Low-Temperature Studies of the NMR Frequency Shift in Superfluid ^{3}He-A

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We report the first measurements of the longitudinal resonance frequency [$\nu_{long}(A)$] in ^{3}He-A in the low-temperature limit, as well as new measurements near T_c. In the low-temperature limit, our data agree well with theoretically predicted behavior. The data allow us to make the first estimates of the BCS cutoff energy as a function of pressure and of the zero-temperature A-phase energy gap.

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The A and B phases of superfluid ^{3}He have been identified as p-wave BCS superfluids, probably the Anderson-Brinkman-Morel (ABM) and Balian-Werthamer (BW) states, respectively [1]. The A phase is energetically preferred to the B phase only in a narrow temperature range below the transition temperature from the normal phase, T_c. The A phase can, however, be supercooled well below [2] the equilibrium transition temperature between the two phases.

Superfluid ^{3}He exhibits a resonant ringing behavior in its magnetization following sudden changes in the applied magnetic field. The field-independent frequency of this longitudinal resonance in the A phase, $\nu_{long}(A)$, is temperature dependent and closely related to the magnitude of the order parameter and the strength of the pairing interaction in the superfluid. For bulk samples [2] $\nu_{long}(A)$ is related to the usual transverse resonance frequency (ν_{trans}) and the Larmor frequency ($\nu_{Lar}=\gamma H$) by $\nu_{long}^2(A)=\nu_{trans}^2-\nu_{Lar}^2$. Using this relation, we measured $\nu_{long}(A)$ as a function of temperature and, with a new type of sample cell that extended A phase supercooling dramatically in low magnetic fields, made the first measurements of $\nu_{long}(A)$ in the low-temperature limit.

We designed our sample cell with the expectation that irregularities in cell surfaces facilitated nucleation of the B phase. Thus, to deeply supercool the A phase, our cell contained the superfluid in two smooth surfaced fused silica tubes (about 10 to 20 cm long), phase isolated from rough surfaces by a magnetic valve. Details of the apparatus and the resultant increase in supercooling have been discussed elsewhere [3]. An NMR coil was placed around each tube, and the coils were attached in parallel within a conventional cw NMR spectrometer. Limited by field gradients from the magnetic valve, the normal liquid ^{3}He signals had linewidths of about 200 Hz in a magnetic field of 28.4 mT applied normal to the tube axes. At this field ν_{trans} was shifted up to several kHz above ν_{Lar}, and the shift could be measured to ± 20 Hz. A third tube contained Pt powder which was used for pulsed NMR thermometry, calibrated against T_c [4]. The cell was attached to a compressional chamber [5] and a capacitance strain gauge so that we could control and measure the sample pressure to within a millibar.

A major concern with this design was the possibility of a heat leak into the tubes causing a temperature gradient along them. Previous experiments [5] conducted with this cryostat using the same heat exchanger showed the temperature of ^{3}He samples to be proportional to the demagnetization field for temperatures above 0.35 mK. Below about 1 mK, the temperature of the Pt thermometer did not show such linearity. But, using earlier measurements [6] of the thermal boundary resistance between liquid ^{3}He and this supply of Pt powder, the deviation could be accounted for by a 2-pW heat leak directly into the Pt sample, possibly due to rf heating of the Pt. The absence of a significant heat leak into the ^{3}He sample tubes was further supported by the frequency shifts of the NMR signals in the two sample tubes at 34.2 bars being identical to within 30 Hz (equivalent to $\Delta T \sim 3\ \mu$K) near T_c, where temperature gradients would be largest due to the low thermal conductivity. Also, the temperature dependence of the shifts closely matched previous measurements [2] between T_c and 1.6 mK. Thus, above 0.8 mK, we assumed that in equilibrium all three tubes were at the same temperature to within $\pm 5\ \mu$K, as indicated by the Pt thermometer corrected for the heat leak. Below 0.8 mK, we used the proportionality of the temperature to the demagnetization field to obtain the temperature to $\pm 10\ \mu$K. Thermal equilibrium was determined by the criteria that the resonant frequency remain constant for several thermal relaxation times of the samples.

