Electric Dipole Moment of the Top Quark in Higgs-Boson-Exchange Models of *CP* Nonconservation

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The leading contribution to the electric and the chromoelectric dipole moments of the top quark is calculated in Higgs-boson-exchange models of *CP* nonconservation. The dipole moments are typically of the order of 10^{-20} $e$cm and 10^{-20} $g$cm, respectively, and arise at one-loop order through neutral-Higgs-boson exchange. Several two-loop contributions are estimated to be smaller by about 2 orders of magnitude for the electric case and about 1 order of magnitude smaller for the chromoelectric case. The q^2 dependence of the dipole moment form factor is given for possible application to experimental searches.

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It is clear that in the coming years study of the properties of the top quark will receive a high priority from both the experimental and the theoretical perspective. The fact that the top is so heavy that it can rapidly undergo an electroweak decay to $b+W$ before hadronization sets in is especially significant. This should enable us to probe its fundamental properties directly with a minimal amount of the masking effects of QCD. That is in sharp contrast to other quarks wherein intricate bound-state effects get involved and, more often than not, are very difficult to handle. In this regard, we recall the many difficulties that arise in interpreting the electric dipole moment (EDM) of the neutron in terms of the underlying short-distance theory responsible for *CP* violation.

The objective of this study is to suggest that the EDM and the chromoelectric dipole moment (CEDM) of the top quark, in a wide class of extensions of the standard model (SM), are likely to be very large, i.e., about 10^{-20} $e$cm and 10^{-20} $g$cm, respectively. Not only are these numbers larger than in the SM by perhaps as much as 10 orders of magnitude [1], but more importantly, they may also be measurably large for experiments with hadron-hadron colliders [2] such as the Superconducting Super Collider (SSC) and the CERN Large Hadron Collider (LHC) or with future electron-positron colliders [3] such as the proposed SLAC Next Linear Collider (NLC). As is well known, *CP* violating effective interactions such as EDM and CEDM can be experimentally searched through *CP*-odd observables, e.g., triple product correlations or energy asymmetry. Using the generally advertised parameters of the SSC, LHC, and NLC and with the top mass in the range of 90–200 GeV, exploratory phenomenological studies indicate that the attainable upper limits for top CEDM and EDM are about 10^{-19}–10^{-20} $g$cm and 10^{-17}–10^{-18} $e$cm, respectively [2,3].

Our main purpose in this Letter is to present an evaluation of the dominant contribution to the dipole moments of the top quark in models with an arbitrary number of Higgs doublets and singlets satisfying natural flavor conservation (NFC) constraints [4]. We recall that, in such models, the rates for flavor changing neutral current (FCNC) transitions are held at an acceptably low level, in a natural way, by imposing a discrete symmetry [5]. This symmetry allows only all charge $+\frac{2}{3}$ quarks to get their masses from interactions with one Higgs doublet, say ϕ_1. Similarly, all charge $-\frac{1}{3}$ quarks interact only with another Higgs doublet, say ϕ_2. *CP* violation is triggered via scalar exchanges between quarks being driven by the imaginary parts of the complex quantities (e.g., $\tilde{Z}_{1n}$) defined as [6]

$$\frac{1}{(v_1)^2}\langle\varphi_1^0\varphi_1^0\rangle_q \equiv \sum_n \frac{\sqrt{2}G_F\tilde{Z}_{1n}}{q^2-m_{H_n}^2}, \tag{1}$$

$$\frac{1}{(v_1v_2^*)^2}\langle\varphi_2^+\varphi_1^{+*}\rangle_q \equiv \sum_n \frac{\sqrt{2}G_F Z_n}{q^2-m_{H_n'}^2}, \quad \text{etc.} \tag{2}$$

The summation here runs over all the mass eigenstates of neutral or charged scalar particles in the theory, G_F is the Fermi constant, and v_1,v_2 are the vacuum expectation values of φ_1^0 and φ_2^0, respectively. Note also that $\langle\chi\eta\rangle_q$ stands, for any pair of scalar fields, χ and η, for the momentum-dependent quantity [6]

$$\langle\chi\eta\rangle_q = \int d^4x\,\langle 0|T[\chi(x)\eta(0)]|0\rangle e^{-iqx}. \tag{3}$$

