Planck-Scale Physics and Neutrino Masses

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We discuss gravitationally induced masses and mass splittings of Majorana, Zeldovich-Konopinski-Mahmoud, and Dirac neutrinos. Among other implications, these effects can provide a solution of the solar neutrino puzzle. In particular, we show how this may work in the 17 keV neutrino picture.

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It is commonly accepted, although not proven, that the higher-dimensional operators induced through the quantum gravity effects are likely not to respect global symmetries. This is, at least in part, a product of one's experience with black holes and wormholes. If so, it becomes important to study the impact of such effects on various global symmetries of physical interest. Recently, attention has been drawn to the issues of Peccei-Quinn symmetry [1] and global non-Abelian symmetry relevant for the textures [2]. Here, instead, we study the possible impact of gravity on the breaking of lepton flavor and lepton number, and, more precisely, its impact on neutrino (Majorana) masses. It is clear that all such effects, being cut off by the Planck scale, are very small, but, on the other hand, even small neutrino mass can be of profound cosmological and astrophysical interest.

We start with a brief review of a situation in the standard model where such effects can induce, as pointed out by Barbieri, Ellis, and Gaillard [3] [in the language of SU(5) grand unified theory (GUT)], large enough neutrino masses to explain the solar neutrino puzzle (SNP) through the vacuum neutrino oscillations. We also derive the resulting neutrino mass spectrum and the mixing pattern, which have important implications for the nature of the solar neutrino oscillations. From there on we center our discussion on the impact of the mass splits between the components of Dirac and Zeldovich-Konopinski-Mahmoud (ZKM) [4] neutrinos. Our main motivation is the issue of SNP, but we will discuss the cosmological implications as well. The important point resulting from our work is a possibility to incorporate the solution of the SNP in the 17 keV neutrino picture in a simple and natural manner. Finally, we make some remarks on the seesaw mechanism and also mirror fermions in this context.

Neutrino masses in the standard model.—Suppose for a moment that no right-handed neutrinos exist, i.e., the neutrinos are only in left-handed doublets. Barring accident cancellations (or some higher symmetry), the lowest-order neutrino mass effective operators are expected to be of dimension five:

$$\frac{\alpha_{ij}}{M_{\rm Pl}}\,(l_i^T C\tau_2\boldsymbol{\tau} l_j)\,(H^T\tau_2\boldsymbol{\tau} H), \qquad (1)$$

where $l_i = (\nu_{iL}\ e_{iL})^T$, H is the usual $\mathrm{SU}(2)_L\times\mathrm{U}(1)$ Higgs doublet, $M_{\rm Pl}$ is the Planck mass $\approx 10^{19}$ GeV, and α_{ij} are unknown dimensionless constants. The operator (1) was first written down by Barbieri, Ellis, and Gaillard [3] who, as we have mentioned before, based their discussion on SU(5) GUT, although strictly speaking it only involves the particle states of the standard model. If gravity truly breaks the lepton number and induces terms in (1), the neutrino may be massive even in the minimal standard model. This important result of Ref. [3] seems not to be sufficiently appreciated in the literature. As was estimated in [3], for $\alpha_{ij}\sim 1$ one gets for the neutrino masses $m_\nu \sim 10^{-5}$ eV, which is exactly of the required order of magnitude for the solution of the SNP through the vacuum neutrino oscillations.

We would like to add the following comment here. One can imagine the gravitationally induced neutrino masses of Eq. (1) as arising due to virtual black holes or wormholes. In this case, the Higgs scalar and a lepton are absorbed and then reemitted by black hole or wormhole (or two leptons annihilate into two Higgs particles). Since the notion of lepton charge is lost inside these objects, the emitted lepton need not be of the same flavor as the absorbed one; moreover, as gravity does not distinguish between different neutrino flavors, one can expect all such amplitudes to be equal to each other: $\alpha_{ij}=\alpha_0$. If so, the neutrino mass matrix must take the democratic form with all its matrix elements being equal to $m_0 \sim \alpha_0 \times (10^{-5}\ \mathrm{eV})$.

At first sight, such an assumption results in a contradiction since the emerging neutrino mass matrix is basis dependent whereas the gravity is expected not to discriminate between different lepton bases. We will argue, however, that there is no contradiction and our assumption is self-consistent.

First of all, as can be readily seen, no basis-independent Majorana mass matrix is possible at all. The neutrino mass term in Eq. (1) is in the sextet representation of the flavor SU(3) symmetry which does not allow an invariant mass matrix. Technically this means that the flavor transformations of the mass matrix M are of the form UMU^T and not UMU^{-1} [5]. Therefore a question naturally arises: In which basis does the neutrino mass matrix take the "democratic" form (if it does)?

