Quantum Nucleation of Vortices in the Flow of Superfluid ^{4}He through an Orifice

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Flow measurements in ultrapure ^{4}He through a micron-size orifice at millikelvin temperatures show, for the first time, the transition from thermal to quantum nucleation of nanometer-size vortices below a crossover temperature of 0.147 K. These observations establish the close relationship between this type of critical flow and negative-ion motion in superfluid ^{4}He and strongly suggest that the underlying mechanism, the nucleation of vortex half rings, is identical.

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There is continuing interest in the problem of critical velocities in superfluid ^{4}He and the nucleation of quantized vortices [1-5]. From a number of independent studies, it has become apparent that the critical velocity v_c through apertures less than about one micron in size increases in a near-linear fashion as the temperature is reduced below 1 K, following a relation of the form $v_c = v_{c0}(1-T/T_0)$ with $T_0 \simeq 2.45$ K. Slightly different temperature dependences have been reported recently [6-8], indicating possible variations with the nature of the substrate and the measuring technique. This temperature variation of v_c in a range where the internal variables of the superfluid such as the superfluid density ρ_s or the vortex core radius a_0 are independent of temperature is a distinctive feature which we have attributed [1,9] to the nucleation of vortices by thermal activation. The rate of such processes is expressed by the Kramers formula:

$$\Gamma_{\rm cl} = \frac{\omega_0}{2\pi} \exp\left\{ -\frac{E_a(u_s)}{k_B T} \right\}. \tag{1a}$$

The problem here, tackled first in the vicinity of the lambda point by Iordanskii and by Langer and Fisher (ILF) [10], is to determine the activation energy E_a in terms of the superfluid velocity at the nucleation site u_s, and the attempt frequency $\omega_0/2\pi$.

As the energies involved are small, the nucleated vortices are of very small size, of the order of a few nm. It can then be expected that, below some crossover temperature T_q, thermal nucleation will give way to quantum tunneling, in which case the rate becomes temperature independent and is given by [11]

$$\Gamma_q = \frac{\omega_0}{2\pi}\left(864\pi \frac{E_a}{\hbar\omega_0}\right)^{1/2} \exp\left\{ -\frac{36}{5}\frac{E_a}{\hbar\omega_0}\left[1+\frac{45\zeta(3)}{2\pi^3\omega_0\tau}\right]\right\}, \tag{1b}$$

ω_0 being related, within small corrections in the case of low-damping systems ($\omega_0\tau \ll 1$), to the thermal attempt frequency [11]. The crossover temperature T_q between the thermal and the quantum regimes is quite generally related to the attempt frequency ω_0 [11]:

$$k_B T_q = \hbar\omega_0/2\pi. \tag{2}$$

The existence of such thermal and quantum nucleation phenomena has been established both theoretically [12] and experimentally [13] for negative ions moving at high speed ($\sim$55 m/s) through superfluid ^{4}He. We report here the first observation of the saturation of the critical velocity v_c as T is lowered below 150 mK in ultrapure ^{4}He at zero pressure. We interpret this effect as evidence for vortex nucleation by quantum tunneling, bringing significant and novel information on this problem, in particular on ω_0 which is otherwise an elusive quantity to determine.

These new experimental results were obtained using the same double-hole hydromechanical resonator, operated at a frequency $\omega/2\pi$ of 12.5 Hz, with the same micro-orifice and procedures as those described in Ref. [9], but with two significant improvements which may explain why the saturation of v_c went unnoticed in our earlier work [14]. First, the entire experimental setup has been moved to new, more massive and rigid surroundings with improved vibration damping and sound proofing to achieve a very high degree of decoupling from external mechanical disturbances [15]. Second, refined signal filtering and processing techniques have been implemented which permit a full analysis of the cell operation, including a drastic rejection of spurious signals and a determination of both the trapped bias current and of the initial state of circulation of the loop between the two holes of the resonator [15,16].