The dominant contribution to the top quark EDM in such a scenario originates from the one-loop diagram of Fig. 1, where *CP* violation arises from the exchange of a neutral Higgs boson φ_1^0. The static (i.e., at $q^2=0$) dipole moment from this figure is given by

$$\begin{aligned} d_t(0) &= \frac{2\sqrt{2}}{3(4\pi)^2}G_F m_t e\sum_n \mathrm{Im}\tilde{Z}_{1n} f\left(\frac{m_{H_n}^2}{m_t^2}\right) \\ &= (1.4\times10^{-19}\,e\,\mathrm{cm})\frac{m_t}{100\ \mathrm{GeV}}\sum_n \mathrm{Im}\tilde{Z}_{1n} f\left(\frac{m_{H_n}^2}{m_t^2}\right), \end{aligned} \tag{4}$$

where

$$f(r)=\begin{cases}1-\dfrac{r}{2}\ln r+\dfrac{r^2-2r}{\sqrt{r(4-r)}}\left[\arctan\left(\dfrac{2-r}{\sqrt{r(4-r)}}\right)+\arctan\left(\dfrac{r}{\sqrt{r(4-r)}}\right)\right], & \text{if } r<4,\\ 3-4\ln 2, \quad \text{if } r=4, & \\ 1-\dfrac{r}{2}\ln r-\dfrac{r^2-2r}{\sqrt{r(r-4)}}\ln\left(\dfrac{\sqrt{r}-\sqrt{r-4}}{2}\right), \quad \text{if } r>4, & \end{cases} \tag{5}$$

where r stands for $m_{H_n}^2/m_t^2$, for any value of n. For $r\gg 4$, $f(r)$ approaches

$$(1/r)(\ln r-\tfrac{3}{2}) \tag{6}$$

asymptotically.

We note that $\tilde{Z}_{1n}$'s and Z_n's satisfy some important sum rules [6], for example,

$$\sum_n \mathrm{Im}\tilde{Z}_{1n}=0\,. \tag{7}$$

As a consequence of Eq. (7), d_t will vanish if all the neutral Higgs bosons are degenerate in mass; no such degeneracy is, of course, expected. For illustrative purposes, we will assume, as usual [6], that the lightest Higgs bosons dominate the sum in Eq. (4). Henceforth, we only keep the lightest Higgs bosons in Eq. (4) and denote its mass by m_H and its CP violating parameters by $\mathrm{Im}\tilde{Z}_{10}$, $\mathrm{Im}Z_0$, etc. For example, if $m_H=2m_t$ and $m_t=125$ GeV, then d_t is about 4×10^{-20} e cm, assuming $\mathrm{Im}\tilde{Z}_{10}$ is of order 1. Indeed, as Weinberg has emphasized [6], in the absence of new physical constraints, the most plausible value of the CP nonconservation parameters, such as $\mathrm{Im}\tilde{Z}_{1n}$, is of order unity. In the rest of this paper, we will set $\mathrm{Im}\tilde{Z}_{10}$, $\mathrm{Im}Z_{10}$, etc., equal to $+1$ to enable us to present the numerical results conveniently.

Figure 2(a) shows the static dipole moment $d_t(0)$ as a function of $r_0\equiv m_H^2/m_t^2$. We see that $d_t(0)$ increases, almost linearly with m_t, for a fixed r_0, and unless $r_0<1$, $d_t(0)$ varies slowly with Higgs boson mass.

Properties of the top quark are most likely to be explored at high energy and momenta, thus static quantities like $d_t(0)$ are not the most accessible, from an experimental point of view. The EDM form factor at high q^2 may instead be more relevant. We, therefore, examine the q^2 dependence of d_t. Thus,

$$d_t(q^2)=\frac{2\sqrt{2}}{3(4\pi)^2}G_F m_t e\sum_n \mathrm{Im}\tilde{Z}_{1n} f\left(\frac{m_{H_n}^2}{m_t^2},\frac{q^2}{m_t^2}\right), \tag{8}$$

where

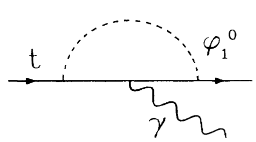

FIG. 1. Dominant one-loop contribution to d_t.

