# Circumstantial Evidence for Transverse Flow in 200A GeV S+S Collisions

E. Schnedermann and U. Heinz
*Institut für Theoretische Physik, Universität Regensburg Postfach 10 10 42, D-8400 Regensburg, Germany*
(Received 8 July 1992)

The hadronic spectra of 200A GeV S+S collisions are analyzed assuming thermal emission of particles including resonance decays. While the rapidity distributions suggest a longitudinal flow almost independent of the temperature, the transverse momentum spectra exhibit an ambiguity between transverse flow and temperature, which cannot be resolved by the spectra of the heavier particles. However, in a theoretical model for the global hydrodynamic expansion with a dynamical freeze-out criterion, we find that almost inevitably a sizable transverse flow develops, which is in quantitative agreement with the data.

PACS numbers: 25.75.+r

It has been argued that in nuclear collisions at high energies the necessary temperatures ($\approx 200\,\mathrm{MeV}$) might be reached for the formation of a new phase, called quark-gluon plasma (QGP), where quarks and gluons can move around over larger distances. The first round of experiments has been performed in the past years at the BNL (Brookhaven National Laboratory) with beam energies of 14.5A GeV and at CERN with 200A GeV. The interpretation of the data, which have become available recently, in terms of plasma formation versus conventional hadronic matter is unfortunately not straightforward, so that on the path to a final interpretation we first have to develop a consistent picture of the reaction as a whole.

For this purpose we want to sketch the evolution of the strongly interacting hadronic matter by using relativistic hydrodynamics. The validity of our approach depends crucially on (local) thermalization, which is expected to be reached shortly ($\approx 1\,\mathrm{fm}/c$) after the collision. However, this has not been proven yet, so our approach is to find out what the consequences of thermalization are and check if they contradict the experimental data.

In the first part we observe that almost all hadronic spectra can be described in terms of one temperature together with the longitudinal and the transverse flow velocity, though the amount of transverse flow remains unknown. In the second part we develop a simplified model of hydrodynamics which gives us the time evolution of the system starting from given initial conditions. Of course, not all of the latter are known *a priori*, but in the third part we show how we can combine the model with the data analysis to gain new information on the initial conditions. We find that in all reasonable cases a considerable amount of transverse flow is generated.

Let us begin with a phenomenological discussion of the data. By now several sets of experimentally measured hadronic spectra exist from different collaborations. We restrict ourselves to the 200A GeV collisions at CERN, because they exhibit a clear anisotropy in the momentum spectra, and thus require the cylindrically (rather than spherically) symmetric geometry to be tested in this paper. Though a similar analysis for the BNL data might be possible as well, it is not attempted here, since the rapidity spectra there suggest that a spherically symmetric model might be more appropriate [1].

Furthermore we confine ourselves to the spectra from $^{32}S+^{32}S$ measured by the NA35 Collaboration [2] at CERN, since they constitute an almost complete hadronic picture, with data on many different particle species ($\pi^-$, $K^0_S$, $p$, $\Lambda$, $\overline{\Lambda}$) from the same detector. Moreover, the equal mass nuclei, in combination with a central trigger, allow for a minimization of the nonparticipating spectator nucleons, further reducing complications in interpreting the data.

The rapidity distribution $dn/dy$ of a thermal source moving with rapidity $y_0$ and emitting particles of rest mass $m_0$ can be computed straightforwardly as

$$\left(\frac{dn}{dy}\right)_{\mathrm{th}} = \frac{V}{(2\pi)^2} T^3 \exp\left(-\frac{m_0}{T}\cosh(y-y_0)\right) \times\left(\frac{{m_0}^2}{T^2} + \frac{m_0}{T}\frac{2}{\cosh(y-y_0)} + \frac{2}{\cosh^2(y-y_0)}\right) . \tag{1}$$