Measurements of the temperature dependence of the transverse resonance frequency were made at 5, 12, 21, 29.4, and 34.2 bars. All of the data were converted to longitudinal resonance frequencies using Larmor frequencies measured in the normal phase, corrected for the shifts due to the fringing field from the demagnetization stage. Although we could supercool the A phase farther than had been previously possible in low magnetic fields, the B phase always nucleated relatively close to T_c at the lower pressures, limiting our data to the high-temperature regime. For the 29.4- and 34.2-bar data, however, we could measure $\nu_{long}(A)$ in the low-temperature limit as described below.

The theoretical form for the longitudinal resonance frequency derived by Leggett [7] in both the ABM and BW states is [8]

$$\nu_{long}^2=\alpha(3\pi\gamma^4 h^2)\langle R^2\rangle[\psi]^2/\chi\,, \qquad (1)$$

where γ is the gyromagnetic ratio for ^{3}He, h is Planck's constant, $\langle R^2\rangle$ is a renormalization constant which accounts for the full difference between ^{3}He atoms and quasiparticles in the dipole Hamiltonian, α is $\frac{1}{5}$ for the ABM state and $\frac{1}{2}$ for the BW state, and χ and ψ are the magnetic susceptibilities and magnitudes of the order parameter of the respective states. The temperature dependent $[\psi]^2$ is given by

$$[\psi]^2=\tfrac{1}{4}[2N(0)]^2\int\frac{d\Omega}{4\pi}\left[\Delta\int_{-\varepsilon_c}^{\varepsilon_c}\frac{\tanh[(E/k_BT)/2]}{2E}d\varepsilon\right]^2, \tag{2}$$

where $2N(0)$ is the density of states at the Fermi surface, the first integral is over the Fermi surface, Δ ($=\Delta[\mathbf{k},T]$) is the absolute magnitude of the superfluid energy gap, ε_c is the BCS cutoff energy which prevents the second integral from diverging, and $E=(\varepsilon^2+\Delta^2)^{1/2}$ with ε the normal-state quasiparticle energy. While the identification of the A phase with the ABM state is still under some scrutiny, we will assume that they are equivalent in the following analysis, an assumption which was validated at melting pressure by previous NMR measurements [2].

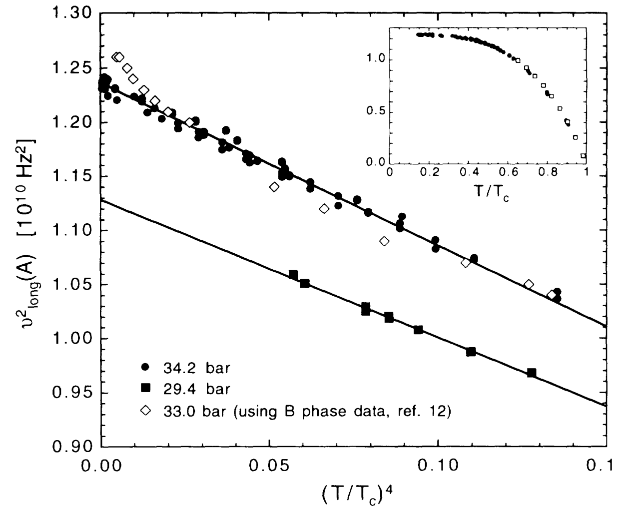

FIG. 1. Low-temperature $\nu^2_{\rm long}(A)$ plotted against $(T/T_c)^4$ at 34.2 and 29.4 bars. The lines are fits described in the text. Also shown are values derived from B-phase data at 33.0 bars from Ref. [12]. Inset: The full temperature range at 34.2 bars vs T/T_c with high-temperature data from Ref. [6] shown by the open squares.