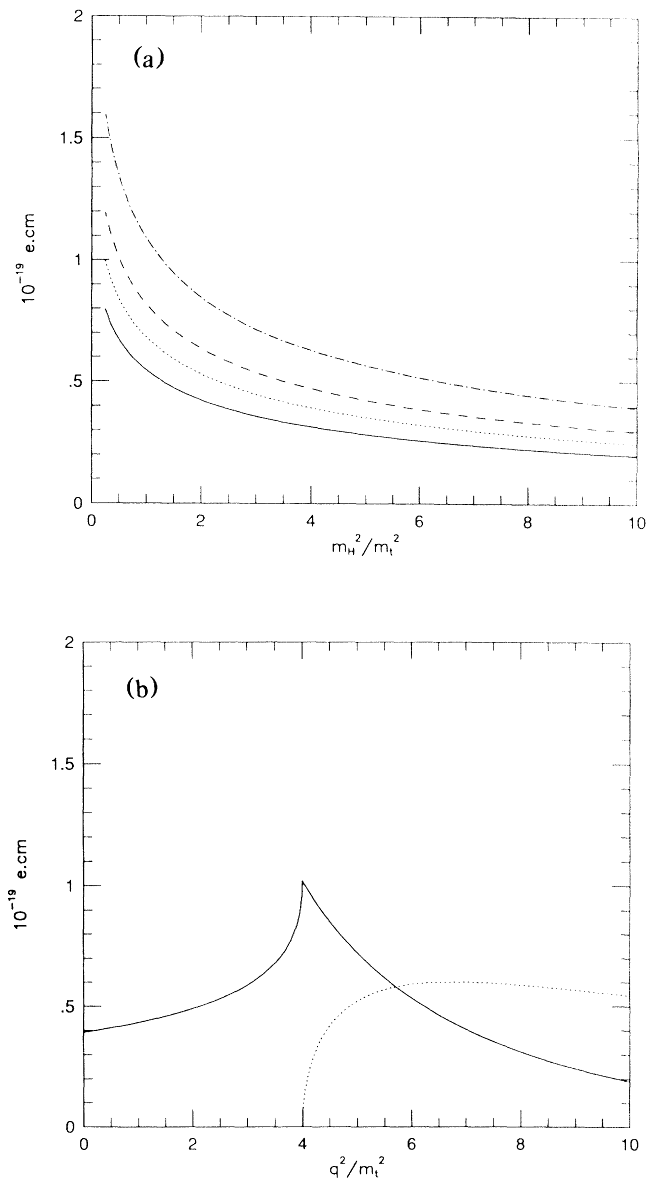

FIG. 2. (a) $d_t(0)$ vs m_H^2/m_t^2: upwards from bottom, $m_t=100$, 125, 150, and 200 GeV, respectively. Note that m_H is the mass of the lightest Higgs boson and all ImZ's are taken to be 1. (b) $d_t(q^2)$ vs q^2/m_t^2, for $m_H=2m_t=250$ GeV; real (solid) and imaginary (dotted) parts are shown. See also the caption to (a).

$$f(r,s)=\int_0^1 dx\int_0^{1-x} dy\,\frac{x+y}{(x+y)^2+(1-x-y)r-xys-i\epsilon}\,. \tag{9}$$

When q^2 is greater than $4m_t^2$, $d_t(q^2)$ will develop an imaginary part. Both the real and imaginary parts are shown in Fig. 2(b) where, for illustration, we have again chosen $m_t=125$ GeV and $m_H=2m_t$. The real part of the form factor peaks to a value slightly larger than 1×10^{-19} e cm at threshold and decreases quickly for $q^2\geq 4m_t^2$. The imaginary part peaks around $q^2/m_t^2\approx 6m_t^2$, decreasing very slowly for higher q^2. Indeed, both the real and imaginary parts may be most accessible in this region, i.e., $q^2/m_t^2\approx 6$.