According to this formula the width of the distribution should be limited regardless of temperature by $\Gamma^{\mathrm{FWHM}}_{\mathrm{th}} \leq 1.76$, in contrast to the data on the negatively charged hadrons which give $\Gamma^{\mathrm{FWHM}}_{\mathrm{exp}} \approx 3.3 \pm 0.1$ [2]. This points to a strong anisotropy in the spectra [3], which could be explained by imposing a strong flow in the direction of the colliding nuclei. The resulting rapidity spectrum can be described by the superposition of many thermal emitters, each being boosted by the longitudinal velocity $v_L = \tanh\eta$ and distributed uniformly in the boost angle $\eta$ within the limits $\eta_{\mathrm{min}}$ and $\eta_{\mathrm{max}}$, resembling limited boost invariance [4]:

$$\frac{dn}{dy}(y) \propto \int_{\eta_{\mathrm{min}}}^{\eta_{\mathrm{max}}} d\eta \left(\frac{dn}{dy}\right)_{\mathrm{th}}(y-\eta) \quad . \tag{2}$$

Here $\eta_{\mathrm{min}} = -\eta_{\mathrm{max}}$, if we write all rapidities relative to the center-of-mass system of the colliding nuclei at $y_{\mathrm{lab}} = 3$. The limits can be extracted from the measured rapidity distributions.

It is possible [5] to fit all measured $y$ spectra [2] by this expression, with the exception of the protons which have a non-negligible leading particle component which is not part of the thermalized fire cylinder. We found that all produced particle species are nicely described ($\chi^2/N_{\rm DF} \leq 4$) by a longitudinal fluid rapidity $\eta_{\rm max} \approx 1.7$, corresponding to $v_{L,\rm max} = 0.94\,c$, independent of the assumed temperature. Including the decays of heavier resonances after freeze-out, e.g., $\Delta \to N\pi$, does not alter this picture.

As soon as one includes such a large longitudinal flow to explain the data, the possibility of transverse flow should also be taken into account, albeit perhaps at a somewhat more moderate level. It is well known [1, 6], i.e., that the inverse slope of the transverse momentum spectra $dn/m_T\,dm_T$ in general does not give the temperature of the emitter, as would be true for a static thermal source, but rather reflects an apparent temperature, including the blueshift from possible transverse collective motion of the thermal source.

We will parametrize the transverse flow by an *ad hoc* velocity profile in the transverse coordinate $r$ [1]

$$\beta_r(r) = \beta_s \left(\frac{r}{R}\right)^n \tag{3}$$

where we fix the shape of the profile with $n$. Using values of $n = 1$ or 2, we checked that the influence of the assumed shape on our results is minor, so we will here show only the results with $n = 2$.

We compute the spectra like in [7], assuming an emitting hypersurface located at constant global time $t =$ const with the Lorentz factor $\gamma_r(r) = 1/\sqrt{1-\beta_r{}^2(r)}$:

$$\frac{dn}{m_T\,dm_T} \propto m_T \int_0^R r\,dr\,K_1\left(\frac{\gamma_r m_T}{T}\right) I_0\left(\frac{\gamma_r \beta_r p_T}{T}\right). \tag{4}$$

$K_1$ and $I_0$ are the modified Bessel functions. Since also resonance decays play an important role in the $m_T$ spectra, especially in the $\pi^-$ spectra at low $m_T$ ($\lesssim 500\,\rm MeV$), we account for the most prominent mesonic and baryonic decay channels along the lines of [8].

Attempting fits to the $\pi^-$ $m_T$ spectra for different values of transverse flow $\beta_s$, we find that for all $\beta_s \lesssim 0.8\,c$ (corresponding to a mean transverse velocity $\langle\beta_r\rangle \lesssim 0.4\,c$) a good fit can be obtained ($\chi^2/N_{\rm DF} \leq 3$), using the temperature as a free parameter.

It is commonly believed [1, 9] that the ambiguity in the $\pi^-$ spectrum can be resolved by looking at the spectra of heavier particles, because the flow is more efficient in giving the heavier particles kinetic energy via their rest mass. Consequently they would need less thermal energy and their fitted emitter temperatures should decrease more rapidly with increasing flow than the ones for the lighter pions. Unfortunately, the decay contributions blur this picture, especially for the pions. In fact, the current data lead for all particle species to similar fit values of the temperature over a large region in $\beta_s$ (see Fig. 1 and [5]). From Fig. 1 we have to conclude that we cannot extract the amount of collective transverse flow directly from the S+S data.

While within such a purely phenomenological analysis this ambiguity cannot be resolved, we can proceed further by theoretical means. The parameters extracted from the spectra should reflect the last stage of the heavy-ion reaction when the thermal picture is breaking down and the hadrons decouple from each other. By requiring consistency of these parameters at freeze-out with the dynamical evolution, especially the buildup of collective flow from thermal pressure during the earlier stages of the collision, we can remove some of the remaining freedom in the explanation of the data.