If charged leptons were massless (i.e., if their Yukawa couplings to the Higgs particle did not exist), all the flavor bases would have been equivalent to each other with no physical difference between them. Clearly, the situation is unambiguous in this case, and the neutrino mass terms of Eq. (1) are defined with respect to any basis. It is only *after* these terms are introduced that the difference between the flavor bases arises.

In the realistic case of nonvanishing lepton Yukawa couplings the global flavor SU(3) symmetry is reduced to $U(1)_e \times U(1)_\mu \times U(1)_\tau$. Thus, *before* the neutrino mass terms of Eq. (1) are introduced, there is only one physically distinguished basis: the basis of charged lepton mass eigenstates which coincides with the weak eigenstate basis. Therefore one can conclude that the neutrino mass terms in Eq. (1) should be defined with respect to this basis [6].

Let us discuss now the phenomenological consequences of the democratic neutrino mass matrix. For three neutrino generations this pattern implies two massless neutrinos and a massive neutrino with $m_\nu = 3m_0$. The survival probability of ν_e due to the $\nu_e \to \nu_\mu, \nu_\tau$ oscillations is

$$P(\nu_e \to \nu_e; t) = 1 - \frac{8}{9}\sin^2\left(\frac{m_\nu^2}{4E}t\right). \quad (2)$$

For $m_\nu \sim 10^{-5}$ eV, the oscillations length $l = 4\pi E/m_\nu^2$ for the neutrinos with the energy $E \sim 10$ MeV is of the order of the distance between the Sun and the Earth. Since Eq. (2) describes the large-amplitude oscillations, one can in principle get a strong suppression of the solar neutrino flux. This, so-called "just so," oscillation scenario leads to well defined and testable consequences [7].

We should add that if the relevant cutoff scale in (1) would be 1 or 2 orders of magnitude smaller than $M_{\rm Pl}$ (as can happen in the string theory, where the relevant scale may be the compactification scale), m_ν could be as large as 10^{-4}–10^{-3} eV. One can distinguish two cases then. For $10^{-10} < m_\nu^2 < 10^{-8}$ eV^2 we are faced with the conventional vacuum oscillations for which Eq. (2) gives the averaged ν_e survival probability $\bar{P}(\nu_e \to \nu_e) \simeq 5/9$. For $m_\nu^2 > 10^{-8}$ eV^2 the Mikheyev-Smirnov-Wolfenstein (MSW) effect [8] comes into the game. Although all three neutrino flavors are involved, one can readily make sure that the resonant oscillation pattern can be reduced to an effective two-flavor one. If the adiabaticity condition is satisfied, the neutrino emerging from the Sun is the massive eigenstate $\nu_3 = (\nu_e + \nu_\mu + \nu_\tau)/\sqrt{3}$. Thus, $P(\nu_e \to \nu_e) = 1/3$ in this case.

ZKM and Dirac neutrinos.—The major point of the result (1) is that the emerging neutrino masses are on the borderline of the range needed for the solution of the SNP. For $\alpha_{ij} \ll 1$, the mechanism would not work. The situation changes drastically if there is an additional mechanism of generating neutrino mass. This is particularly interesting in the case of ZKM or Dirac neutrinos. By ZKM we generically denote any situation with degenerate active neutrino flavors ν_{iL} and ν_{jL} when the lepton charge $L_i - L_j$ is conserved. In other words, the resulting state is a four-component neutrino $\nu_{\rm ZKM} = \nu_{iL} + (\nu_{jL})^c$. In the conventional Dirac picture one has $\nu_{\rm D} = \nu_L + n_R$, where n_R is sterile and the conserved charge is just the particular lepton flavor defined through ν_L.

In any case, if one desires to have the oscillations between the components of $\nu_{\rm ZKM}$ ($\nu_{\rm D}$), one has to break the degeneracy, i.e., induce Majorana masses which violate the conserved charge in question. This, as before, can be achieved through the gravity induced operators of (1) and also in the same manner by

$$\frac{1}{M_{\rm Pl}}(n_R^T C n_R)\left[\alpha_n H^\dagger H + \beta_n S^2\right], \quad (3)$$

where S is any $SU(2)_L \times U(1)$ singlet scalar field, which may or may not be present. Here, as throughout our analysis, we assume no direct right-handed neutrino mass such as $n_R^T C n_R S$ which is implicit in our assumption of having a Dirac state [9].

