Stochastic Broadening of the Separatrix of a Tokamak Divertor

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The plasma in a modern tokamak is bounded by a separatrix between magnetic field lines that form toroidal magnetic surfaces, on which the plasma is confined, and open field lines that divert the plasma exhaust to so-called divertor plates. This separatrix is sharp in an ideal tokamak, but we show that magnetic perturbations create a stochastic region of open field lines inside the ideal separatrix which contains approximately 6 times the magnetic flux as does the strike points of these lines on the divertor plates. Since magnetic field lines are a $1\frac{1}{2}$ degree of freedom Hamiltonian system, the behavior of field lines in a tokamak divertor is archetypal for the similar Hamiltonian systems.

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The plasma in a modern tokamak [1] is bounded by a separatrix between magnetic field lines that form toroidal magnetic surfaces, on which the plasma is confined, and open field lines that divert the plasma exhaust to so-called divertor plates (Fig. 1). An ideal tokamak is axisymmetric, and the separatrix in an ideal tokamak is a sharp surface. Asymmetries in the magnetic field cause the last confining magnetic surface to lie inside the ideal separatrix and create a layer of open field lines between the last confining surface and the ideal separatrix, which we call the stochastic layer. Near the X point, the width of the stochastic layer is proportional to the square root of the toroidally asymmetric part of the magnetic field. The asymmetric part of the field may be caused by deviations in the tokamak coils from axisymmetry, magnetic perturbations due to plasma instabilities, or the effects of special coils introduced to control the width of the stochastic region.

An excessive concentration of heat on the divertor plates is a major issue in large, long-pulse tokamaks. In an axisymmetric divertor the spreading of the heat is due to plasma transport that is often modeled by a diffusion coefficient D which is of order 1 m^2/s. During the time τ that the plasma takes to flow to the divertor chamber, the plasma diffuses across open magnetic field lines that contain a flux which is proportional to $(D\tau)^{1/2}$. The open field lines that contain flowing plasma have approximately the same amount of magnetic flux near the main body of the plasma and near the divertor plates.

By adding asymmetric magnetic perturbations, one can widen the region of heat deposition on the divertor plates. However, we find that the magnetic flux of the open field lines in the stochastic region is roughly a factor of 6 larger than the width in flux of these field lines when they strike the divertor plate. In effect, the open field lines of the stochastic region encircle the plasma roughly 6 times before striking the divertor plates. The relatively thick region of plasma flowing along open field lines in the stochastic region may provide an important shielding of the main plasma from impurities.

Magnetic field line trajectories are the trajectories of a $1\frac{1}{2}$ degree of freedom Hamiltonian [2,3]. Consequently, the tokamak separatrix and divertor are archetypal for the behavior of a region of bounded Hamiltonian trajectories surrounded by a region of open trajectories. Plasmas flow rapidly along magnetic field lines and diffuse slowly across. The study of magnetic field line behavior and the closely related plasma behavior are important not only for their tokamak application but also for their application to other Hamiltonian, or near-Hamiltonian, systems.

The behavior of field lines near a tokamak separatrix has been studied previously. Early work [4] used the Cherikov island overlap criterion, which is a reasonable approximation for calculating the stochastic region but overestimates the spreading of heat on the divertor plates. Recently Pomphrey and Reiman [5] integrated magnetic trajectories in a field produced by wires. They noted the extremely thin spiral regions of impact of magnetic field lines started near the last closed surface on the divertor plates.

The basic features of trajectories near a separatrix are generic for a Hamiltonian system. Consequently, these features can be studied by an area-preserving map. The major advantage is that field line trajectories can be followed many orders of magnitude faster with a map than with an explicit integration. The speed of map iterations allows one to study the structures formed by the perturbations in more detail and to investigate the effects of qualitatively different types of perturbations.

