Single Vortex Tunneling in a Superconducting Film

In a recent Letter, Liu, Haviland, Glazman, and Goldman [1] have convincingly observed an unusual low-temperature dependence of the resistance, $R=R_0 \times\exp(T/T_0)$, in superconducting films in small magnetic fields. The authors proposed a model based on vortex tunneling in the presence of strong damping to explain the data which cannot be explained by other known models of vortex motion. The purpose of the present Comment is to point out the inconsistency of the overdamping model in explaning the data and to propose instead a single vortex tunneling model in the weak damping limit to explain the data.

The overdamping model in Ref. [1] implies that $R_Q/R_N \gg 1$, where R_N is the normal sheet resistance believed responsible for damping and $R_Q=\hbar/e^2=4.1$ kΩ is the quantum resistance. The data of Ref. [1] show that the normal sheet resistance R_N varies widely from weak to strong damping regions according to the above condition. This shows that the overdamping model is not consistent with the data and indicates the damping due to R_N may not play a significant role here. Nevertheless, this inconsistency does not necessarily suggest that single vortex tunneling has not been observed in Ref. [1]. Instead an alternative model, single vortex tunneling without damping, is possible for understanding the experiment. In this model, R_N does not play the role, if any, of the damping for a vortex.

Now we describe the model. The argument in Ref. [1] leading to the independent single vortex tunneling model is still applied here. The vortex mass M is determined by the equality [2] between the product Mv^2 and the vortex core energy which is equal to the condensation energy times the space occupied by a vortex core. Here v is the propagating velocity of the collective mode, the Bogoliubov-Anderson mode. Following the BCS theory [2] the vortex core energy is, near T_c,

$$Mv^2=\frac{\pi^2}{4}N(0)\hbar^2v^2d\left(1-\frac{T}{T_c}\right). \quad (1)$$

Here $N(0)$ is the density of states of electrons at the Fermi surface, T_c is the superconducting transition temperature, and d is the thickness of the film which is required to be smaller than the coherence length, the problem being effectively in two dimensions. From the BCS theory [3,4], in the clean limit the propagating velocity is $v=v_F/\sqrt{3}$ with v_F the Fermi velocity, and in general v decreases as disorder increases. The disorder also decreases T_c near the superconductor-insulator transition. In the determination of the vortex core energy the contribution from the electromagnetic field is ignored. This is valid in the extreme type-II superconductor situation, which we have assumed for the present problem.

For simplicity we assume that vortices are pinned by holes, and the measuring current is small such that it has no influence on the potential barrier U. Then potential barrier is simply the vortex core energy, that is, $U=Mv^2$. The tunneling rate Γ_t of a single vortex tunneling from one hole to another over distance r_0 is dominated by the exponent, $\ln\Gamma_t=-2r_0\sqrt{2MU}/\hbar$. As argued in Ref. [1], the resistance $R_t\propto\Gamma_t$ in the tunneling regime. With the average distance $\langle r_0\rangle$ between holes, the resistance in the tunneling regime is $\ln R_t=-2\langle r_0\rangle\sqrt{2MU}/\hbar$. Putting all the relations together, the resistance is

$$\ln R_t=-\frac{\pi^2}{\sqrt{2}}\hbar d\langle r_0\rangle N(0)v\left(1-\frac{T}{T_c}\right). \quad (2)$$

We immediately obtain the pronounced exponential temperature dependence. The experimental data in Ref. [1] can be satisfactorily explained by Eq. (2). The resistance according to Eq. (2) also shows a dependence on the thickness of the film, the density of states, and the transition temperature, which is testable experimentally. There is no explicit dependence on the normal sheet resistance. In the situation where damping affects the vortex tunneling, the present results, Eq. (2), can serve as an upper bound for the resistance due to the quantum tunneling of a vortex because the tunneling rate decreases in the presence of dissipation. In conclusion, the inconsistency of the overdamping model in explaining the single vortex tunneling data of Ref. [1] has been pointed out.

This work was supported by the U.S. National Science Foundation under Grant No. DMR 89-16052.

Ping Ao
Department of Physics, FM-15
University of Washington
Seattle, Washington 98195

Received 19 May 1992
PACS numbers: 74.60.Ge, 74.40.+k, 74.75.+t, 74.90.+n

[1] Y. Liu, D. B. Haviland, L. I. Glazman, and A. M. Goldman, Phys. Rev. Lett. **68**, 2224 (1992).
[2] P. Ao, "Quantum Decay of Supercurrent in 2D as a QED Process" (to be published).
[3] J.-M. Duan and A. J. Leggett, Phys. Rev. Lett. **68**, 1216 (1992). Comprehensive references on the mass of a vortex can be found here as well as in Ref. [2]. It should be pointed out that, after consideration of the Fermi-liquid effects and the difference between amplitude and phase motion, there will be a factor of $\ln L$ difference between the masses estimated in this reference and Ref. [2], with L the size of the film.
[4] N. R. Werthamer, in *Superconductivity*, edited by R. D. Parks and M. Dekker (Gordon and Breach, New York, 1969), Vol. 1.