If the photon in Fig. 1 is replaced by a gluon, then Fig. 1 would produce a chromoelectric dipole moment instead of EDM. Clearly, the CEDM (i.e., d_t^c) is immediately obtained by replacing $(\frac{2}{3}e)$ with g (the QCD coupling constant) in Eqs. (4) and (8). Thus $d_t^c\simeq 1.5 d_t$, where d_t is in e cm and d_t^c is in g cm.

CP violation through neutral-Higgs-boson exchange, which is necessary to drive the EDM and the CEDM of the top quark, can happen in theories with two Higgs doublets, with or without additional singlets [7]. On the other hand, *CP* violation through the charged-Higgs-boson exchange becomes operational when there are more scalar fields than just two Higgs doublets [8]. It can also contribute to the top dipole moments through the one-loop graph similar to Fig. 1. The effects due to charged-Higgs-boson exchange are nevertheless suppressed by a factor of m_b^2/M^2, where M^2 is a linear combination of m_t^2 and m_H^2. Thus the charged-Higgs-boson contribution is likely to be subleading compared to the neutral-Higgs-boson exchange.

To ensure that Fig. 1 is indeed the dominant contribution to the top quark EDM, we have also computed a set of two-loop diagrams shown in Fig. 3. We recall that the two-loop graphs [Figs. 3(a) and 3(b)] with the top loop [9] and with the W loop [9,10], respectively, are important for the EDM of the electron. The purely scalar exchange two-loop graph [Fig. 3(c)] has not been considered so far but could be important in the current context given the large Yukawa couplings. It is found that although they are important, these two-loop graphs are about 2 orders of magnitude smaller than the result from the one-loop neutral-Higgs-boson exchange. For example, the top EDM [i.e., $d_t(0)$] is about -2×10^{-22} e cm and about 4×10^{-22} e cm from Figs. 3(a) and 3(b), respectively, for $m_H=2m_t=250$ GeV. Note that in obtaining the above results, all powers of the external fermion mass (i.e., m_t) are kept, although in the existing calculations [9,10] for these graphs, which were applied to the electron (or the light quark), only the lowest power of m_e (or the light quark mass) needed to be retained.

Figure 3(c) is rather interesting since all exchanged particles are neutral scalars. In the case of the EDM for all the observed quark and leptons, Fig. 3(c) is entirely negligible. Since the top quark is much heavier, the contribution of this graph could be important. We have evaluated these types of diagrams, taking the three-scalar vertices from the term

$$\tfrac{1}{2}\eta(\phi_1^\dagger\phi_i)^2+\tfrac{1}{2}\eta^*(\phi_i^\dagger\phi_1)^2\,,$$

where ϕ_i is any other doublet but ϕ_1. It is found that, for example, $d_t(0)$ is about 10^{-23} e cm for $m_H=2m_t=250$ GeV and $d_t(0)\simeq10^{-22}$ e cm for $m_H=m_t=250$ GeV, assuming η of order 1 and all the vacuum expectation values are of the same order. Although the contribution of the two-loop scalar graph seems to increase quite rapidly with the top quark mass (for a fixed m_H), it still remains about an order of magnitude smaller than the one-loop case for $m_t\leq 500$ GeV.

There are many two-loop graphs contributing to CEDM of the top quark. Among them is the diagram considered in Ref. [11], by replacing photon lines in Fig. 3(a) with gluon lines. It gives $d_t^c(0)\simeq -1\times10^{-21}$ g cm for $m_H=2m_t=250$ GeV and $\alpha_s(m_t)\simeq0.1$, about an order of magnitude smaller than that from Fig. 1 [12].

