Quantum Demolition Measurement of Photon Statistics by Atomic Beam Deflection

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(Received 20 April 1992; revised manuscript received 9 October 1992)

We consider the deflection of a resonant two-level atom by a quantized electromagnetic field using the Jaynes-Cummings Hamiltonian. We show that a joint measurement of the atomic momentum and an appropriate field variable allows us to reconstruct the original photon statistics even for this demolition Hamiltonian. We demonstrate that the momentum distribution of atoms scattered at the nodes of the standing wave also follows the original photon statistics of the field. In this sense a recent experiment on the optical Stern-Gerlach effect [T. Sleator *et al.*, Phys. Rev. Lett. **68**, 1996 (1992)] measures the intensity fluctuations of the standing wave.

PACS numbers: 42.50.Vk, 03.65.Bz, 32.80.Pj, 42.50.Dv

How can we extract information about a quantum system? A first step is to couple it to a meter, that is a system, which should be in a state approaching the classical limit [1]. A readout of this meter will then provide information about the quantum system. A fashionable example illustrating this measurement concept is atomic deflection [2] from a quantized electromagnetic field [3]: The quantum system to be measured is a single mode of the electromagnetic field and the meter is a resonant two-level atom interacting with this field. The momentum of the deflected atom is strongly correlated to the photon statistics of the quantum field [4].

But apart from a sensitive meter an effective measurement also needs a good measurement strategy. This strategy depends on the choice of the interaction. The quantum nondemolition (QND) scheme [5] relies on an interaction between field (system) and atom (meter) which does not provoke quantum transitions in the system [6]. Only the phases of the energy eigenstates change. Therefore the interaction preserves the photon statistics, which then can be extracted via measurement of the atomic variables [7].

In contrast a demolition interaction provokes transitions and changes the original photon distribution. In this case the meter is not able to give the original statistics in all details, and we need a more intricate strategy. In this paper we present such a strategy and show that a joint measurement [8] of the atomic momentum and the appropriate field variable allows us to reconstruct the photon statistics $W_m = |w_m|^2$ even for a demolition Hamiltonian. This joint measurement selects those scattering events in which the atoms have not changed the field appreciably, that is, it restricts the data analysis to those atoms which have passed through the nodes of the field. Moreover, we show that a mechanical mask with narrow slits at the nodes also creates a momentum distribution of the scattered atoms which follows precisely the photon statistics. We emphasize that this demolition strategy has a big advantage compared to the nondemolition one: It is based on a resonant interaction which is much stronger and therefore easier to detect experimentally [9], as has already been shown in a recent experiment [10] which scatters atoms at a node of a standing laser field.

Consider the deflection of a two-level atom by a standing, resonant light field in a state $|\psi\rangle = \sum_{m=0}^{\infty} w_m |m\rangle$, given in terms of number states $|m\rangle$ [11]. A de Broglie wave of the atom in the ground state $|b\rangle$ enters the light field perpendicular to the wave vector. We describe the interaction between the field and the atom by the resonant Jaynes-Cummings Hamiltonian

$$H_{\mathrm{int}} = \mu \mathcal{E}_0 \sin(kx)(\sigma_+ a + \sigma_- a^\dagger) , \tag{1}$$

where μ and $\mathcal{E}_0$ denote the dipole moment and the "electric field per photon," respectively. The annihilation and creation operators a and $a^\dagger$ allow the excitation of the field mode of wave vector $k = 2\pi/\lambda$. The Pauli matrices σ_+ and σ_- account for the atomic transitions. The electromagnetic wave is aligned along the x direction. In the Raman-Nath regime we ignore the contribution of the kinetic energy of the atoms, assuming that the transverse displacement during the interaction time τ is small compared to the wavelength λ.

For the spatial distribution of the atom before it enters the light field we take $|\Phi(x)\rangle = f(x)|b\rangle$, where $f(x)$ describes the normalized transverse distribution of the de Broglie wave. The state $|\Psi\rangle$ of the combined system of atom and field before the interaction is the direct product of the states $|\psi\rangle$ and $|\Phi\rangle$, that is, $|\Psi(t=0)\rangle = |\psi(t=0)\rangle\, |\Phi(x)\rangle$. After the interaction it reads

$$|\Psi(x,\tau)\rangle = f(x) \sum_{m=0}^{\infty} w_m \left\{ \cos[\kappa\sqrt{m}\sin(kx)]|m\rangle|b\rangle - i \sin[\kappa\sqrt{m}\sin(kx)]|m-1\rangle|a\rangle \right\} , \tag{2}$$