Solving Eqs. (1) and (2) for the A phase in the low-temperature limit, one obtains [4,9]

$$\nu^2_{\rm long}(A)[0]=0.327[2N(0)\Delta_{A0}]^2\langle R^2\rangle(0.581-1.68y+1.333y^2)/\chi, \tag{3}$$

where Δ_{A0} is the A-phase energy gap in mK at $T=0$ averaged over the Fermi surface ($\Delta_{0A}=\sqrt{2/3}(\Delta_{\max})$ $=1.657k_BT_c$ in the weak-coupling limit), $2N(0)$ is in units of 10^{51} J^{-1}m^{-3}, and $y=\ln(\Delta_{A0}/\varepsilon_c)$. Given [10] that the A-phase energy gap approaches its zero-temperature value as $(T/T_c)^4$, we get [11]

$$\nu^2_{\rm long}(A)[T]=\nu^2_{\rm long}(A)[0][1-C(T/T_c)^4]. \tag{4}$$

Our data at 29.4 and 34.2 bars are plotted as a function of $(T/T_c)^4$ in Fig. 1. The 34.2-bar data strongly support Eq. (4), and the 29.4-bar data are certainly consistent with a $(T/T_c)^4$ temperature dependence. The solid lines are fits to the data of the above form with $\nu^2_{\rm long}(A)[0]=1.237\times10^{10}$ Hz^2 and $C=1.22$ at 34.2 bars, and $\nu^2_{\rm long}(A)[0]=1.129\times10^{10}$ Hz^2 and $C=1.13$ at 29.4 bars. Also shown are results Wölfle [12] compiled for $\nu^2_{\rm long}(A)$ at 33.0 bars from B-phase data taking the ratio of the ABM and BW forms of Eq. (1). As shown by the open symbols in Fig. 1, the magnitudes of his values compare well with ours, supporting the identification of the A and B phases with the ABM and BW states at high pressures. Wölfle's data show a qualitatively different behavior, however, which we attribute to a possible underestimation of χ_B, the admixture of f-wave pairing potentially altering the value of α at low temperatures [13], and/or his assumption that $|\psi|_A/|\psi|_B=1$.

Since both χ and $2N(0)$ are well known [4,14], we can use our low-temperature A-phase results and Eqs. (3) and (4) to solve for Δ_{A0} if we know both $\langle R^2\rangle$ in the low-temperature limit and ε_c, which should be temperature independent. In BCS theory, the ratio ε_c/T_c is a measure of the strength of the pairing interaction. However, unlike s-wave BCS states in ordinary superconductors where ε_c can be physically equated with the Debye energy, superfluid ^{3}He has no obvious energy scale to which ε_c is related. This is especially true when ε_c is used in the present context of the dipole interaction which could require a different cutoff from the simple BCS pairing interaction [13]. Originally, Leggett [7] estimated ε_c to be 0.7 K and most early workers [8,9] assumed it to be independent of pressure. Patton and Zaringhalam [15] derived a form which related ε_c to T_c and the strength of the pairing interaction through the exponential of a sum of Landau parameters. Their form is, however, quite sensitive to the higher-order parameters and thus somewhat unreliable for predicting ε_c. It has also been suggested [13] that $\varepsilon_c\sim E_{\rm sf}$, the spin fluctuation energy given by $E_{\rm sf}=(1+Z_0/4)E_F^*$, where E_F^* is the Fermi energy computed with the effective mass and Z_0 is a Landau parameter. The uncertainty in ε_c is such that a recent review [1] placed the bounds on ε_c to be $0.040<\varepsilon_c<1.0$ K. The other parameter, $\langle R^2\rangle$, is also undetermined and is usually [1,8,9] taken to be near unity at all pressures. Fomin, Pethick, and Serene [16] have, however, calculated $\langle R^2\rangle$ to be 1.6 at saturated vapor pressure and 1.7 at melting pressure.