Our theoretical tool to study the dynamics is relativistic hydrodynamics, which has a long history in its application to high energy hadron data [10–12]. For such a description to be valid, the mean scattering times between the particles have to be short enough to keep up with the hydrodynamical expansion. We will take this for granted during the initial stages of the evolution, but will come back to this issue later when discussing the freeze-out process. The hydrodynamical equations involve the local conservation of energy and momentum

$$\partial_\mu T^{\mu\nu} = 0 \tag{5}$$

where $T^{\mu\nu}$ is the energy-momentum tensor. These partial differential equations are hard to solve for a realistic scenario, which for central collisions (azimuthal symmetry) should involve at least 1+2 dimensions.

Instead we prefer to work with a global version of these equations, which is obtained by integrating the local equations over a space-time region $\Omega$, thus turning them into global conservation laws:

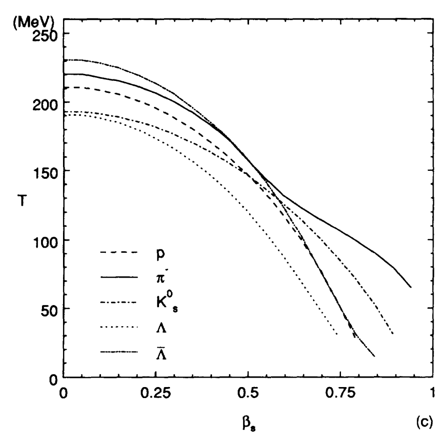


FIG. 1. Phenomenological correlation between temperature and transverse flow velocity at freeze-out. Every point on these lines corresponds to a good fit of the computed $m_T$ spectrum [5] (including resonance decays and flow) to the measured spectrum [2] of the respective particle species.

$$\int_\Omega \partial_\mu T^{\mu\nu} d^4\Omega = \oint_{\partial\Omega} T^{\mu\nu} d^3\Sigma_\mu = 0 \,. \quad (6)$$

We assume azimuthal symmetry and also arrange for longitudinal comoving coordinates $\tilde{t}$ and $\zeta$ (with $\partial t/\partial\tilde{t} = \gamma_z$ and $\partial z/\partial\tilde{t} = \gamma_z\beta_z$), in order to account for the strong time dilatation in the longitudinal direction. $\tilde{t}$ thus acquires the meaning of the proper time of a fluid element on the $z$ axis, while $\zeta$ as the longitudinal coordinate is attached to the volume elements. We also adjust the shape of the initial hypersurface to smoothly interpolate between a full stopping (Landau-like) and a partial transparency (Bjorken-like) scenario. The limits for the integration region $\Omega$ are chosen in a cylindrical coordinate system, taking care that the effect of the inner pressure on the momentum integrals will not be averaged out because of $\phi$ symmetry [5].

The integrals (6) are evaluated by using the ideal fluid decomposition of the energy-momentum tensor $T^{\mu\nu} = (\varepsilon + P)\,u^\mu u^\nu - Pg^{\mu\nu}$, i.e., neglecting viscosity effects. Further simplifications are achieved by imposing cylindrical symmetry [i.e., taking $R(\tilde{t})$ independent of $\zeta$], parametrizing the velocity fields in a simple way, and assuming box profiles for all thermal quantities (at $S/A =$ const to eliminate $\mu_b$ as an independent variable):

$$\beta_r(\tilde{t}, r, \zeta) = \beta_s(\tilde{t}) \left(\frac{r}{R}\right)^n , \quad (7)$$

$$\beta_z(\tilde{t}, r, \zeta) = \tanh[\alpha(\tilde{t})\,\zeta], \quad \eta(\tilde{t}, \zeta) = \alpha(\tilde{t})\,\zeta, \quad (8)$$

$$T(\tilde{t}, r, \zeta) = \begin{cases} T(\tilde{t}) & 0 \le r \le R \text{ and } -\mathcal{Z} \le \zeta \le \mathcal{Z} \\ 0, & \text{otherwise} . \end{cases} \quad (9)$$