where $\kappa = \mu\mathcal{E}_0\tau/\hbar$ denotes the interaction parameter. Note that the interaction of the atom with the field has created a highly entangled state. Fourier transformation over x gives us the amplitude of the final state in the momentum representation:

$$|\tilde{\Psi}(\wp,\tau)\rangle = \sum_{m=0}^{\infty} w_m \{c_m(\wp)|m\rangle|b\rangle + s_m(\wp)|m-1\rangle|a\rangle\} , \tag{3}$$

where

$$c_m(\wp) = \sum_{\ell=-\infty}^{\infty} \frac{J_{2\ell}(\kappa\sqrt{m})}{\sqrt{2\pi}k} \int_{-\infty}^{\infty} d\theta f\left(\frac{\theta}{k}\right) e^{i(\wp+2\ell)\theta}, \tag{4}$$

$$s_m(\wp) = \sum_{\ell=-\infty}^{\infty} \frac{J_{2\ell+1}(\kappa\sqrt{m})}{\sqrt{2\pi}k} \int_{-\infty}^{\infty} d\theta f\left(\frac{\theta}{k}\right) e^{i(\wp+2\ell+1)\theta}. \tag{5}$$

Here $J_\ell(x)$ is the ℓth Bessel function, $\wp = p/\hbar k$ denotes the atomic momentum scaled in photon momentum, and $\theta = kx$ is the scaled length. We show that due to the entanglement between the momentum $\wp$, the internal atomic states, and the field states we can reconstruct the original photon statistics of the field.

Now we calculate the probability amplitude to find the atom with momentum $\wp$ in an internal state $|j\rangle$ and the field in a given reference state $|\varphi\rangle = \sum_{m=0}^{\infty} \varphi_m|m\rangle$. The corresponding scalar product reads

$$\langle j|\langle\varphi|\tilde{\Psi}(\wp,\tau)\rangle = \sum_{m=0}^{\infty} w_m\{ \varphi_m^* c_m(\wp)\langle j|b\rangle + \varphi_{m-1}^* s_m(\wp)\langle j|a\rangle\} . \tag{6}$$

The conditional probability $\overline{W}(\wp,\varphi,j)$ to find the atom with momentum $\wp$ provided the measurement of the field has given the value φ follows from Eq. (6) after normalization with respect to φ, that is,

$$\overline{W}(\wp,\varphi,j) = |\langle j|\langle\varphi|\tilde{\Psi}(\wp,\tau)\rangle|^2 \left(\sum_{\wp,j}|\langle j|\langle\varphi|\tilde{\Psi}(\wp,\tau)\rangle|^2\right)^{-1} .$$

We ignore the internal state of the atom, that is, trace over $|a\rangle$ and $|b\rangle$, and arrive at

$$\overline{W}(\wp,\varphi) = \mathcal{N}\left\{\left|\sum_{m=0}^{\infty} w_m\varphi_m^* c_m(\wp)\right|^2 + \left|\sum_{m=0}^{\infty} w_m\varphi_{m-1}^* s_m(\wp)\right|^2\right\} ,$$

where $\mathcal{N}$ accounts for the normalization.

Assume that the amplitudes φ_m of the reference state $|\varphi\rangle$ vary slowly with m in the region where w_m essentially differs from zero. In this case we can factor them out of the sum and arrive at

$$\overline{W}(\wp,\varphi) = \mathcal{N}\left\{\left|\sum_{m=0}^{\infty} w_m c_m(\wp)\right|^2 + \left|\sum_{m=0}^{\infty} w_m s_m(\wp)\right|^2\right\} . \tag{7}$$

Now we compare this expression to the momentum distribution for the case when we ignore the information stored in the field. We trace in Eq. (3) over the field as well as the atomic variables and find

$$W(\wp) = \sum_{m=0}^{\infty} |w_m|^2 c_m^2(\wp) + \sum_{m=0}^{\infty} |w_m|^2 s_m^2(\wp). \tag{8}$$

In contrast to Eq. (7) the probability $W(\wp)$, Eq. (8), does not contain any cross terms $w_n w_m^*$. Hence all interference properties are lost in this result.

To bring out the importance of these interference terms we now consider a rectangular transverse distribution of atoms which is constant over many wavelengths of the standing light field. We perform the integrations in Eqs. (4) and (5) and the momentum distributions $\overline{W}(\wp,\varphi)$ and $W(\wp)$, Eqs. (7) and (8), read

$$\overline{W}(\wp,\varphi) = \mathcal{N} \sum_{\ell=-\infty}^{\infty} \delta(\wp-\ell) \left|\sum_{m=0}^{\infty} w_m \, J_\wp(\kappa\sqrt{m})\right|^2 \tag{9}$$

and [4]

$$W(\wp) = \sum_{\ell=-\infty}^{\infty} \delta(\wp-\ell) \sum_{m=0}^{\infty} |w_m|^2 J_\wp^2(\kappa\sqrt{m}) . \tag{10}$$

We note that the spatial periodicity of the standing wave has led to quantization of momentum in $\hbar k$.

