On Testing *CPT* Symmetry in *B* Decays

M. Kobayashi

KEK, National Laboratory for High Energy Physics, 1-1 Oho, Tsukuba, Ibaraki 305, Japan

A. I. Sanda

Department of Physics, Rockefeller University, New York, New York 10021
and Department of Physics, Nagoya University, Chikusa ku, Nagoya 464-01, Japan

(Received 18 February 1992)

It is known that the existing experimental limit for *CPT* violation is rather poor—the 10% level at best. For this reason, we study experimental tests for *CPT* violation in *B* decays. We show that $B\bar{B}$ mixing as well as *CP*-violating asymmetries can reveal *CPT* violation. We also point out that, with possible *CPT* violation, further experimental study is necessary in order to obtain the value for $\Delta m/\Gamma$ from the rate of same-sign dilepton events.

PACS numbers: 11.30.Er, 13.20.Jf, 14.40.Jz

The *CPT* theorem follows from the general properties of local renormalizable field theory [1]. The strongest experimental test for the violation of this symmetry is often stated as [2]

$$|(M_{\bar{K}^0}-M_{K^0})|/M_{K^0}\leq 4\times 10^{-18}. \qquad (1)$$

On the basis of both theory and experiment, any search for *CPT* violation seems rather unnecessary. This is false.

Indeed, Eq. (1) implies that *CPT*-violating interaction is very weak compared to the strength of QCD. Note that $M_{\bar{K}^0}\neq M_{K^0}$ requires both *CP* and *CPT* violations. Thus, it is more appropriate to compare the strength of allowed *CPT* violation to the strength of *CP* violation. Presently, this is done by measuring the phases of η_{00} and η_{+-}. A particular combination sensitive to *CPT* violation is [3]

$$\delta_\perp \approx |\eta|(\tfrac{2}{3}\phi_{+-}+\tfrac{1}{3}\phi_{00}-\phi_\epsilon)=(1.3\pm 0.8)\times 10^{-4},$$

where

$$\phi_\epsilon \equiv \tan^{-1}(2\Delta m/\Gamma_S)=(43.7\pm 0.2)^\circ. \qquad (2)$$

It is commonly stated that $\arg\epsilon=\phi_\epsilon$. This is criticized by Lavoura [4]. He points out that the 3π and the leptonic channels may contribute to the phase of ϵ. Without further experimental information on *CP* violation in these channels we can only conclude that $39.5^\circ<\arg\epsilon<47.4^\circ$ at the 90% confidence level. Comparing $\delta_\perp$ with the size of ϵ, and considering its theoretical error, we conclude that *CPT* is tested only at the 10% level.

Further theoretical studies are also necessary. The proof of the *CPT* theorem makes use of the properties of asymptotic states [1]. The proof may not be applicable for QCD—quarks and gluons are confined, and they do not appear in the set of asymptotic states. Also, if we are dealing with a low-energy effective theory of some more fundamental theory, it may not be renormalizable. At this time, the fundamental nature of the *CPT* theorem can be questioned.

In this paper, we investigate how *CPT* can be systematically probed in *B* decays. We show that all previous considerations concerning *CP* violation and B-$\bar{B}$ oscillation will be modified if there is *CPT* violation.

First we briefly discuss the analysis of Lee and Wu [5]. The $P^0\bar{P}^0$ mass matrix is given by

$$M-i\tfrac{1}{2}\Gamma=\begin{pmatrix} M_{11}-i\frac{1}{2}\Gamma_{11} & M_{12}-i\frac{1}{2}\Gamma_{12}\\ M_{12}^*-i\frac{1}{2}\Gamma_{12}^* & M_{22}-i\frac{1}{2}\Gamma_{22}\end{pmatrix}. \qquad (3)$$

The off-diagonal matrix elements are related by the Hermiticity of the Hamiltonian. Now write

$$\tfrac{1}{2}\Gamma+iM=D+i(E_1\sigma_1+E_2\sigma_2+E_3\sigma_3), \qquad (4)$$

where

$$\sigma_1=\begin{pmatrix}0&1\\1&0\end{pmatrix},\quad \sigma_2=\begin{pmatrix}0&-i\\i&0\end{pmatrix},\quad \sigma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}, \qquad (5)$$

$$E_1=\mathrm{Re}M_{12}-\tfrac{1}{2}i\,\mathrm{Re}\Gamma_{12},$$
$$E_2=-\mathrm{Im}M_{12}+\tfrac{1}{2}i\,\mathrm{Im}\Gamma_{12}, \qquad (6)$$
$$E_3=\tfrac{1}{2}(M_{11}-M_{22})-\tfrac{1}{4}i(\Gamma_{11}-\Gamma_{22}).$$