The ^{4}He samples used in these experiments were purified in the laboratory using both conventional distillation and the heat flush effect [17] to a concentration x_3 of ^{3}He impurities of 0.9 parts per 10^9 (ppb) measured [18] on a gaseous sample collected after completion of the experiments. Typical results for $v_c(T)$ from $\sim$15 mK up to the highest temperature at which our superconducting displacement sensor operates are shown in Fig. 1. The plateau in v_c at low temperature is the characteristic and reproducible behavior of ultrapure ^{4}He in this cell, as is the constant slope decrease at high temperature, v_c extrapolating to zero at $T_0 = 2.45$ K as in all our previous

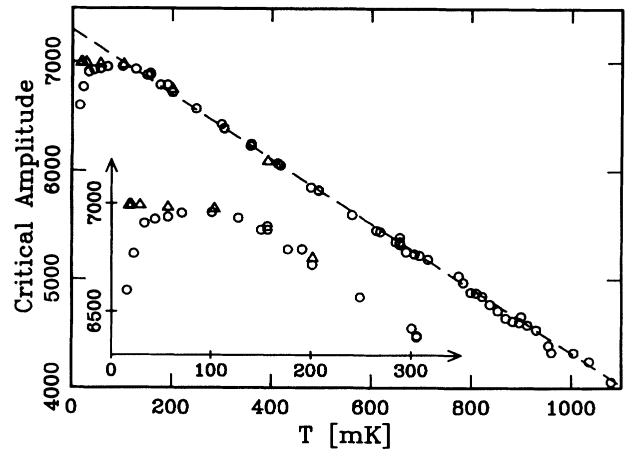

FIG. 1. Critical resonator amplitude, in arbitrary units, vs temperature. The inset shows the low-temperature droop due to residual ^{3}He impurities (○) as compared to the plateau characteristic of ultrapure ^{4}He behavior (△).

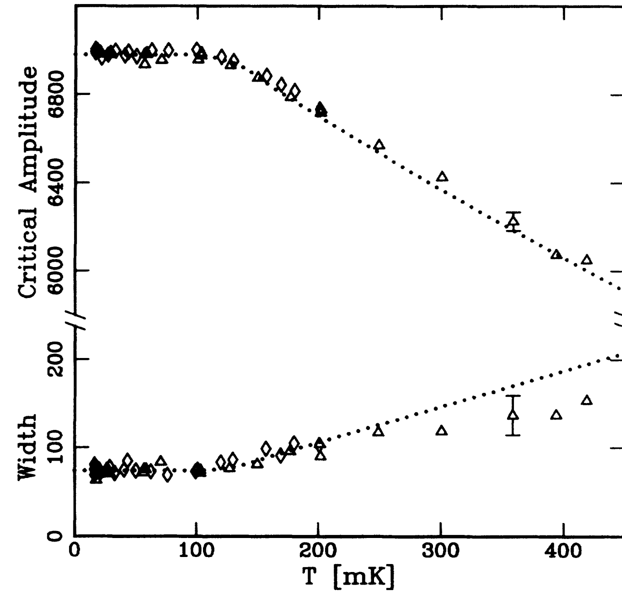

FIG. 2. Critical resonator amplitude and statistical width, in arbitrary units, of the critical transition vs temperature in ultrapure ^{4}He for two runs. Critical amplitudes have been normalized by the size of the 2π phase slips, set to 36.0 units, and the contribution of trapped circulation has been taken into account. The dotted lines are model calculations, as explained in the text.

measurements. The inset in Fig. 1 displays the effect of ~ 7 ppb of ^{3}He impurities [19]. The mean critical velocity and the width of its statistical distribution (see below) for the highest purity sample ($x_3 \simeq 0.9$ ppb) are shown in Fig. 2. It is experimentally established that the plateau is due neither to the lack of thermal equilibrium nor to residual impurities.

It is more difficult to rule out the effect of some extrinsic apparatus vibration. All our efforts to permanently alter the plateau level or the crossover temperature by changing the experimental conditions have failed: We believe that we are observing a phenomenon intrinsic to the superfluid and that the fluctuations taking over below T_q must be of a quantum nature. This point of view is further promoted by the consideration of the following vortex nucleation model which describes the transition from the thermal to the quantum nucleation regimes.

Along the lines of our previous attempts [1,9] and following the ideas of Muirhead, Vinen, and Donnelly [12], we consider the nucleation of a vortex half ring perpendicular to a smooth plane wall, with an axis opposite to the local superfluid velocity $\mathbf{u}_s$, that is, we simply extend to wall nucleation the ILF theory [10] in which the vortex "free" energy is written as

