Structure-Function Approach to Vector-Boson Scattering in pp Collisions

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(Received 23 June 1992)

We discuss weak-vector-boson scattering, at next-to-leading order in QCD, within the framework of hadronic structure functions. We use this approach to calculate the Higgs-boson production cross section via vector-boson fusion at the CERN Large Hadron Collider and the Superconducting Super Collider; we find a modest increase over the leading-order prediction. We also give expressions for the distribution of vector bosons in a proton (effective-W approximation) including $O(\alpha_s)$ corrections.

PACS numbers: 13.85.Qk, 12.38.Bx

The CERN Large Hadron Collider (LHC) and the Superconducting Super Collider (SSC) will be the first machines capable of studying weak-vector-boson scattering, where the "initial" vector bosons are radiated from the quarks and antiquarks which reside in the proton. It is believed that this process will yield clues to the mechanism which breaks the electroweak symmetry [1], which is one of the outstanding puzzles of high-energy physics. For example, in the standard Higgs model of electroweak-symmetry breaking, the Higgs boson is copiously produced via vector-boson fusion [2,3].

Weak-vector-boson scattering may be calculated in terms of the charged-current and neutral-current hadronic structure functions $F_i(x,Q^2)$ $(i=1,2,3)$. A similar approach had been applied to two-photon processes [4], and was generalized to vector-boson scattering by Lindfors [5]. In this paper we discuss the necessary conditions for the factorization implicit in this approach to hold. We find that the factorization is valid to $O(\alpha_s)$, and it therefore allows a considerable simplification of the calculation of the $O(\alpha_s)$ correction to vector-boson scattering. The correction is simply incorporated by employing the QCD-corrected expressions for the structure functions in the parton model. The resulting cross section is differential in the vector-boson scattering subprocess.

As usual, we write the hadronic tensor $W_{\mu\nu}$ in terms of the three structure functions:

$$MW_{\mu\nu}(x,Q^2)=F_1(x,Q^2)\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)+\frac{F_2(x,Q^2)}{P\cdot q}\left(P_\mu-\frac{P\cdot q}{q^2}q_\mu\right)\left(P_\nu-\frac{P\cdot q}{q^2}q_\nu\right)-i\frac{F_3(x,Q^2)}{2P\cdot q}\epsilon_{\mu\nu\rho\sigma}P^\rho q^\sigma\,, \quad (1)$$

where M is the proton mass, P_μ is the proton four-momentum, q_μ is the vector-boson four-momentum, $Q^2=-q^2$, and $x=Q^2/2P\cdot q$. Vector-boson scattering in pp collisions is then calculated by contracting the hadronic tensors at each vertex with the tensor corresponding to the square of the vector-boson-scattering subprocess, as in Fig. 1. One finds [5]

$$d\sigma=\frac{1}{2S}4\frac{g_{V_1}^2}{8}\frac{g_{V_2}^2}{8}\frac{1}{(Q_1^2+M_{V_1}^2)^2}\frac{1}{(Q_2^2+M_{V_2}^2)^2}M^2W_{\mu\nu}(x_1,Q_1^2)\mathcal{M}^{\mu\rho}\mathcal{M}^{*\nu\sigma}W_{\rho\sigma}(x_2,Q_2^2)$$
$$\times\frac{d^3P_{X_1}}{(2\pi)^3 2E_{X_1}}\frac{d^3P_{X_2}}{(2\pi)^3 2E_{X_2}}ds_1 ds_2 d\Gamma(2\pi)^4\delta^4(P_1+P_2-P_{X_1}-P_{X_2}-P_{VV})\,, \quad (2)$$

where $g_W=g$, $g_Z=g/\cos\theta_W$, S is the square of the total machine energy, $s_i=(P_i+q_i)^2$ is the squared invariant mass of the remnant of proton i, and $d\Gamma$ is the vector-boson-scattering phase space.