The role of the interference terms comes out in Fig. 1, where we compare the momentum distributions $\overline{W}(\wp,\varphi)$, Eq. (9), and $W(\wp)$, Eq. (10), for the case of a highly squeezed and displaced state [12]. We note that $\overline{W}(\wp)$ follows closely the photon statistics W_m, shown as the lower curve of Fig. 1, while only a few features of W_m survive in $W(\wp)$. This is a consequence of the fact that the kernel of the sum Eq. (9), $J_\wp(\kappa\sqrt{m})$, due to its oscillatory character, acts more like a delta function [13] than the kernel $J_\wp^2(\kappa\sqrt{m})$ in Eq. (10), which extends over a broader region. We replace in Eq. (9) summation by integration; then the asymptotic expression [4]

$$J_\wp(\kappa\sqrt{m}) \cong \sqrt{\frac{2}{\pi}}(\kappa^2 m - \wp^2)^{-1/4} \cos(S - \tfrac{1}{4}\pi)\Theta(\kappa\sqrt{m} - \wp)$$

immediately suggests for the conditional distribution

$$\overline{W}(\wp,\varphi) \cong \mathcal{N}\left(\frac{2\wp}{\kappa^2}\right)^2 W_{m=(\wp/\kappa)^2}, \tag{11}$$

whereas the total distribution $W(\wp)$, Eq. (10), reduces in the same approximation [4] to

$$W(\wp) \cong \int_{(\wp/\kappa)^2}^{\infty} \frac{dm}{\pi\kappa} \frac{W_m}{\sqrt{m - (\frac{\wp}{\kappa})^2}} = \frac{2}{\pi\kappa} \int_0^{\infty} dy \, W_{m=(\wp/\kappa)^2 + y^2}.$$

We emphasize that the direct mapping of W_m on $W(\wp)$ requires large κ values. In some sense this implies the

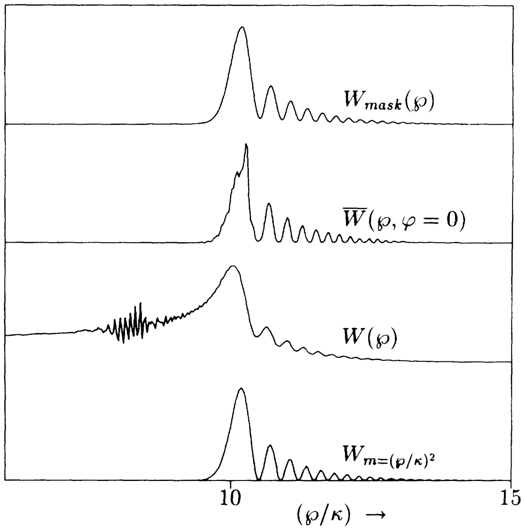

FIG. 1. The photon statistics of a squeezed displaced state of squeezing parameter $s = 50$ and displacement parameter $\alpha = 10$ (lower curve) and its readout via the momentum distributions of deflected atoms, Eqs. (9), (10), and (12) with interaction parameter $\kappa = 110$. The curve $\overline{W}(\wp, \varphi = 0)$ corresponds to a joint measurement of the atomic momentum and the field phase, Eq. (9), whereas the distribution $W(\wp)$, given by Eq. (10), ignores the field phase. The top curve, $W_{\text{mask}}(\wp)$, Eq. (12), gives the momentum distribution of atoms filtered by a mask of slit width $d = \lambda/10$ placed at the nodes of the standing wave. The joint measurement strategy gives an adequate readout, while ignoring the field phase results in an less effective readout as well as in additional rapid oscillations. We note that there is a modulation on the left side of the first maximum of $\overline{W}(\wp, \varphi = 0)$ and the period of the oscillations is slightly different from W_m. The latter results from a beating between the oscillations in the Bessel function and the oscillations in W_m in the sum Eq. (9). The jagged structure at the first maximum is a result of the discrete summation over m and is an early manifestation of the revival structure in $W(\wp)$ around $\wp/\kappa = 5$. This rapid contribution becomes more dominant for increasing κ. The momentum distribution W_{mask} obtained by the mask does not show the rapid part and the period of the oscillations in $\wp$ matches those in W_m.

transition to a classical meter. The experimentally accessible values for κ reach from $\kappa = 1$ [14] in the optical regime to $\kappa = 120$ in the microwave domain [6].