$$\mathcal{E}_v = \mathcal{E}_0 - \mathcal{P}_0 v . \tag{3}$$

The half-ring energy and impulse are given in terms of its radius r by $\mathcal{E}_0 = r\eta/4$ and $\mathcal{P}_0 = r^2/4$ when we use dimensionless units such that lengths are scaled by the core parameter b, energies by $E^* = \rho_s \kappa_4^2 b$, and velocities by $v^* = \kappa_4/2\pi b$ so that $v = u_s/v^*$. The core parameter b is related to the core radius a_0, for a hollow-core classical ring [20], by $b = e^2 a_0/8$ so that the quantity η is $\ln r$. These classical hydrodynamic expressions are valid for bulk vortices and for $r \gg 1$. The deviations as $r \rightarrow 1$ have been studied by Jones and Roberts [21] who find, as confirmed by Dalfovo [4], that $\mathcal{E}_0$ and $\mathcal{P}_0$ do not decrease with r as rapidly as for the classical ring and that vortex rings in the bulk cease to exist altogether for $r < 1$. More directly relevant to our problem is the work of Sonin [22] on the behavior of vortex filaments close to walls, who reached the conclusion that their energy per unit length goes to zero at the wall as the square of the distance from the wall with a certain characteristic length. This dynamic healing length, also considered in Ref. [23], is not well defined in the presence of vortices and is comparable to and probably larger than the static healing length computed by Hills and Roberts [24]. Thus, for computational convenience as well as to at least in part take into account the results of Refs. [21,22] at $r \gtrsim 1$, we have put in the following: $\eta = \frac{1}{2}\ln(1+r^2)$. This regularization artifice has the important physical consequence that the vortex self-velocity $\partial\mathcal{E}_0/\partial\mathcal{P}_0$ goes through a maximum $v_{c0} = 0.432$ at $r_m = 1.26$ and decreases to zero as r. A surface vorticity sheet in a metastable state builds up at the wall and constitutes the system which either tunnels through or hops over the energy barrier [25]. None of the complexities of the interplay between the imposed superflow, the surface vorticity, and the nucleating vortex are included in Eq. (3). We shall adjust b phenomenologically to approximate reality, and hence, we convey to it a physical significance which clearly differs from that of the core parameter and which is more like the length of the vortex.

Vortex "free" energies given by Eq. (3) are shown for several velocities in the inset of Fig. 3. The activation energy $\mathcal{E}_a(v)$ is the difference, when $v \leq v_{c0}$, between the

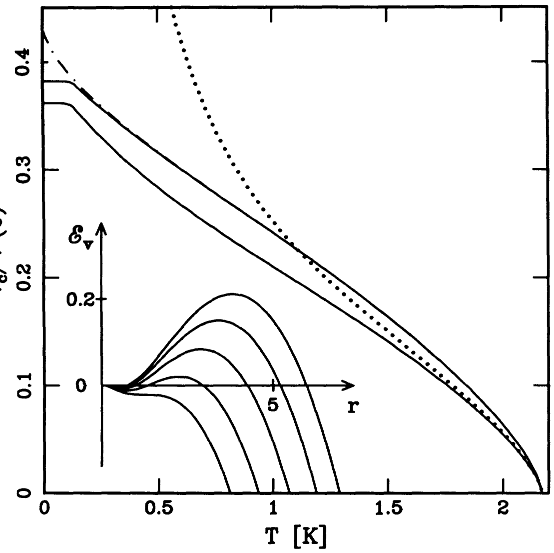

FIG. 3. Critical velocities, normalized to $v^* = \kappa_4/2\pi b_0$, computed in terms of the temperature for two values of b_0, 9.44 Å (top, solid curve) and 4.72 Å (bottom, solid curve), as explained in the text. The dash-dotted curve corresponds to the top plain curve without quantum corrections. The dotted curve is the outcome of the present model for half-ring nucleation when using the same approximations as ILF [10]. The inset shows the vortex reduced "free" energy given by Eq. (3) in terms of the reduced half-ring radius r with v equal to 0.308, 0.320, 0.347, 0.385, and $0.432 = v_{c0}$ from top to bottom. These values of v are taken at the temperatures of 2.17, 1, 0.5, 0.147, and 0 K, respectively, in the (classical) case of the dash-dotted curve.

values of the minimum and the maximum of $\mathcal{E}_v(r)$. It must be evaluated numerically except for $\epsilon = (1 - v^2/v_{c0}^2)^{1/2} \ll 1$ where it can be expanded in a power series of ϵ,

$$\mathcal{E}_a = \tfrac{2}{3} \mathcal{E}_J \epsilon^3 , \tag{4}$$

with $\frac{2}{3}\mathcal{E}_J = \frac{1}{2} v_{c0} r_m^{3/2} (r_m^2 + 1)^{1/2} (3r_m^2 - 1)^{-1/2}$. Equation (4) holds in a quite general way about the point where the system ceases to be metastable and "runs away" spontaneously. This situation has been studied extensively [11] in the context of superconducting junctions and macroscopic quantum tunneling, in which case $\mathcal{E}_J$ is the "Josephson-junction" energy. We can thus use the results obtained in this closely related field to solve the vortex nucleation problem. Then, the nucleation rate Γ at all temperatures between the two limits of Eqs. (1a) and (1b) can be obtained using the work of Grabert, Olschowski, and Weiss [26].