The simplest map that represents a divertor configuration has a single positive parameter k,

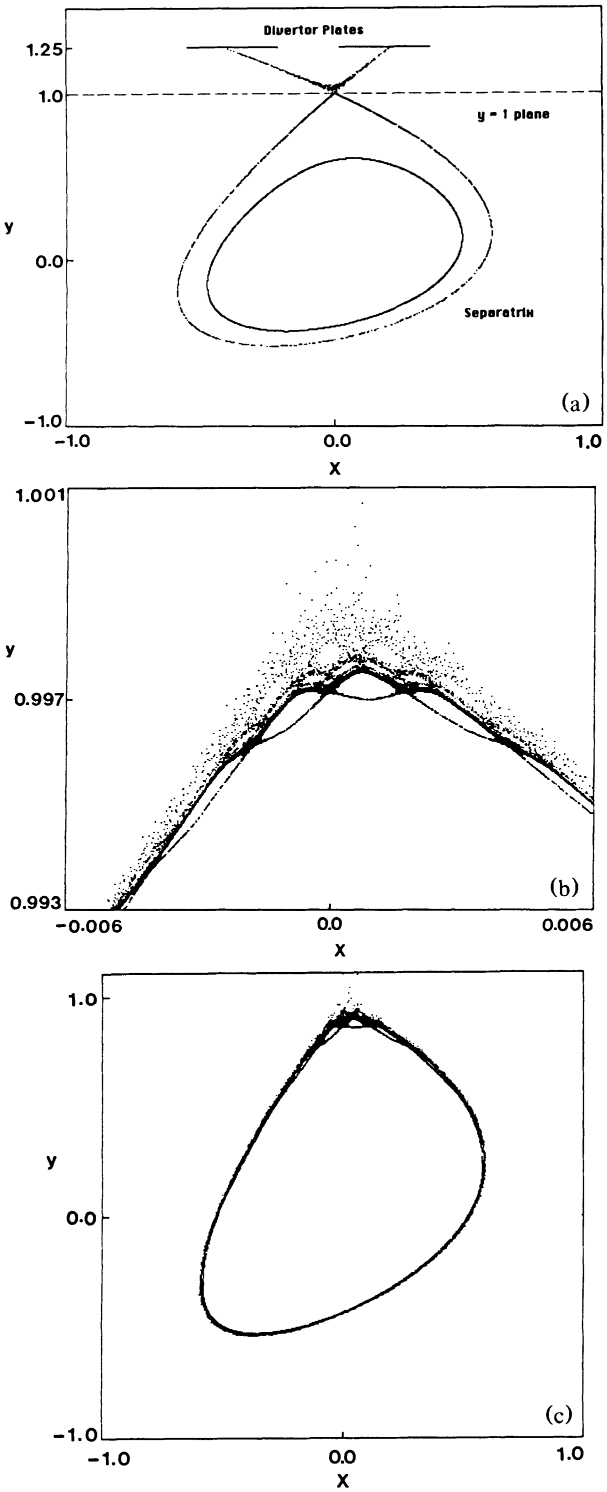

FIG. 1. (a) Phase portrait for the simple single-null tokamak divertor map when the map parameter $k=0.6$. The portrait shows the separatrix, $y=1$ plane, and the divertor plates. (b) Stochastic scrape-off layer near the X point when $k=0.6$. This portrait is made from iterations of the initial condition $x_0=0$, $y_0=0.9971$. (c) Phase portrait when $k=0.9$. For this k, the last good surface passes near $x=0$, $y=0.854$. The stochastic layer here is made from iterations with initial condition $x_0=0$, $y_0=0.85492$.

$$x_{n+1}=x_n-ky_n(1-y_n)\,, \tag{1}$$

$$y_{n+1}=y_n+kx_{n+1}\,, \tag{2}$$

an O point at $x=0$, $y=0$, and an X point at $x=0$, $y=1$ [Figs. 1(a)-1(c)]. The map provides a good representation of the magnetic field lines of a single-null tokamak divertor. Toroidal asymmetries can be simulated using k or by additional terms [6]. Here, toroidal asymmetries will be simulated using k, but the results are not qualitatively changed by other representations. The smaller k, the closer the last confining magnetic surface passes to the X point. For $k=0.6$, the closest approach is $x=0$, $y=0.997$ [Figs. 1(a) and 1(b)]; for $k=0.9$, the closest approach is $x=0$, $y=0.854$ [Fig. 1(c)]. Each iteration of the map corresponds to a toroidal angular advance ζ_0. Near the O point the rotational transform of the field lines per iteration $1/q_1$ is

$$\sin(2\pi/q_1)=k(1-k^2/4)^{1/2}\,. \tag{3}$$

One obtains a transform of roughly unity near the axis and of $\frac{1}{3}$ near the last confining magnetic surface if ten iterations of the map constitute a single toroidal circuit,