Although neither $\langle R^2\rangle$ nor ε_c are well-known quantities, we will follow previous authors [7–9,12] and assume that they are the same in both the A and B phases. The error introduced by this assumption could be as much as $\sim 0.1\langle R^2\rangle$, but a detailed theoretical treatment has not yet been developed [11]. With this assumption, since the B-phase gap at $T=0$ (Δ_{B0}) is known, we can obtain $\langle R^2\rangle$ as a function of ε_c by solving Eqs. (1) and (2) for the B phase in the low-temperature limit. The resulting relation is [4,8]

$$v_{\rm long}^2(B)[0]=2.562[2N(0)\Delta_{B0}]^2\langle R^2\rangle[\ln(2\varepsilon_c/\Delta_{B0})]^2/\chi_B\,, \tag{5}$$

where Δ_{B0} is in mK and $2N(0)$ is in units of 10^{51} $\mathrm{J^{-1}m^{-3}}$. Taking zero-temperature limiting values at 34.2 bars of $\chi_B=(0.325\pm0.005)\chi_N$ [8,17], $v_{\rm long}^2(B)[0]=(1.01\pm0.02)\times10^{11}$ Hz^2 [8,18–20], and $\Delta_{B0}=(1.08\pm0.02)\times[1.76k_BT_c]$ [21,22], Eq. (5) becomes

$$\langle R^2\rangle=(27.0\pm1.7)/[\ln(2\varepsilon_c/\Delta_{B0})]^2\,. \tag{6}$$

To solve for Δ_{0A}, we now substitute for $\langle R^2\rangle$ in Eq. (3) the expression in Eq. (6), obtaining $v_{\rm long}^2(A)[0]$ as a function of only $\ln(\varepsilon_c)$ and Δ_{0A}. Then, using our values for $v_{\rm long}^2(A)[0]$, we can solve numerically for Δ_{0A} as a function of ε_c. We find that Δ_{0A} is remarkably insensitive to ε_c, which appears only within the logarithms; a variation of ε_c between 0.05 and 1 K changes Δ_{0A} by about 1%. Thus the uncertainty in Eq. (6) makes the particular choice of ε_c irrelevant to the resultant value of Δ_{0A}. We obtain $\Delta_{0A}=4.60\pm0.30$ mK ($=[1.85\pm0.12]T_c$) at 34.2 bars and, through a parallel calculation, $\Delta_{0A}=4.50\pm0.24$ ($=[1.85\pm0.10]T_c$) at 29.4 bars. The error bars are largely due to uncertainties in the B-phase quantities. The values are consistent with strong-coupling effects at high pressures in the A phase, which are indicated by a large enhancement of $\Delta C/C_N$, the ratio of the A-phase specific-heat jump to the normal-phase specific heat at T_c, over the BCS prediction [4]. Further development of the theory, in particular, an analytic expression for the measured coefficient (C) of the T^4 term in the temperature dependence of $v_{\rm long}^2(A)$, should place additional limits on Δ_{A0} and give new information about $\langle R^2\rangle$ [11].

In the Ginzburg-Landau regime (close to T_c) Eqs. (1) and (2) predict $v_{\rm long}^2(A)$ to be linear in $1-T/T_c$ with the slope given by [7,8]

$$\frac{\partial[v_{\rm long}^2(A)]}{\partial(T/T_c)}=-\frac{\pi}{10}\gamma^2(1+\tfrac{1}{4}Z_0)[2N(0)]\frac{\Delta C}{C_N}\langle R^2\rangle(k_BT_c)^2\left(\ln\frac{1.14\varepsilon_c}{k_BT_c}\right)^2. \tag{7}$$

Previous measurements of this slope by the Helsinski [8] and the La Jolla [23] groups did not agree, which both groups credit to problems with the temperature scales. A third measurement [24] with very high precision (due to an effective linewidth of 0.5 Hz) was made at melting pressure and given in terms of the pressure along the melting curve which is easily converted to the current temperature scale [4].

To compare our data with these earlier workers, we must take into account as they [8,23] did that the actual temperature dependence of $v_{\rm long}^2(A)$ is not linear except just below T_c. We found, however, that the curvature was insignificant for $T>0.9T_c$ and pressures below 34.2 bars. We thus took the slope to be the mean of $v_{\rm long}^2(A)/(1-T/T_c)$ measured at $T>0.9T_c$, with an uncertainty of 1 standard deviation of the mean. As shown in Fig. 2, our data agree well with the Helsinki data and extrapolate to a new measurement [25] at 1.03 bars by Halperin and co-workers. At 34.2 bars we have shown the high precision result at melting pressure [24] rather than our own and used that datum in the analysis below.