Critical velocities are computed as in Refs. [1,10] by assuming that, when Γ exceeds some rate Γ_{obs} which depends on the method of observation, critical events are being observed and the critical velocity v_c is reached:

$$\mathcal{E}_a(v_c) = \gamma k_B T / E^* , \tag{5}$$

with $\gamma = \ln \Gamma / \Gamma_{obs}$. For energy barriers of the form (4), γ can be computed explicitly, and to logarithmic accuracy, is equal to $\ln(0.1\omega_0/\omega)$. The reduced critical velocity v_c is obtained by solving numerically Eq. (5), except when expansion (4) is valid, in which case it can be expressed analytically as

$$v_c = v_{c0} \left[1 - \left\{ \frac{3}{2} \frac{k_B T}{E^* \mathcal{E}_J} \gamma \right\}^{2/3} \right]^{1/2} . \tag{6}$$

The temperature dependence of v_c, shown in Fig. 3, comes in an explicit way from Eq. (5) but also from the quantum corrections [26], and from the implicit dependence of ρ_s and a_0, taken from Refs. [27,28], respectively. In the numerical calculations, we have adjusted $E^*(0)$ in such a way that the temperature variation of v_c between 0.2 and 0.5 K is a near-straight line which intersects the temperature axis at $T_0 = 2.45$ K, yielding $E^*(0) = 98.7$ K. This value is also used for the computed curve for v_c in Fig. 2. The lower curve in Fig. 3, whose overall slope between 0.4 and 1.8 K also leads to $T_0 = 2.45$ K, is obtained for $E^*(0) = 49.3$ K.

The statistical width of the critical transition is defined as in Ref. [1] and, when expansion (4) holds, is expressed by

$$\Delta v_c = v_{c0} \frac{2}{\ln 2} \frac{x(1-x^2)}{3(\frac{1}{2} + \gamma)x^2 - 1} , \tag{7}$$

with $x = v/v_{c0}$. Equation (7) depends only on the critical velocity and on γ. As shown in Fig. 2, it yields a fair description of the measured width: The same value of γ accounts both for T_q and the temperature variation of the width. This constitutes a check on the consistency of our interpretation.

The value of b at $T = 0$ for the top (bottom) curve in Fig. 3 is $b_0 = 9.44$ (4.72) Å. The corresponding value of v^* is 16.8 (33.6) m/s and that of the critical velocity on the plateau is 6.5 (12.2) m/s. As discussed in Refs. [1,5], local velocities u_s are, due to geometrical effects, enhanced with respect to the average velocities v_s through the orifice and can be expected to reach 10 to possibly 40 m/s. We have at present no *direct* measurement of u_s which would pinpoint the value of b and permit further refinement of the model.

Surface defects of a size comparable to that of the nucleated vortex can be expected to alter the value of b and thus introduce a measure of variance with experimental situations, as would the change of the van der Waals attraction from substrate to substrate.

In the case of nucleation on negative ions considered in Refs. [12,13], the value of b_0 was found to be ~ 1 Å at 15 bars [13]: Dynamic healing effects are small on ions. The present analysis yields $v_{c0} = 50$ m/s in agreement with the experimental ion velocity of 55 m/s. The value of T_q of 147 mK corresponds, by Eq. (2), to $\omega_0/2\pi = 1.9 \times 10^{10}$ Hz which is of the same magnitude as the vortex "cyclotron" frequency considered by Muirhead, Vinen, and Donnelly [12].

To conclude, we note that, although our model is rudimentary, the range of values which we find for b_0 and v_{c0} certainly falls within expectations and that the qualitative features of the data are well reproduced. This leaves little doubt that we are indeed dealing here with the thermal and quantum nucleation of nanometer-size vortex rings, as is already known to be the case in ion propagation [12,13].

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