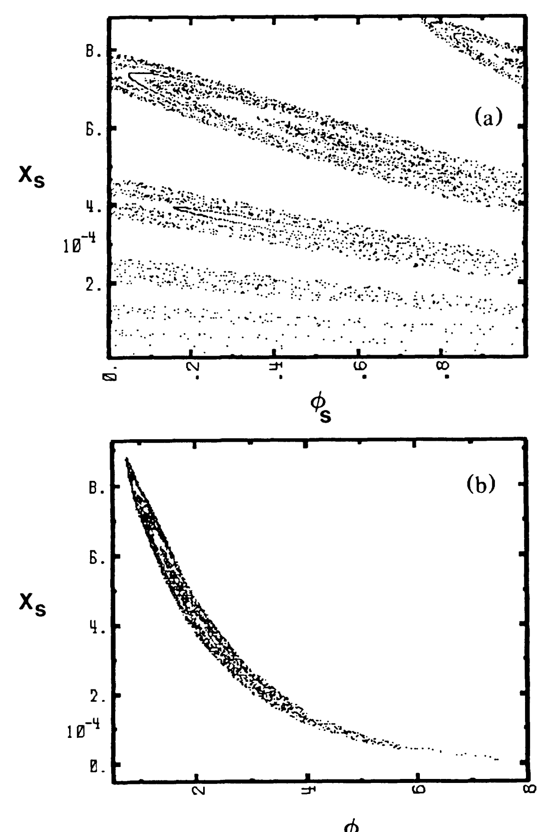

FIG. 2. (a) Footprint of the strike points on the divertor plate when the expansion coefficient $\delta=0$. Trajectories start in the stochastic scrape-off layer at $x_0=0$ and $0.997<y_0<1$. (b) When the footprint for $\delta=0$ is unfolded in the ϕ direction, we get a sharply defined, narrow, helical stripe.

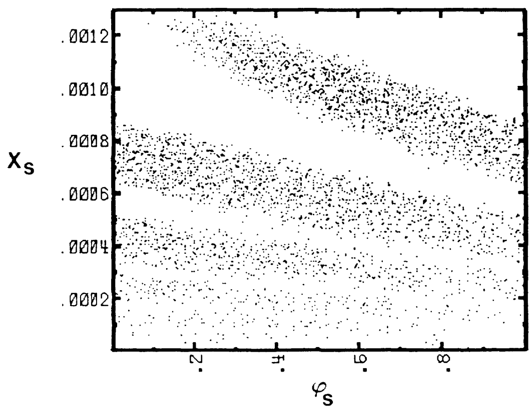

FIG. 3. Footprint of the strike points when $\delta = 10^{-13}$.

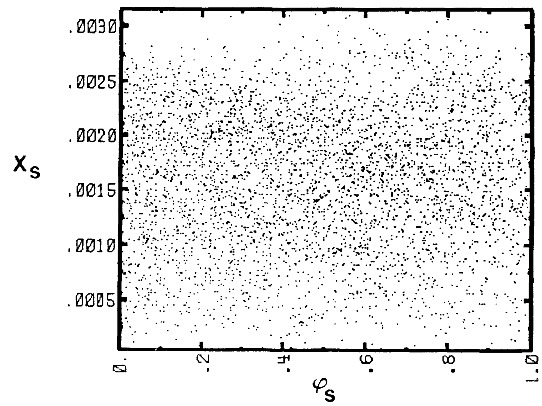

FIG. 4. Footprint when $\delta = 10^{-11}$. Now the footprint is a continuum, whose width scales as $\delta^{1/4}$.

$\zeta_0 = 2\pi/10$.

The width of the stripe on the divertor plates is uniquely determined by the stripe formed by field lines as they cross the plane $y=1$, which passes through the X point. The toroidal location ζ at which field lines cross the x-ζ plane is determined using a continuous analog to the discrete map. Letting $\phi = \zeta/\zeta_0$, $0 \le \phi \le 1$, the (area-preserving) continuous analog is

$$x(\phi) = x_n - k y_n (1 - y_n)\phi , \tag{4}$$

$$y(\phi) = y_n + k x(\phi)\phi . \tag{5}$$

These equations are the solutions of a 1 degree of freedom Hamiltonian system, with canonical coordinate x, canonical momentum y, and time parameter ϕ. This model permits us to investigate the Hamiltonian mechanics problem of a broadened separatrix.