At lowest order, no color is exchanged between the protons, the remnants of the protons are color singlets. It follows that no QCD corrections due to gluon exchange between the first proton (or its remnant) and the second proton (or its remnant), or between either proton (or their remnants) and the products of the vector-boson-scattering subprocess, occur at $O(\alpha_s)$. In terms of quark diagrams, one sees that diagrams with a gluon exchanged between quarks from different protons, or between a quark and the products of the vector-boson-scattering subprocess, do not interfere with the tree diagram once a color summation is performed. Therefore, the $O(\alpha_s)$

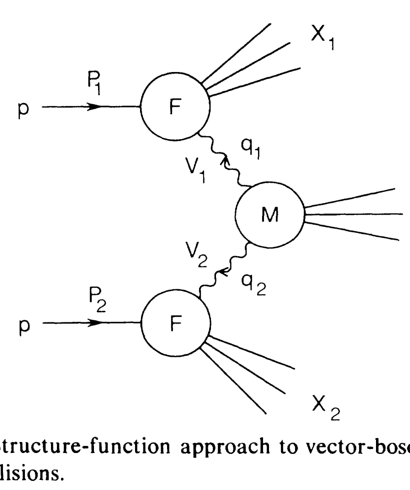

FIG. 1. Structure-function approach to vector-boson scattering in pp collisions.

correction to vector-boson scattering may be factorized into the corrections to the structure functions and the corrections to the vector-boson-scattering subprocess. This factorization is no longer true at $O(\alpha_s^2)$, however. The structure functions are given at next-to-leading order in QCD by [6]

$$\begin{aligned}F_1(x,Q^2) &= \sum_i (C_{Vi}^2+C_{Ai}^2)\left\{\int_x^1 \frac{dy}{y}[q_i(y,Q^2)+\bar{q}_i(y,Q^2)]\left[\delta(1-z)-\frac{4}{3}\frac{\alpha_s(Q^2)}{\pi}z\right]\right. \\ &\qquad \left. -2\frac{\alpha_s(Q^2)}{\pi}\int_x^1\frac{dy}{y}g(y,Q^2)z(1-z)\right\}, \\ F_2(x,Q^2) &= 2x\sum_i (C_{Vi}^2+C_{Ai}^2)[q_i(x,Q^2)+\bar{q}_i(x,Q^2)], \\ F_3(x,Q^2) &= 4\sum_i C_{Vi}C_{Ai}\left\{\int_x^1\frac{dy}{y}[-q_i(y,Q^2)+\bar{q}_i(y,Q^2)]\left[\delta(1-z)-\frac{2}{3}\frac{\alpha_s(Q^2)}{\pi}(1+z)\right]\right\},\end{aligned} \tag{3}$$

where $z=x/y$, the sum runs over the flavors of all quarks and antiquarks which contribute to a given structure function, $C_{Vi}=C_{Ai}=1/\sqrt{2}$ for $W^\pm$, and $C_{Vi}=T_{Li}^3-2e_i\sin^2\theta_W$, $C_{Ai}=T_{Li}^3=\pm\frac{1}{2}$ for Z. These expressions correspond to the deep-inelastic scattering (DIS) factorization scheme. The corresponding expressions in the modified minimal-subtraction ($\overline{\rm MS}$) scheme are obtained by replacing the quark and antiquark distribution functions in the leading-order terms by [7]

$$\begin{aligned}q_i(x,Q^2) &= q_i^{\overline{\rm MS}}(x,Q^2)+\frac{1}{4}\frac{\alpha_s(Q^2)}{\pi}\int_x^1\frac{dy}{y}g(y,Q^2)\left[[z^2+(1-z)^2]\ln\left(\frac{1-z}{z}\right)+8z(1-z)-1\right] \\ &\quad +\frac{2}{3}\frac{\alpha_s(Q^2)}{\pi}\int_x^1\frac{dy}{y}q_i(y,Q^2)\left[(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+-\frac{3}{2}\frac{1}{(1-z)_+}-\frac{1+z^2}{1-z}\ln z+3\right. \\ &\qquad \left. +2z-\left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-z)\right],\end{aligned} \tag{4}$$

where the "plus" prescription is defined as usual (see, e.g., Ref. [6]).