With the profiles fixed in this way, the three integral equations (6) for $\nu = 0, r, z$ are now rewritten into *ordinary* differential equations, representing global energy conservation and the global increase of longitudinal and transverse momentum caused by the internal pressure. The problem has been reduced to solve for the time dependences of the "variational parameters" $T(\tilde{t})$, $\beta_s(\tilde{t})$, and $\alpha(\tilde{t})$ which determine the absolute heights of these profiles. The numerical solution of this problem is about a factor of 1000 faster than solving the full local hydrodynamical equations (5). We have checked [5] by direct comparison with such solutions [12] that, while locally the profiles (7)–(9) are not always a good representation of the true ones, the global feature of the expansion, including the flow characteristics and cooling curves, are reproduced. The remaining small differences are a worthwhile sacrifice considering the enormous gain in numerical speed without which the following multiparameter study would not have been possible.

In this form, entropy conservation $\partial_\mu(su^\mu) = 0$ (or alternatively baryon number conservation since $S/A =$ const), which is an exact result of local hydrodynamics in the absence of shocks, is not included in our equations and can thus be used as a test for the accuracy of our variational ansatz (7)–(9). We find less than 10% entropy creation over the whole lifetime of the fireball. Alternatively, we can combine the two equations for radial and longitudinal momentum into one and introduce entropy conservation as an additional equation [5], the final results being very similar (see Fig. 2). The results have been obtained with a realistic hadron resonance gas equation of state [1, 5]. Including a phase transition to QGP does not change the results qualitatively [5].

Since the expansion leads to a more and more dilute system, the interactions among the particles eventually die out, thus violating the condition of local thermal equilibrium. We set the end point to the hydrodynamic evolution by comparing two time scales $\tau_{\text{exp}} = \tau_{\text{sca}}$, where the expansion time scale $\tau_{\text{exp}} = (\partial_\mu u^\mu)^{-1}$ is given by the dynamics of the system and works against the scattering time scale $\tau_{\text{sca}}$, computed from the local densities and cross sections of the particles (see [1, 7] for details).

We now use these equations to evolve the system from the initial conditions towards the freeze-out point. We set the initial transverse velocity to $\beta_{s,0} = 0$ and the initial box radius to $R_0 = 4$ fm similar to the size of the S nucleus. From the known total energy loss of the incoming nuclei $E_{\text{loss}} = 313 \pm 38\,\text{GeV}$ [2] we can restrict the longitudinal size $Z_0$ depending on the other initial parameters. Remaining unknowns are the initial longitudinal flow $\eta_0$ and energy density $\varepsilon_0$.

We can use above the phenomenological result that the rapidity spectra suggest a longitudinal flow rapidity of $\eta_f = 1.7$ to select pairs of initial parameters $(\varepsilon_0, \eta_0)$ which dynamically reproduce this freeze-out value. We find that for all values of the initial flow between $0 \le \eta_0 \le 1.7$ a corresponding $\varepsilon_0$ can be found, which yields a good fit to the $dn/dy$ data at freeze-out.

For each one of these "allowed" pairs $(\varepsilon_0, \eta_0)$ the model predicts the corresponding freeze-out values $T_f$ and $\beta_{s,f}$.

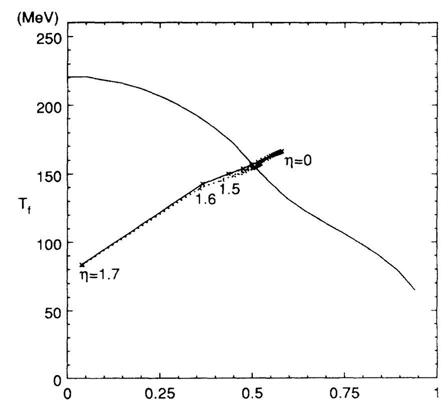


FIG. 2. Comparison of the theoretical line ($\eta_0 = 0, \ldots, 1.7$) with the phenomenological curve ($\pi^-$) in freeze-out temperature $T_f$ and transverse flow $\beta_{s,f}$. The intersection at $\beta_{s,f} \approx 0.5\,c$ can be interpreted as evidence for collective transverse flow. The global hydrodynamic system without explicit entropy conservation is indicated by the dashed line.