The quantities on the right-hand side of Eq. (7) have all been measured [4,9,14,26] except for ε_c and $\langle R^2\rangle$ as discussed above. From our data we obtain the product $\langle R^2\rangle[\ln(1.14\varepsilon_c/T_c)]^2$ as a function of pressure. Using this product, we can then calculate ε_c for given values of $\langle R^2\rangle$. In Fig. 3 we plot ε_c assuming $\langle R^2\rangle$ varies linearly with molar volume between the two estimates from Ref. [16]. The error bars reflect the uncertainties in our data and in the other parameters. If we set $\langle R^2\rangle=1$ for all pressures, we obtain values of ε_c which are closer in magnitude to $E_{\rm sf}$, as is also shown in Fig. 3. Despite the differences between the two sets of values for ε_c, they show similar pressure dependence and the magnitudes are close enough to suggest new limits on the strength of the superfluid pairing interaction. Furthermore, if we take the values of ε_c computed above from the data taken near T_c with $\langle R^2\rangle=1$ and use them in Eq. (6) to estimate $\langle R^2\rangle$ at $T=0$, we get $\langle R^2\rangle[T=0]=1.11\pm0.15$ at 34.2 bars and 1.19 ± 0.23 at 29.4 bars. The small changes between $T=T_c$ and $T=0$ are within the uncertainties. This suggests that $\langle R^2\rangle$ depends only weakly if at all on temperature, and also supports the assumption that $\langle R^2\rangle$ is the same for the two phases.

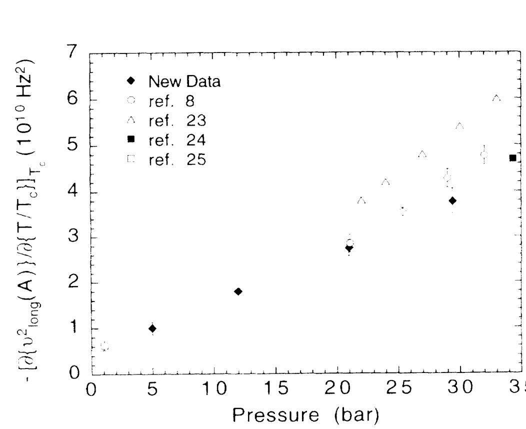

FIG. 2. Slope of $v_{\rm long}^2(A)[T]$ near T_c as a function of pressure from this and other experiments.

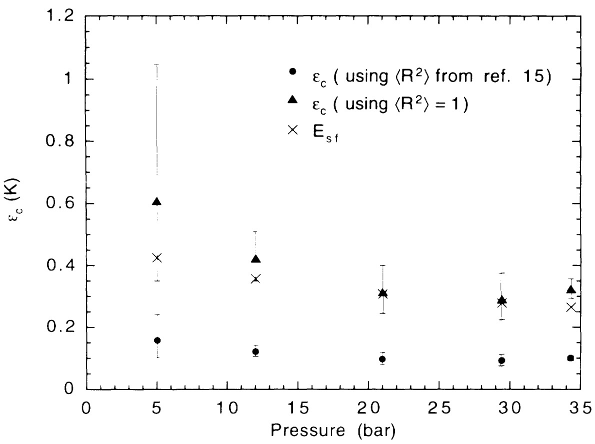

FIG. 3. Derived values of ε_c and E_{sf} vs pressure, as discussed in the text.

In summary, our measurements of the *A*-phase longitudinal resonance frequency in the low-temperature limit and near T_c allow us to obtain the $T=0$ energy gap in the *A* phase. We also evaluate the BCS cutoff energy for superfluid ^{3}He and make the first experimental estimate of its pressure dependence. A higher-order theoretical expression for the low-temperature behavior of $\nu_{\rm long}^2(A)$, combined with our results, would add greatly to understanding the microscopic dynamics of superfluid ^{3}He.

Future experiments could study the pressure dependence of Δ_{A0} and compare it to theoretical predictions [21]. More extensive measurements, in particular at low pressures, should also prove valuable toward a deeper theoretical understanding of ε_c and $\langle R^2 \rangle$.

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