If the structure functions, over the relevant ranges of x and Q^2, were available from deep-inelastic scattering, they could be used directly, bypassing the parton model altogether. The typical Q^2 in vector-boson-scattering processes is M_V^2, which is within the reach of the DESY ep collider HERA, but only for $x\gtrsim 0.1$ [8]. The possibility of using the measured structure functions directly will thus be limited to very high invariant masses for the vector-boson-scattering subprocess, $M_{VV}\sim 0.1\sqrt{S}$. CERN e^+e^- collider LEP combined with LHC would be able to reach down to $x\sim 0.01$ at this value of Q^2 [9].

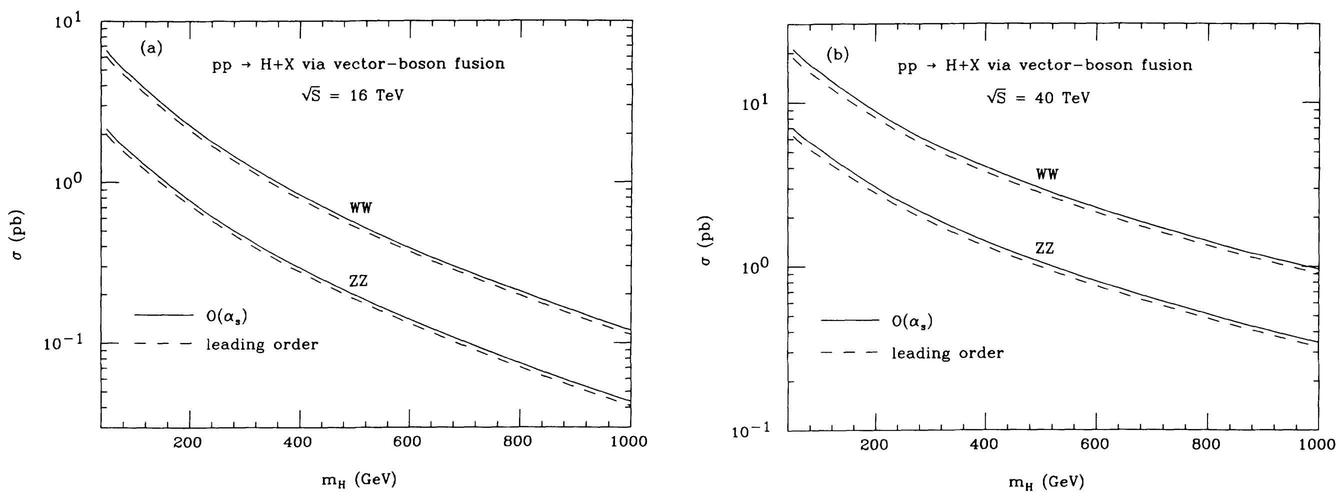

FIG. 2. Total cross section for Higgs-boson production via vector-boson fusion at the (a) LHC and (b) SSC, vs the Higgs-boson mass. The solid curves are calculated at next-to-leading order in QCD, using set S1-DIS of Ref. [13]. The dashed curves are calculated at leading order, using the leading-order set of Ref. [13]. The cross sections due to an intermediate W^+W^- and ZZ pair are shown separately.

The validity of this approach relies on the factorization of the cross section that is inherent in Fig. 1. In order for the factorization to hold, two criteria must be satisfied. First, there must be no significant interference, at the parton level, between diagrams in which a quark from one proton and a quark from the other proton scatter into the same final-state quark. Kinematic arguments suggest that this interference is very small [10], and we have verified that this is true in the example below. Second, the vector-boson-subprocess final state must be produced only via this mechanism, or dominantly so, or not interfere with similar final states. Examples include Higgs-boson production [2,3], heavy-fermion production [11], and terms of enhanced electroweak strength in longitudinal-vector-boson scattering [1,12].

