 = c_0 M^2 + b_1 \Lambda^2 + \cdots, \quad \delta\mu_{\rm le}^2 = b_1' \Lambda^2 + \cdots . \qquad (5)$$

In both of these expressions the ellipses represent terms that depend differently on the various small mass ratios $m/\Lambda$, $\Lambda/M$, and $m/M$. The key observation is that these two contributions can only sum to the $\Lambda$-independent result of Eq. (3) if their coefficients are related: $b_1 + b_1' = 0$, etc.

This is the main point. Many of the articles in Ref. [3] use the coefficient $b_1'$ (or an analogous coefficient calculated from the particular process they have considered) to put bounds on anomalous TGV's. However, this term has no physical significance—it is exactly canceled by a counterterm from the high-energy piece of the calculation. The important term, $c_0 M^2$, which gives the true dependence on the high-energy physics scale and thus could be used to bound the new physics, *cannot* be calculated purely from the low-energy Lagrangian. The coefficients $b_1'$ and $c_0$ are simply unrelated.

This example also illustrates the problem with equating the existence of quadratic divergences in the effective theory with the degree of fine tuning in the underlying theory. Naturalness is instead better analyzed simply using dimensional analysis, in which it is realized that the coefficients of low-dimension operators in the effective Lagrangian can, on dimensional grounds, receive contributions that are proportional to positive powers of the heavy mass scale $M$. The $c_0 M^2$ term of Eq. (3) furnishes one such example. The danger of trying to give this type of physical interpretation to quadratic divergences is perhaps most starkly illustrated in dimensional regularization, where quadratically divergent graphs involving light particles are proportional to powers of a small light-particle mass $m^2$, rather than to $\Lambda^2$.

Similar remarks apply to the quartic and higher $\Lambda$ dependence that has also been reported in some of the articles in Ref. [3] and elsewhere. People often distinguish two types of these divergences. There are both bona fide higher divergences as well as those that are really quadratic or lower divergences "in disguise." The distinction arises because whereas some effective interactions have coefficients that are proportional to powers of $1/M$, others may be suppressed only by powers of $1/\Lambda$. Typically terms are suppressed by $M$ rather than $\Lambda$ if they violate the selection rules of the renormalizable interactions of the low-energy theory. The Fermi interaction is a familiar example of this kind. Clearly any inverse powers of $\Lambda$ appearing in an effective coupling can partially cancel low-energy-loop divergences and so weaken their overall cutoff dependence. The same does *not* happen for terms that are suppressed by inverse powers of $M$, however. We need make no such distinction since neither carries any physical meaning—the $\Lambda$ independence of the final result ensures that all such divergences are simply canceled by counterterms generated by the high-energy part of the theory. This is equally true for divergences like $\Lambda^4/M^2$ or $\Lambda^4/m^2$, both of which can and do legitimately arise in calculations. The fallacy that cutoffs track heavy masses is particularly clear for the latter of these since a behavior of the form $M^4/m^2$ may be ruled out by general arguments.

A commonly occurring situation where a low-energy divergence *does* [1] reliably track the dependence on a heavy mass is when the divergence is logarithmic. In this case the argument just given leads to a different conclusion for a dimensionless observable $A$. Separating the low- and high-energy contributions to $A$ gives

$$A = A_{\rm le} + A_{\rm he} = a_0 \ln\left(\frac{M^2}{m^2}\right) + \cdots$$

while

$$A_{\rm he} = a_0' \ln\left(\frac{M^2}{\Lambda^2}\right) + \cdots, \quad A_{\rm le} = a_0'' \ln\left(\frac{\Lambda^2}{m^2}\right) + \cdots . \qquad (6)$$

In this case the cancellation of the cutoff dependence requires the condition $a_0 = a_0' = a_0''$, allowing the coefficient of the logarithm within the full theory to be determined from the coefficient of the low-energy logarithmic divergence.

At a practical level it is clear from the above that computing one-loop amplitudes with TGV's in a low-energy effective Lagrangian can only reliably determine any logarithmic dependence on the scale of new physics, $M$. All bounds on anomalous TGV's [3] which rely on quadratic or higher divergences in such calculations are therefore considerably overestimated.

All of these issues are particularly simple in dimensional regularization. The beauty of dimensional regularization is that it completely dispenses with cutoffs for controlling loop integrations and so no confusion between the cutoff $\Lambda$ and the heavy-physics scale $M$ is possible. This leads to a real distinction between the use of cutoffs and dimensional regularization in differentiating the high-energy and low-energy parts of an underlying theory. When using a cutoff, all frequencies above the scale $\Lambda$ are integrated out. This includes not only the heavy physics at scale $M \gg \Lambda$, but also the high-frequency components of the light fields. In dimensional regularization, one instead integrates out *only* the heavy physics; the momentum of the light fields in loops is still allowed to run to infinity. There is no conflict with the low-energy expansion, however, since dimensionally regularized divergent graphs in the low-energy theory are only proportional to light-particle masses. This last point also allows the matching between the effective and the underlying theories to be made at the heavy scale $M$ rather than at much lower scales as would be required with a cutoff.

All of these points are encapsulated in the "decoupling substrate" renormalization scheme [7], which consists of minimal subtraction when renormalizing between particle thresholds, supplemented by the explicit removal of heavy degrees of freedom as the renormalization point is lowered below the corresponding mass. The "integrating out" of the heavy particles is in practice implemented as a set of matching conditions for the appropriate effective couplings at these thresholds. The resulting couplings may then be used as initial conditions for the renormalization-group equations in the low-energy theory below the threshold. In this way one sees that the logarithmic dependence on the heavy mass scale $M$ simply reflects the effects of operator mixing as the effective interactions are renormalized down from the scale $M$ to low energies. In fact, this is probably the easiest way to calculate the logarithmic dependence on $M$, and hence to put reliable bounds on new physics.