The behavior of field line trajectories that cross the last confining surface at a random point can be assessed by adding a small area-expanding step to each iteration of the discrete map, $x_{\mathrm{new}} = (1 + R\delta) x_{\mathrm{old}}$, with R a random number between 0 and 1 and δ the expansion coefficient. For $k = 0.6$, we find that for $\delta \ll 10^{-13}$ the strike points form helical stripes that are independent of δ [Figs. 2(a) and 2(b)]. For larger values of δ the strike points form a single stripe (Figs. 3 and 4), which has a width that scales as $\delta^{1/4}$.

Particles that diffuse across the last confining surface of a tokamak with a divertor can follow open magnetic field lines to the divertor plates. The strike points of these field lines on the divertor plates lie in discrete stripes. The region between the last confining magnetic surface and the separatrix of the ideal divertor is called the stochastic layer. The open magnetic field lines of the stochastic layer enter the region from one divertor plate and leave striking another divertor plate after an integer number of complete poloidal circuits, N_p. The discrete stripes on the divertor plates correspond to the number of poloidal circuits made by open field lines. The area of the stripes scales as $1/N_p^2$. The open field lines that approach

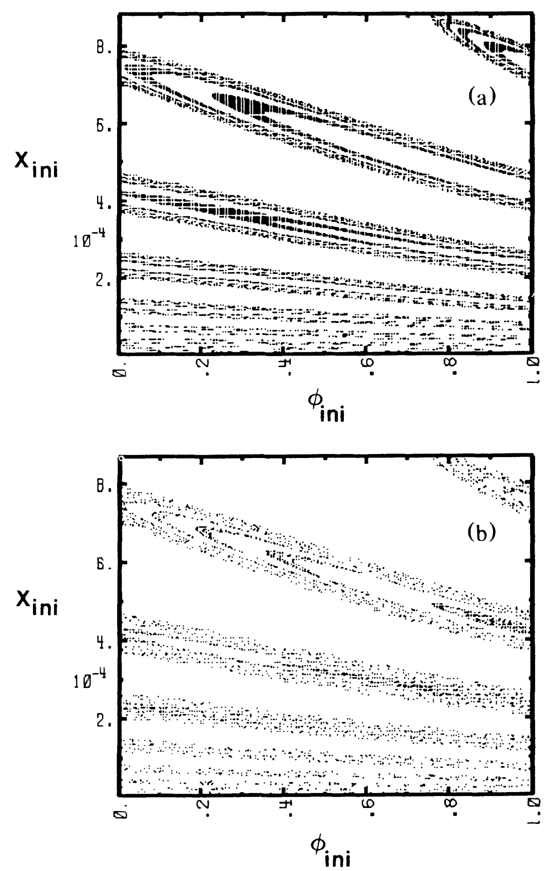

FIG. 5. (a) The points in the helical strip corresponding to the number of poloidal circuits, N_p, between 1 and 2 show concentric-ring-like structure. We reverse integrate a very large number of trajectories starting on the divertor plate on the positive x side. Trajectories start on uniformly distributed grid points in a rectangle exactly accommodating the helical strip. In this figure, we show the grid points whose reverse trajectories strike the divertor plate on the negative x side with $1 \le N_p < 2$. (b) Grid points whose reverse trajectories strike the other side of the divertor plate with $2 \le N_p < 3$.

the last confining surface make many poloidal circuits before striking a divertor plate in a stripe that is much narrower than the stochastic layer ($\approx 15\%$) but nonetheless of nonzero width. An open field line that makes many poloidal circuits in going from one divertor plate to another in essence densely fills the stochastic layer except for the regions in the stochastic layer that are occupied by magnetic islands. Within the helical stripe, the strike points corresponding to a fixed value of poloidal transits, N_p, show a structure of concentric rings [Figs. 5(a) and 5(b)]. The dwelling time in the toroidal direction for the field lines in the stripe has a fractal structure. The distribution of points in the stripe in the x direction is skewed towards the X point, while the distribution in the ϕ direction is skewed in the direction of increasing ϕ. The distribution in the transverse direction is Gaussian.

The complicated filling of the stochastic region by the open field lines gives an associated complexity to the ambipolar electric potential that exists along any open magnetic field line that is embedded in a plasma. Although we have not calculated the particle transport from the $\mathbf{E}\times\mathbf{B}$ drifts associated with the ambipolar potential, we expect a diffusive effect that would smear the stripe structures.

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