As an explicit example of this approach, we calculate Higgs-boson production via vector-boson fusion at the LHC and SSC. The differential cross section is given by Eq. (2), with [5]

$$\begin{aligned}
M^2 W_{\mu\nu}(x_1,Q_1^2)\mathcal{M}^{\mu\rho}\mathcal{M}^{*\nu\sigma}W_{\rho\sigma}(x_2,Q_2^2) = g_V^2 M_V^2 \\
\times \Bigg\{ F_1(x_1,Q_1^2)F_1(x_2,Q_2^2)\left[2+\frac{(q_1\cdot q_2)^2}{q_1^2 q_2^2}\right] \\
+\frac{F_1(x_1,Q_1^2)F_2(x_2,Q_2^2)}{P_2\cdot q_2}\left[\frac{(P_2\cdot q_2)^2}{q_2^2}-M^2+\frac{1}{q_1^2}\left(P_2\cdot q_1-\frac{P_2\cdot q_2}{q_2^2}q_1\cdot q_2\right)^2\right] \\
+\frac{F_2(x_1,Q_1^2)F_1(x_2,Q_2^2)}{P_1\cdot q_1}\left[\frac{(P_1\cdot q_1)^2}{q_1^2}-M^2+\frac{1}{q_2^2}\left(P_1\cdot q_2-\frac{P_1\cdot q_1}{q_1^2}q_1\cdot q_2\right)^2\right] \\
+\frac{F_2(x_1,Q_1^2)F_2(x_2,Q_2^2)}{P_1\cdot q_1 P_2\cdot q_2}\left(P_1\cdot P_2-\frac{P_1\cdot q_1 P_2\cdot q_1}{q_1^2}\right. \\
\left.-\frac{P_2\cdot q_2 P_1\cdot q_2}{q_2^2}+\frac{P_1\cdot q_1 P_2\cdot q_2 q_1\cdot q_2}{q_1^2 q_2^2}\right)^2 \\
+\frac{F_3(x_1,Q_1^2)F_3(x_2,Q_2^2)}{2P_1\cdot q_1 P_2\cdot q_2}(P_1\cdot P_2 q_1\cdot q_2-P_1\cdot q_2 P_2\cdot q_1)\Bigg\}. \qquad (5)
\end{aligned}$$

For W^+W^- fusion one must sum over the W^+ (and the W^-) being emitted from either proton.

We show in Fig. 2 the total cross section for Higgs-boson production at the LHC and SSC through $O(\alpha_s)$, using the next-to-leading-order parton distribution functions of Ref. [13], set S1-DIS. Also shown are the leading-order cross sections, evaluated using the leading-order parton distribution functions of Ref. [13], which were fitted to the same data as set S1-DIS. The next-to-leading-order cross section at the SSC is 12% larger at $m_H=100$ GeV, and 6% larger at $m_H=800$ GeV, than the leading-order cross section. (A heavy Higgs boson is sufficiently wide that one may also want to consider non-resonant contributions to vector-boson pair production. As mentioned above, terms of enhanced electroweak strength may be obtained via the structure-function approach. However, terms of ordinary electroweak strength cannot, since they involve diagrams which do not factorize.) The corresponding increase at the LHC is 8% and 6%. Most of the increase is due to the parton distribution functions, not to the explicit $O(\alpha_s)$ corrections to the structure functions, because the total cross section is dominated by F_2, which receives no QCD correction in the DIS scheme. To the extent that the next-to-leading-order parton distribution functions provide a better fit to the available data, and a more reliable extrapolation to high Q^2, the next-to-leading-order cross sections in Fig. 2 represent the best predictions possible at this time.