To put the numbers that we obtain for the one-loop dominant contribution (i.e., $\approx10^{-20}$ e cm) in perspective, we recall that in the SM an elementary fermion gets its contribution to the EDM only at three loops [13] rendering $d_t^{\rm SM}$ extremely small, perhaps $\leq10^{-30}$ e cm [1,14]. It would certainly be useful to examine the issue of the top EDM in other specific extensions of the SM,

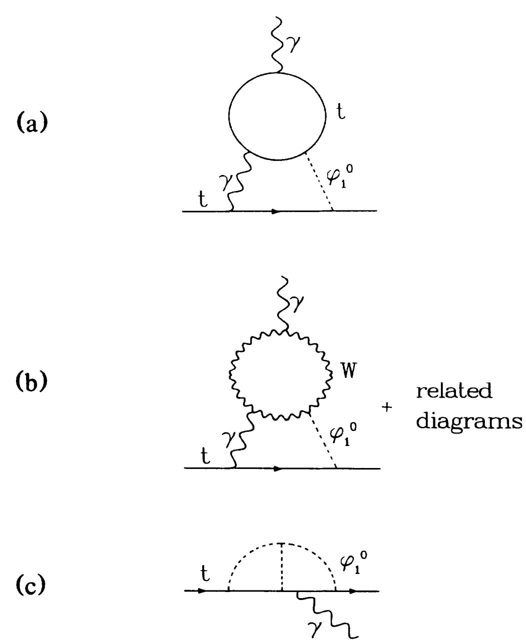

FIG. 3. Two-loop diagrams: (a) top loop, (b) W loop, and (c) all scalars.

such as left-right symmetric gauge theories, supersymmetry, etc. These and many other extensions of the SM, as a rule, entail an enlargement of the Higgs sector beyond the SM. Furthermore, unless some additional physical constraints are enforced, it is difficult in these models to have just the Kobayashi-Maskawa phase as the sole source of *CP* violation. One should therefore expect the EDM of the top quark to be significantly larger than in the SM. Be that as it may, an important issue that needs to be addressed is that of experimental measurability of the top dipole moments to enable us to enrich the pool of valuable constraints on *CP* violating parameters. In this context, as we mentioned earlier, hadron colliders with planned parameters should have a useful impact on theory as the dipole moments that are possible in some theoretical extensions of the SM appear within experimental reach [2].

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[14] Strong *CP* violation in SM can also contribute to top EDM. However, it is expected to be extremely small due to its nonperturbative nature. A very simplistic estimate of d_t^θ can be made by chirally rotating the quark fields such that the θ angle is absorbed in the quark mass matrix. The chiral rotation will also rotate part of the anomalous magnetic moment into EDM. Thus d_t^θ can at most be $\theta\sigma_t$, where σ_t is the anomalous magnetic moment of top quark. Assuming the one-loop gluon exchange makes the dominant contribution to σ_t, then $\sigma_t \sim 10^{-18}\ e\,\text{cm}$, yielding $d_t^\theta \ll \theta \times 10^{-18}\ e\,\text{cm}$. Thus $d_t^\theta \ll 10^{-27}\ e\,\text{cm}$, taking θ to be about 10^{-9}.

[15] W. Bernreuther, T. Schroder, and T. N. Pham, Heidelberg Report No. HD-THEP-91-39 (to be published). For the benefit of the reader we briefly point out the key differences between their work and ours: Their work focuses on the *CP* violating dipole moment form factors in $e^+e^- \to t\bar{t}$. It includes a discussion of the Z exchange contribution (which we did not address). However, they did not deal with the CEDM which is of relevance to SSC and LHC. Also they specifically use a two-Higgs-doublet model whereas our discussion is for an arbitrary number of Higgs doublets. Nevertheless, the numbers for the EDM obtained in their work are in general agreement with ours. We also address the size of the two-loop corrections. The article by Bernreuther, Schroder, and Pham also contains an extensive list of useful references to related issues that the reader may wish to consult.

[16] We also take the opportunity to mention that *CP* violation in top quark physics originating from neutral-Higgs-boson exchanges in an extended Higgs sector, similar to the class of models we are using, is also recently discussed by C. R. Schmidt and M. E. Peskin, Report No. SLAC-PUB-5788 (to be published), and by B. Grzadkowski and J. F. Gunion, Report No. UCD-92-7 (to be published). Also for a general discussion of *CP* violating effects in top quark see G. L. Kane, G. A. Ladinsky, and C.-P. Yuan, Phys. Rev. D **45**, 124 (1992).