Let us focus first on the ZKM case. In general, the terms in (1) will induce the split $\Delta m \leq 10^{-5}$ eV. The oscillation probability depends on $\Delta m^2 \sim m\,\Delta m$. For this to be relevant for the SNP, one of the components ν_i or ν_j must be ν_e and therefore $m \leq 10$ eV [10], or $\Delta m^2 \leq 10^{-4}$ eV^2. Clearly, for any value of $m \geq 10^{-5}$ eV, this can provide a solution to the SNP through the vacuum oscillations [11].

The same qualitative analysis holds true for the Dirac neutrino, the only difference being the additional contribution of (3) to the Majorana masses. The possible presence of the S^2 term (if $\langle S\rangle \neq 0$) could modify drastically the predictions for Δm. Strictly speaking, in a general case no statement is possible at all since $\langle S\rangle$ could be in principle as large as $M_{\rm Pl}$. Of course, in the most conservative scenario of no new Higgs fields above the weak scale, the analysis gives the same result as for the ZKM situation. Oscillations between the components of a Dirac neutrino can also provide a solution to the SNP; however, the experimental consequences for experiments such as SNO or Borexino will be different. Namely, the detection rates in the neutral current mediated reactions will be reduced since the resulting neutrino is sterile.

Another important consequence of the induced mass splitting between the components of a Dirac neutrino is a possibility to have a sterile neutrino brought into the equilibrium through the neutrino oscillations at the time of nucleosynthesis [12]. This has been analyzed at length in Ref. [13], and can be used to place limits on Δm_{ij}^2 and neutrino mixings.

17 keV neutrino.—A particularly interesting application of the above effects finds its place in the problem of 17 keV neutrino [14]. Although the very existence of this neutrino is not yet established, it is tempting and theoretically challenging to incorporate such a particle into our understanding of neutrino physics.

Many theoretical scenarios on the subject were proposed; however, it is only recently that the profound issue of the SNP in this picture has been addressed. The problem is that the conventional scenario of three neutrino flavors ν_{eL}, $\nu_{\mu L}$, and $\nu_{\tau L}$ cannot reconcile laboratory constraints with the solar neutrino deficit. Namely, the combined restriction from the neutrinoless double beta decay and $\nu_e \leftrightarrow \nu_\mu$ oscillations leads to a conserved (or at most very weakly broken) generalization of the ZKM symmetry: $L_e - L_\mu + L_\tau$ [15]. This in turn implies the 17 keV neutrino mainly to consist of ν_τ and $(\nu_\mu)^c$, mixed with the Simpson angle $\theta_S \sim 0.1$ with the massless ν_e. Clearly, in this picture there is no room for the solution of the SNP due to neutrino properties.

It is well known by now that the LEP limit on Z^0 decay width excludes the existence of yet another light active neutrino. However, the same in general is not true for a sterile neutrino $n = n_R$. Of course, once introduced, n (instead of ν_μ) can combine with, say, ν_τ to form ν_{17} or just provide a missing light partner to ν_e needed for the neutrino-oscillations solution to the SNP. The latter possibility has been recently addressed by the authors of Ref. [16].

The introduction of a new sterile state n allows for a variety of generalizations of a conserved lepton charge $L_e - L_\mu + L_\tau$. This will be analyzed in detail in a forthcoming publication [13]. Here we concentrate on the simplest extension $\hat{L} = L_e - L_\mu + L_\tau - L_{n^c}$ [17] and assume the following physical states:

$$\nu_{17} \simeq \nu_\tau + (\nu_\mu)^c, \quad \nu_{\text{light}} \simeq \nu_e + n \tag{4}$$

mixed through θ_S. In the limit of the conserved charge $\hat{L}$, the light state is a Dirac particle and no oscillations are possible which would be relevant for the SNP. Furthermore, the only allowed oscillations are $\nu_e \leftrightarrow \nu_\tau$ and $\nu_\mu \leftrightarrow n^c$ with $\Delta m^2 \simeq (17\text{ keV})^2$. The situation changes drastically even with a tiny breaking of $\hat{L}$ and we show here how the potential gravitational effects in (1) and (3) may naturally allow for the solution of the SNP without any additional assumptions.

Clearly, the main impact of the above effects is to induce the mass splittings between the ν_τ and ν_μ, on one hand, and ν_e and n, on the other hand. Recall that we expect these contributions to be of the order of 10^{-5} eV or so, if no new scale below M_W is introduced. This tells us that

$$\Delta m^2_{\nu_\tau\nu_\mu} \leq 10^{-1}\text{ eV}^2, \quad \Delta m^2_{\nu_e n} \leq 10^{-4}\text{ eV}^2. \tag{5}$$

As we can see, the scenario naturally allows for the solution of the SNP due to the vacuum oscillations [11] and furthermore predicts the $\nu_\mu \leftrightarrow \nu_\tau$ oscillations potentially observable in the near future. There are some indications that these oscillations may have already been observed in the Kamiokande II and Irvine-Michigan-Brookhaven experiments with atmospheric neutrinos [18]. The above values of $\Delta m^2_{\nu_\tau\nu_\mu}$ along with practically maximal mixing perfectly fits the required parameter range [18].