We can define complex numbers E, θ, and ϕ such that

$$E_1=E\sin\theta\cos\phi,\quad E_2=E\sin\theta\sin\phi,\quad E_3=E\cos\theta. \qquad (7)$$

Note that [6]

$$CPT \text{ or } CP \text{ conservation}\Longrightarrow \cos\theta=0,$$
$$CP \text{ or } T \text{ conservation}\Longrightarrow \phi=0. \qquad (8)$$

The mass eigenstates can be written as

$$|P_1\rangle=\frac{1}{[2(1+|\epsilon_1|^2)]^{1/2}}[(1+\epsilon_1)|P^0\rangle+(1-\epsilon_1)|\bar{P}^0\rangle], \qquad (9)$$

$$|P_2\rangle=\frac{1}{[2(1+|\epsilon_2|^2)]^{1/2}}[(1+\epsilon_2)|P^0\rangle-(1-\epsilon_2)|\bar{P}^0\rangle], \qquad (10)$$

and ϵ_1 and ϵ_2 are given by

$$\frac{q_1}{p_1} \equiv \frac{1-\epsilon_1}{1+\epsilon_1} = \tan\frac{\theta}{2} e^{i\phi}, \quad \frac{q_2}{p_2} \equiv \frac{1-\epsilon_2}{1+\epsilon_2} = \cot\frac{\theta}{2} e^{i\phi}. \qquad (11)$$

Here P stands for either a B or a K meson. Now, CPT invariance requires that

$$\frac{q_1}{p_1} = \frac{q_2}{p_2} = e^{i\phi},$$
$$s = \cot\theta = \frac{1}{2}\left(\frac{q_2}{p_2} - \frac{q_1}{p_1}\right) e^{-i\phi} = 0. \qquad (12)$$

For the test of CPT in K decays, we refer the reader to the articles of Lee and Wu [5], and of Tanner and Dalitz [7]. In this Letter, we shall concentrate on B meson decays.

CPT violation in B-$\bar{B}$ mixing effects.— We shall examine the effects of nonvanishing s on the B-$\bar{B}$ mixing effects. Let $N^{\pm\pm}_{C=\pm}(t_1,t_2)$, where t_1 and t_2 are times at which two $l^{\pm}$ are emitted ($t_1 < t_2$ by convention), denote the number of events for

$$(B\bar{B})_{C=\pm} \to l^{\pm} l^{\pm} + \text{anything}. \qquad (13)$$

Let $N^{+-}_{C=\pm}(t_1,t_2)$, where t_1 and t_2 are times at which l^+ and l^- are emitted, respectively, denote that for

$$(B\bar{B})_{C=-} \to l^+ l^- + \text{anything}. \qquad (14)$$

In these definitions, C is the charge conjugation quantum number for the $\bar{B}$ state, and leptons are always understood to be the primary leptons.

First, consider the time-integrated dilepton events for general s and ϕ:

$$R \equiv \left(\frac{N^{++}+N^{--}}{N^{+-}}\right)_{C=-} = \frac{(|e^{-i\phi}|^2+|e^{i\phi}|^2)}{2} \frac{1-a}{|1+s^2|(1+a)+|s|^2(1-a)}, \qquad (15)$$

$$a = \left(\frac{N_{++}-N_{--}}{N_{++}+N_{--}}\right)_{C=-} = \frac{|e^{-i\phi}|^2-|e^{i\phi}|^2}{|e^{-i\phi}|^2+|e^{i\phi}|^2}, \qquad (16)$$

where $\Delta m = m_2 - m_1$, $x = \Delta m/\Gamma$, $a = (1-y^2)/(1+x^2)$, and $y = \Delta\Gamma/2\Gamma$. In deriving these expressions we have assumed that the amplitudes for semileptonic decays satisfy the $\Delta Q = \Delta B$ relation and CPT invariance. Relaxing these limitations can be easily done, but the results become much more complicated.

We now make the following observations:

(1) The measurement of a will lead to the most precise measurement of Imϕ. While it does not lead to any information on CPT symmetry, observation of Imϕ is clear evidence of T violation, together with CP violation. This is a very practical measurement and we expect that information on Imϕ will be available as B decay events accumulate at CLEO and ARGUS. In the meantime, we prepare for a less interesting possibility by assuming that Imϕ is small in most of the discussions below.

(2) Presently, the following time-integrated quantities are accessible to experiments: The experimental values are

$$R = 0.19 \pm 0.04 \text{ (ARGUS)},$$
$$R = 0.15 \pm 0.03 \text{ (CLEO)}. \qquad (17)$$

The present experimental value for $\Delta m/\Gamma$ is determined with the assumption that s, Imϕ, and y are negligible. For illustration purposes, we consider the effect of s:

$$R = \frac{x^2}{2+x^2+2(1+x^2)s^2}. \qquad (18)$$

Figure 1 shows the constraint introduced by R on the s-x plane. It can be seen that

$$s < 2. \qquad (19)$$

As for x, while there is a strict lower bound, there is no upper bound.