The structure-function approach to vector-boson scattering makes it clear that the relevant scale of the structure functions, and therefore of the parton distribution functions, is the Q^2 of the vector boson. Since the typical Q^2 is about M_V^2, the use of this fixed scale is a good approximation, as is well known. Since the parton model breaks down at low Q^2, and the parton distribution functions are available only for $Q^2>4$ GeV^2, we have not included the region $Q^2<4$ GeV^2 in our calculation. We estimate that this region contributes only about (4 GeV^2)/$M_V^2 \sim 10^{-3}$ of the total cross section.

A heavy Higgs boson decays predominantly to W- and Z-boson pairs. The principal background to this signal at the LHC and SSC is W^+W^- and ZZ production via quark-antiquark annihilation. The QCD correction to the ZZ invariant-mass distribution increases the leading-order prediction by about 25% near threshold, up to about 60% at 800 GeV (for a factorization scale $\mu^2=M_{ZZ}^2$) [14]. The corresponding increase for W^+W^- is about 45% near threshold and about 75% at 800 GeV [15]. The absence of a comparable increase in the Higgs-boson production cross section therefore decreases the signal-to-background ratio. At any rate, the more precise estimate of the production cross section obtained from our calculation can only aid in extracting the signal.

In the standard Higgs model, the Higgs boson is also produced at the LHC and SSC from gluon fusion via a top-quark loop [16] with a cross section comparable to or larger than that from vector-boson fusion (for $m_t > 100$ GeV, $m_H < 800$ GeV). The QCD correction to this process is known only for a light ($m_H < 2m_t$) Higgs boson [17], and increases the cross section by 50% at the SSC and 100% at the LHC (for $\mu^2 = m_H^2$). It is important to observe Higgs-boson production via gluon fusion and vector-boson fusion separately, since the former involves the coupling of the Higgs boson to the top quark, while the latter does not. It may be possible to separate the two mechanisms experimentally by tagging the forward jets associated with the vector-boson-scattering process [18].

If the invariant mass of the vector-boson-scattering subprocess is large compared to M_V, the cross section is dominated by the region $Q^2 \lesssim M_V^2$. One can then derive a distribution function for vector bosons carrying a fraction x of the proton's momentum [3,19]. In terms of hadronic structure functions, we find

$$
\begin{aligned}
f_T(x) &= \frac{g_V^2}{32\pi^2}\frac{1}{x}\int_x^1 \frac{dy}{y}\left[F_2(y,M_V^2)\left(1-\frac{x}{y}\right)+F_1(y,M_V^2)\frac{x^2}{y}\right]\ln\left[1+\frac{Sy(y-x)}{M_V^2}\right],\\
f_L(x) &= \frac{g_V^2}{32\pi^2}\frac{1}{x}\int_x^1 \frac{dy}{y}\left[F_2(y,M_V^2)\left(1-\frac{x}{2y}\right)^2-\frac{1}{2}F_1(y,M_V^2)\frac{x^2}{y}\right],
\end{aligned}
\tag{6}
$$

where the subscripts T,L denote the polarization of the vector boson (the two transverse polarizations have been averaged). The expression for $f_L(x)$ agrees with Eq. (22) of Ref. [5]. At leading order in QCD, $F_2(y,Q^2) = 2yF_1(y,Q^2)$ (Callan-Gross relation), and the expressions above reduce to the usual parton-model formulas. The $O(\alpha_s)$ expressions for the vector-boson distribution functions are obtained simply by using the next-to-leading-order expressions for the structure functions, Eq. (3). It has already been shown that there are no terms of $O(\alpha_s\pi)$ (from soft gluons) in the DIS scheme [20] (although there are in the $\overline{\text{MS}}$ scheme), as is evident from Eq. (3). In any case, the $O(\alpha_s\pi)$ terms generally do not dominate at LHC and SSC energies.