*Conclusions.*— Many of the past analyses of anomalous trilinear gauge-boson vertices use effective Lagrangians in which only electromagnetic gauge symmetry is imposed. The neglect of electroweak gauge invariance in this work has been criticized recently by de Rújula and co-workers [2]. We argue here that this criticism is unjustified— these Lagrangians are the unitary-gauge versions of Lagrangians for which $SU_L(2)\times U_Y(1)$ gauge invariance is present, but nonlinearly realized. In this sense any theory containing light $W$'s and $Z$'s may be thought to be *automatically* $SU_L(2)\times U_Y(1)$ gauge invariant.

It is nevertheless true that many loop-generated bounds [3] on these effective couplings are incorrect. The culprit in these calculations is the misuse of cutoffs in estimating the size of loop diagrams. The point is that the cutoff dependence in the low-energy theory does not, in general, give an accurate indication of the true dependence on the heavy physics, although it can do so for a logarithmic divergence. The constraints on anomalous trilinear gauge boson couplings are therefore considerably weakened.

A final question then remains: Given that most of the constraints on anomalous TGV's are invalid, is it possible that these couplings might be large enough (i.e., $\sim 1$) to be seen relatively soon? The main issue which constrains the size of such operators is whether they would invalidate the low-energy expansion itself. An effective Lagrangian is only useful if higher-dimensional operators are suppressed by powers of $E^2/M^2$, where $E$ is a typical external energy and $M$ is the scale of the new physics. Now, if $\kappa$ is the dimensionless coefficient of a dimension-four effective interaction, then its contribution to scattering processes normalized to the lowest-order terms in the effective interaction will be of order $\kappa E^2/v^2$, where $v$ is the electroweak symmetry-breaking scale. For $\kappa\sim 1$ this is *not* small for the energies of practical interest. Rather, these effects are $O(E^2/M^2)$ only if $\kappa\sim O(v^2/M^2)$. These "power counting" limits are, in fact, the strongest bounds on anomalous TGV's. Were $\kappa$ of order 1, this would imply that the scale of new physics is $O(v)$, and it is likely that the new degrees of freedom would be directly observable, instead of through their contributions to TGV's.

Many thanks to Fawzi Boudjema, Steven Godfrey, Yossi Nir, Santi Peris, Xerxes Tata, and German Valencia for helpful criticism. This research was partially funded by funds from the NSERC of Canada and les Fonds FCAR du Québec.

(a)Electronic address: cliff@physics.mcgill.ca
(b)Electronic address: london@lps.umontreal.ca

[1] C. P. Burgess and David London, McGill University-Université de Montréal Reports No. McGill-92/04-UdeM-LPN-TH-83 and McGill-92/05-UdeM-LPN-TH-84, 1992 (to be published).

[2] A. de Rújula, M. B. Gavela, P. Hernandez, and E. Massó, CERN Report No. CERN-Th.6272/91, 1991 (to be published).

[3] M. Suzuki, Phys. Lett. **153B**, 289 (1985); A. Grau and J. A. Grifols, Phys. Lett. **166B**, 233 (1986); J. J. van der Bij, Phys. Rev. D **35**, 1088 (1987); G. L. Kane, J. Vidal, and C.-P. Yuan, Phys. Rev. D **39**, 2617 (1989); H. Neufeld, J. D. Stroughair, and D. Schildknecht, Phys. Lett. B **198**, 563 (1987); Y. Nir, Phys. Lett. B **209**, 523 (1988); J. A. Grifols, S. Peris, and J. Solà, Phys. Lett. B **197**, 437 (1987); Int. J. Mod. Phys. A **3**, 225 (1988); R. Alcorta, J. A. Grifols, and S. Peris, Mod. Phys. Lett. A **2**, 23 (1987); C. Bilchak and J. D. Stroughair, Phys. Rev. D **41**, 2233 (1990); P. Méry, S. E. Moubarik, M. Perrottet, and F. M. Renard, Z. Phys. C **46**, 229 (1990); F. Boudjema, C. P. Burgess, C. Hamzaoui, and J. A. Robinson, Phys. Rev. D **43**, 3683 (1991); R. D. Peccei and S. Peris, Phys. Rev. D **44**, 809 (1991); H. König, Phys. Rev. D **45**, 1575 (1992); D. London, Phys. Rev. D **45**, 3186 (1992); S. Godfrey and H. König, Phys. Rev. D **45**, 3196 (1992).

[4] A light Higgs requires a third, inequivalent possibility, in which the full $SU_L(2)\times U_Y(1)$ symmetry is linearly realized.

[5] M. S. Chanowitz, M. Golden, and H. Georgi, Phys. Rev. D **36**, 1490 (1987).

[6] S. Weinberg, Phys. Rev. **140**, B516 (1965); J. Polchinski, Nucl. Phys. **B231**, 269 (1984).

[7] S. Weinberg, Phys. Lett. **91B**, 51 (1980); F. Gilman and M. Wise, Phys. Rev. D **27**, 1128 (1983); R. Miller and B. McKellar, Phys. Rep. **106**, 169 (1984); I. Hinchliffe, in *TASI Lectures in Elementary Particle Physics; Weak Interactions and Modern Particle Theory*, edited by H. Georgi (Benjamin/Cummings, Menlo Park, 1984).