From the observation of time-integrated quantities alone, we cannot determine x and s separately. These two can be separated by measuring Δm directly from the time profile of the oscillation. This can be done in the asymmetric B factory.

While a general calculation can be done, the result is rather complicated and is not very instructive. Here we limit ourselves to real s and ϕ. Then

$$N^{\pm\pm}_{C=-}(t_1,t_2) = \frac{e^{-\Gamma(t_1+t_2)}}{2(1+s^2)}[1-\cos\Delta m(t_1-t_2)], \qquad (20)$$

$$N^{+-}_{C=-}(t_1,t_2) = \frac{e^{-\Gamma(t_1+t_2)}}{(1+s^2)}[1+2s^2+\cos\Delta m(t_1-t_2)]. \qquad (21)$$

To make the definition precise, we choose $t_1 < t_2$ by convention.

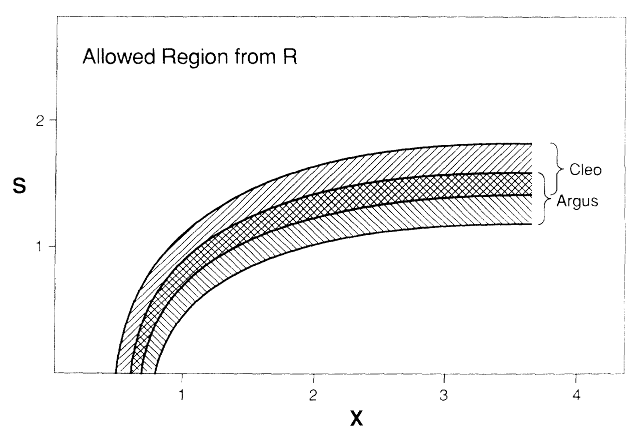

FIG. 1. The allowed region on the x-s plane imposed by ARGUS and CLEO results on R.

We hope that this type of information can also become available from hadron colliders. For this purpose we record here the $C=+$ case:

$$N^{+-}_{C=+}(t_1,t_2)=\frac{e^{-\Gamma(t_1+t_2)}}{(1+s^2)^2}\{1+\cos\Delta m(t_1+t_2)+s^2[1+2\cos\Delta m t_1+2\cos\Delta m t_2-\cos\Delta m(t_1-t_2)]+2s^4\}\,, \tag{22}$$

$$N^{\pm\pm}_{C=+}(t_1,t_2)=\frac{e^{-\Gamma(t_1+t_2)}}{2(1+s^2)^2}\{1-\cos\Delta m(t_1+t_2)+s^2[3-2\cos\Delta m t_1-2\cos\Delta m t_2+\cos\Delta m(t_1-t_2)]\}\,. \tag{23}$$

If $|s|$ is large, it will be detected from the above considerations. Here, we assume $|s|$ is small and consider the detection of Ims. We then have

$$N^{+-}_{\pm}\sim 1+\cos\Delta m(t_1\pm t_2)-2\,\mathrm{Im}s\begin{Bmatrix}\sin\Delta m t_1-\sin\Delta m t_2\\ \sin\Delta m(t_1-t_2)\end{Bmatrix}. \tag{24}$$

We see that if Ims is nonvanishing, there is a tendency to have either l^+ or l^- emitted first, or that Ims leads to an antisymmetric dependence on t_1-t_2.

CPT violation in CP asymmetries.— We now discuss the consequences of *CPT* and *CP* violations on the asymmetries which involve $B\bar{B}$ mixing. Let $N^{\pm}_{(\psi K_S;C=\pm)}$ be the number of events corresponding to the process

$$(B\bar{B})_{C=\pm}\to(l^{\pm}+X)+(\psi K_S)\,. \tag{25}$$

If the *CPT*-symmetry-breaking effect is large, it will be detected in the $B\bar{B}$ mixing. Thus, in analyzing the asymmetry in $N^{\pm}_{(\psi K_S;C=\pm)}$, we assume that the *CPT*-violating parameters are small and real.

With this approximation, we can compute the number of events:

$$N^{\pm}_{(\psi K_S;C=-1)}\sim 1\pm\mathrm{Im}\lambda\sin\Delta m(t_1-t_2)\pm s\,\mathrm{Re}\lambda[\cos\Delta m(t_1-t_2)-1] \tag{26}$$

(in this section, t_1 and t_2 denote times corresponding to the leptonic decay and ψK_S decay, respectively),

$$N^{\pm}_{(\psi K_S;C=+1)}\sim 1\mp\mathrm{Im}\lambda\sin\Delta m(t_1+t_2)\pm s\,\mathrm{Re}\lambda[1-2\cos\Delta m t_1+\cos\Delta m(t_1+t_2)]\,, \tag{27}$$

where $\rho=A(B\to\psi K_S)/A(\bar{B}\to\psi K_S)$ and $\lambda=e^{-i\phi}\rho$.