We are grateful for conversations with S. Dawson, D. Dicus, K. Ellis, W. Giele, J. Morfin, C. Quigg, G. J. van Oldenborgh, and W.-K. Tung. T.H. was supported by an SSC Fellowship from the Texas National Research Laboratory Commission (TNRLC) under Award No. FCFY9116. S.W. was supported by an SSC Fellowship from the TNRLC and by Contract No. DE-AC02-76-CH-00016 with the U.S. Department of Energy.

Note added.— After submitting this paper for publication, we received a report in which the structure-function approach is applied to Higgs-boson production in *ep* collisions [21].

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[1] M. Chanowitz and M. K. Gaillard, Nucl. Phys. **B261**, 379 (1985).
[2] R. Cahn and S. Dawson, Phys. Lett. **136B**, 196 (1984).
[3] G. Kane, W. Repko, and W. Rolnick, Phys. Lett. **148B**, 367 (1984).
[4] M.-S. Chen, I. Muzinich, H. Terazawa, and T. Cheng, Phys. Rev. D **7**, 3485 (1973).
[5] J. Lindfors, Phys. Lett. **167B**, 471 (1986).
[6] G. Altarelli, R. K. Ellis, and G. Martinelli, Nucl. Phys. **B143**, 521 (1978); **B146**, 544(E) (1978); **B157**, 461 (1979).
[7] W. Bardeen, A. Buras, D. Duke, and T. Muta, Phys. Rev. D **18**, 3998 (1978).
[8] J. Feltesse, in *Proceedings of the HERA Workshop, Hamburg, 1987*, edited by R. Peccei (DESY, Hamburg, 1988), p. 33.
[9] J. Blümlein, J. Feltesse, and M. Klein, in *Proceedings of the ECFA Large Hadron Collider Workshop, Aachen*, edited by G. Jarlskog and D. Rein (CERN Report No. 90-10, 1990), p. 830.
[10] D. Dicus and S. Willenbrock, Phys. Rev. D **32**, 1642 (1985).
[11] O. Éboli *et al.*, Phys. Rev. D **34**, 771 (1986); Phys. Lett. B **178**, 77 (1986); S. Dawson and S. Willenbrock, Nucl. Phys. **B284**, 449 (1987).
[12] Z. Kunszt and D. Soper, Nucl. Phys. **B296**, 253 (1988).
[13] J. Morfin and W.-K. Tung, Z. Phys. C **52**, 13 (1991).
[14] J. Ohnemus and J. Owens, Phys. Rev. D **43**, 3626 (1991); B. Mele, P. Nason, and G. Ridolfi, Nucl. Phys. **B357**, 409 (1991).
[15] J. Ohnemus, Phys. Rev. D **44**, 1403 (1991).
[16] H. Georgi, S. Glashow, M. Machacek, and D. Nanopoulos, Phys. Rev. Lett. **40**, 692 (1978).
[17] S. Dawson, Nucl. Phys. **B359**, 283 (1991); A. Djouadi, M. Spira, and P. Zerwas, Phys. Lett. B **264**, 440 (1991).
[18] R. Cahn, S. Ellis, R. Kleiss, and W. J. Stirling, Phys. Rev. D **35**, 1626 (1987); V. Barger, K. Cheung, T. Han, J. Ohnemus, and D. Zeppenfeld, Phys. Rev. D **44**, 1426 (1991); for a recent study, see SDC Technical Design Report No. SDC-92-201, 1992 (to be published).
[19] M. Chanowitz and M. K. Gaillard, Phys. Lett. **142B**, 85 (1984); S. Dawson, Nucl. Phys. **B249**, 42 (1985).
[20] S. Dawson, Phys. Lett. B **217**, 347 (1989).
[21] J. Blumlein, G. J. van Oldenborgh, and R. Rückl, CERN Report No. CERN-TH-6581-92 (to be published).