Notice that although we discussed the case of only one sterile neutrino, any number would do. Also, we should emphasize that the result is completely model independent, as long as one deals with (almost) conserved charge $\hat{L}$. However, a simple model can easily be constructed and will be presented elsewhere [13].

We have seen how gravitation may play a major role in providing neutrino masses and mass splittings relevant for the SNP. Purely on dimensional grounds, at least in the case of only left-handed neutrinos, the Planck-scale physics induced masses and splittings are $\leq 10^{-5}$ eV. The smaller they are, the larger the neutrino masses generated by some other mechanism should be, in order to obtain large enough Δm^2. For this reason a ZKM neutrino is rather interesting, especially with cosmologically relevant m_{ν_e} close to its experimental upper limit ~ 10 eV.

The situation is less clear in the case of a Dirac neutrino, since the gravitationally induced mass term $m_n n^T C n$ could in principle give m_n as large as M_{Pl}. In fact, in this case it is hard to decide whether one is dealing with a Dirac neutrino or actually with a seesaw phenomenon [19]. The situation depends on the unknown aspects at very high energies, i.e., whether or not the scale of the n physics is much above M_W.

The seesaw effects may be even more important if one is willing to promote the whole $\text{SU}(2)_L \times \text{SU}(2)_R \times U(1)$ electroweak symmetry or, in other words, if one is studying a parity conserving theory. This is a natural issue in many GUTs, such as SO(10) or E_6. Normally, in order for the seesaw mechanism to work, one introduces a Higgs field which gives directly a mass to n. In the spirit of our discussion it is clear that gravitation may also do the job and, if so, one would expect $m_n \sim M_R^2/M_{\text{Pl}}$, where M_R is the scale of the $\text{SU}(2)_R$ breaking, i.e., the scale of parity restoration. In other words, even with large M_R, m_n could be quite small allowing far more freedom for light neutrino masses. Of course, the details are model dependent (i.e., M_R scale dependent) and we do not pursue them here.

As we have seen above, all the cases relevant to the 17 keV neutrino lead to the large-mixing-angle solution of the SNP. Before concluding this paper, we would like to offer some brief remarks regarding an interesting possibility of mirror fermions picture providing the desirable MSW solution of this problem.

Imagine a world which mimics ours completely in a sense that these mirror states have their own, independent weak interactions. In other words, let gravitation be the only bridge between the neutrino sectors of the two worlds. One would then have the following new mass operators in addition to those in Eq. (1):

$$\frac{\alpha_0}{M_{\rm Pl}}\,(l_{Mi}^T C\tau_2 \boldsymbol{\tau} l_{Mj})\,(H_M^T\tau_2\boldsymbol{\tau}H_M),$$
$$\frac{\alpha_0}{M_{\rm Pl}}\,(\bar{l}_i\tau_2 H^\dagger)(l_{jM}\tau_2 H_M), \qquad (6)$$

where M stands for mirror particles. Therefore, in addition to the 17 keV neutrino described before there should be an analogous $(\nu_{17})_M$ neutrino in the mirror world, and another massless state much like the massless state in the standard generalized ZKM picture. It turns out that, due to the mixing in Eq. (6), one of these two massless states picks up a mass $\sim \alpha_0 M_W^2 M_{\rm Pl}^{-1}\theta^{-2}$ with $\theta \simeq \langle H\rangle/\langle H_M\rangle$ whereas the other still remains massless. Therefore they can oscillate into each other with the mixing angle θ, and so in principle this can provide a solution of the SNP through the appealing MSW effect. Namely, for $\langle H_M\rangle \simeq$ 1–10 TeV the mass difference Δm^2 could easily be in the required MSW range. The above range of $\langle H_M\rangle$ makes this scenario in principle accessible at the Superconducting Super Collider.

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Note added.—After this work was completed we received a paper by Grasso, Lusignoli, and Roncadelli [20] who also discuss gravitationally induced effects in the 17 keV neutrino picture.

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[6] Actually, the fact that the lepton flavor is forgotten inside the black hole or wormhole does not necessarily imply that the amplitudes in Eq. (1) must be basis independent. To say that the black holes and wormholes destroy a global charge, one must first define this global charge; in our case the lepton charges are defined in the basis in which the mass matrix of charged leptons (or, more precisely, the matrix of Yukawa coupling constants) is diagonal. This singles out a preferred basis.

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