Now we return to the key assumption used in deriving Eq. (7) and ask for a reference state $|\varphi\rangle$ which has slowly varying amplitudes φ_m. For real coefficients w_m it is the London phase state [15] corresponding to zero phase, $|\varphi = 0\rangle = (2\pi)^{-1/2} \sum_{m=0}^{\infty} |m\rangle$, which makes Eqs. (7) and (9) exact. This state brings out the physics of this joint measurement strategy: From all scattering events we keep only those which have not altered the phase of the initial field state. This of course requires that the original field had a rather well-defined phase. This implies that the initial photon distribution is rather broad compared to the oscillations of the Bessel functions. A field being initially in a single number state $|m_0\rangle$ violates this condition. Since the Fock state does not have a well-defined phase a joint measurement cannot improve over an ordinary measurement. The conditional momentum distribution $\overline{W}(\wp, \varphi) = J_\wp^2(\kappa\sqrt{m_0}) = W(\wp)$ is therefore identical to $W(\wp)$.

An experiment based on this joint measurement strategy must compare the phase of the field state after the interaction to that of the initial state. Only when they coincide do we record the momentum of the scattered atom. Such an experiment might be difficult to realize. However, the key feature of this joint measurement strategy is to select those scattered atoms which created a minimum of disturbance of the field. These are apparently the atoms which have passed through the nodes of the field and hence the selection caused by the joint measurement procedure plays the role of a spatial filter. A mask put in front of the electromagnetic field which transmits atoms only in the vicinity of the nodes of the standing wave works in the same way [16] and therefore gives an adequate readout of the photon statistics. Indeed in this case the initial spatial distribution $f(x)$ consists of rectangular peaks of width $d \ll \lambda$ centered at the nodes of the field. We perform the integrals in Eqs. (4) and (5) and arrive at the momentum distribution of the scattered atom:

$$W_{\text{mask}}(\wp) = \sum_{\ell=-\infty}^{\infty} \delta(\wp - \ell) \sum_{m=0}^{\infty} |w_m|^2 g_m(\wp, kd) , \tag{12}$$

where

$$g_m(\wp, kd) = 2 \left| \sum_{r=-\infty}^{\infty} J_{r-\wp}(\kappa\sqrt{m}) \frac{\sin(rkd/2)}{r\sqrt{\pi kd}} \right|^2 . \tag{13}$$

The top curve of Fig. 1 shows this momentum distribution W_{mask} for atoms sent through such a mask. It reproduces precisely the photon statistics. The mathematical reason lies in the behavior of g_m for $kd \ll 1$: The function $r^{-1}\sin(rkd/2)$ varies slowly over the oscillations of the Bessel function which then acts as a delta function, and Eq. (13) reduces to

$$g_m(\wp, kd) \cong (2\pi)^{-1} \frac{\sin^2[(|\wp| - \kappa\sqrt{m})kd/2]}{(|\wp| - \kappa\sqrt{m})^2\,(kd/2)} . \tag{14}$$

In the limit of large κ, g_m is a distribution sharply peaked at $|\wp| = \kappa\sqrt{m}$, and hence selects the term $W_{m=(\wp/\kappa)^2}$ out of the sum Eq. (12).

We note that Eq. (12) is valid provided that (i) the deflection of the atoms in the light wave is small compared to the wavelength, i.e., $\kappa\frac{(\hbar k)^2}{2M}\frac{\tau}{\hbar} \ll 1$, and (ii) the width d of the slit cannot be too small as to avoid diffraction, i.e., $\hbar k\tau/M \ll d$. In order to achieve an adequate readout the function g_m has to be narrow compared to the typical variations of the photon distribution, i.e., $1 \ll \kappa kd$. All three conditions have been satisfied in a recent experiment on the optical Stern-Gerlach effect using a beam of metastable He atoms [10]. There a single slit of width $d = 2\ \mu$m has been put in front of a standing light wave of period $\lambda = 15\ \mu$m. Although in this experiment the scattering does not occur from a quantized field, but from a laser field, we think that it is the first indication that deflection of atoms at the nodes is a sensitive probe for the intensity fluctuations in the wave: The finite width of the side peaks in Fig. 3(b) of Ref. [10] reflects the fluctuations of the scattering wave.

We conclude by noting that this photon-distribution-from-node-scattering method is not restricted to the on-resonance Hamiltonian but of course could also be applied to the QND off-resonance case [8]. Moreover, this method does not require small cavity decay rates since only one atom has to be scattered and not many consecutive atoms.

This work was supported by the Deutsche Forschungsgemeinschaft.

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when we employ the fact that z is large and expand $\sin\theta$ in a Taylor series.

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