Finally, we record here the time-integrated asymmetry:

$$a_{(\psi K_S;C=-)}\equiv\frac{N^{+}_{(\psi K_S;C=-)}-N^{-}_{(\psi K_S;C=-)}}{N^{+}_{(\psi K_S;C=-)}+N^{-}_{(\psi K_S;C=-)}}=-s\,\mathrm{Re}\lambda\frac{x^2}{1+x^2}\,. \tag{28}$$

We note that the effect is linear in s in contrast with the dilepton mode discussed in the previous section.

Finally, if Ims, and Imϕ are kept, Eq. (28) becomes

$$a_{(\psi K_S;C=-)}=(1-a)[\mathrm{Im}\phi-\mathrm{Re}(\lambda^{-1}s^*)]+O((\mathrm{Im}\phi)^2,s^2)\,. \tag{29}$$

Conclusion.— We point out that the experimental limit on *CPT* violation must be presented with some care. Note that *CPT*-violating observables often vanish unless *CP* is violated. Thus the limit on a *CPT*-violating interaction should be compared to that of a *CP*-violating interaction. Looking at the experimental evidence for *CPT* conservation along this line, we see that the evidence is not very conclusive.

We have seen that both B-$\bar{B}$ mixing, and *CP*-violating effects get modified if *CPT* violation is present. We now suggest a procedure to systematically probe *CPT* violation.

(1) The presence of a term proportional to s signals both *CP* and *CPT* violation.

(2) If $|s|$ is large, the previous experimental determination of x will be modified as seen in Eq. (18).

(3) Imϕ can be measured by the dilepton ratio given in Eq. (16). Nonvanishing Imϕ, of course, does not imply *CPT* violation. If large Imϕ is found, this is a sign for physics beyond the standard model. In what follows, we treat the less interesting possibility by assuming that Imϕ is small.

(4) The effect of x and s can be separated by observing the time dependence of equal-sign and unequal-sign dilepton events, as seen in Eqs. (20) and (21).

(5) For small s, the effects of *CPT* violation given in Eqs. (26) and (27) are linear in s, while that in $B\bar{B}$ mixing, Eq. (18), is quadratic in s. Thus *CP* asymmetries may be more sensitive to the presence of small s than the dilepton decays.

(6) The observation of time-integrated asymmetry, Eq. (28), in the decay product of $\Upsilon(4S)$ implies either the nonvanishing Imϕ or *CPT* violation.

(7) Ims can be measured by studying the time ordering of l^+ and l^- in the opposite-sign dilepton events given in Eq. (24).

(8) The time dependence of various decay modes gets modified once *CPT*-violating interactions are included. An analysis such as that presented in Ref. [8] should be repeated.

(9) Finally, noting that $\Delta m=2E$, it is simple to show that

$$2\frac{E_3}{M_B}=\frac{\Delta m}{\Gamma_B}\cos\theta\frac{\Gamma_B}{M_B}\sim x\cos\theta\times 10^{-13}\,. \tag{30}$$

So, for reasonable values of $x \cos\theta$, we expect $M_B - M_{\bar{B}}$ to be $O(10^{-13})$.

This study illustrates the point that measurement capabilities of B decay properties must be pushed to the limit. It may be appropriate to think about second-generation B factories (beyond the ones under consideration at present) and dedicated fixed-target experiments.

A major part of this work was complemented while one of the authors (A.I.S.) was visiting L. T. Sanda at the Memorial Sloan Kettering Cancer Center. He thanks L. T. Sanda for the guidance received there. We also thank S. Stone for discussions. This work was supported in part by Department of Energy Contract No DE-AC0287ER-40325.B and NSF Contract No. INT 90-16750.

[1] G. Luders, Ann. Phys. (N.Y.) **2**, 1 (1957), and references therein.

[2] Particle Data Group, J. J. Hernández *et al.*, Phys. Lett. B **239**, 1 (1990).

[3] R. Carosi *et al.*, Phys. Lett. B **237**, 303 (1990).

[4] L. Lavoura, Carnegie-Mellon University report, 1991 (to be published). See also [5] for earlier work along this line.

[5] T. D. Lee and C. S. Wu, Annu. Rev. Nucl. Sci. **16**, 471 (1966).

[6] See also H. Lipkin, Phys. Lett. B **219**, 474 (1989).

[7] See also N. W. Tanner and R. Dalitz, Ann. Phys. (N.Y.) **171**, 463 (1986).

[8] M. Gronau, Phys. Rev. Lett. **63**, 1451 (1989); Phys. Lett. B **233**, 479 (1989).