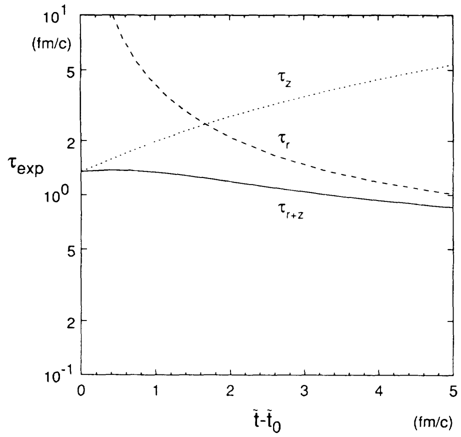


FIG. 3. The transverse, longitudinal, and total expansion time scale $\tau_{\text{exp}}$ for an initial longitudinal flow $\eta_0 = 1.3$. For $\eta_0 = 0$ the curves look qualitatively similar, with $\tau_{\text{exp}}$ near freeze-out being $\approx 20\%$ smaller.

These are shown in Fig. 2, with the initial flow (i.e., degree of transparency) $\eta_0$ parametrizing the curve. For $\eta_0 = 0$ (i.e., full stopping) we see that a large amount of transverse flow is generated, causing the system to freeze out rather early at high temperatures. As nuclear transparency is switched on ($\eta_0 > 0$), slightly less transverse flow is generated, causing the system to stay together longer and cool down a little further.

Surprisingly, the strong longitudinal flow soon ceases to dominate the expansion time scale (see Fig. 3), due to the rapidly increasing length of the cylinder. (In the boost invariant scenario we would obtain a time scale which grows linearly with the proper time $\tau_{\text{exp}} = \tilde{t}$.) Instead, transverse dilution begins to dominate the expansion time scale, in spite of the rather slow growth of the transverse radius, due to its two-dimensional nature, and finally dictates the freeze-out point. [In the nonrelativistic limit $\tau_{\text{exp}} \approx (n+1)\beta_s/R$ [1].]

However, only for $\eta_0 \gtrsim 1.5$ (in which case nearly all of the observed longitudinal flow would be primordial and no real hydrodynamic evolution would occur) do $T_f$ and $\beta_{s,f}$ deviate drastically from the value calculated from the full stopping assumption. The reason is that for all reasonable parameters $0 \leq \eta_0 \leq 1.5$ the hydrodynamical evolution in the partially transparent case is a nearly exact replica of the corresponding late stages of the evolution in the Landau-like scenario [5].

We compare now the theoretical curve in Fig. 2 to the phenomenological one for the pions from Fig. 1, which has the best statistics. The two curves show an opposite overall tendency: While the freeze-out enforces high temperatures at large flow velocities, the slope of the measured spectra can only be reproduced if larger flow velocities are compensated by lower emitter temperatures. The intersection around $\beta_{s,f} \approx 0.5\,c$ has to be taken within the hydrodynamical framework as evidence for the existence of transverse flow. Of course, this conclusion depends on our strong model assumption of hydrodynamical behavior and has to be treated with caution, until thermalization is proven by a microscopic kinetic analysis.

We would like to thank J. Sollfrank for helping us with the implementation of the resonance decays. This work was supported by DFG and by BMFT.

[1] K.S. Lee and U. Heinz, Z. Phys. C **43**, 425 (1989); K.S. Lee, U. Heinz, and E. Schnedermann, Z. Phys. C **48**, 525 (1990).
[2] S. Wenig, PhD. thesis, Universität Frankfurt [GSI Report No. GSI-90-23, 1990 (unpublished)]; S. Wenig (private communication); NA35 Collaboration, J. Bartke *et al.*, Z. Phys. C **48**, 191 (1990).
[3] H. v. Gersdorff *et al.*, Phys. Rev. C **39**, 1385 (1989).
[4] Yu.M. Sinyukov, V.A. Averchenkov, and B. Lörstadt, Z. Phys. C **49**, 417 (1991).
[5] E. Schnedermann, Ph.D. thesis, Universität Regensburg, 1992; E. Schnedermann and U. Heinz (to be published).
[6] P.J. Siemens and J.O. Rasmussen, Phys. Rev. Lett. **42**, 880 (1979).
[7] U. Heinz, K.S. Lee, and E. Schnedermann, in *Quark-Gluon Plasma*, edited by R.C. Hwa (World Scientific, Singapore, 1990).
[8] J. Sollfrank, P. Koch, and U. Heinz, Phys. Lett. B **252**, 256 (1990); Z. Phys. C **52**, 593 (1991).
[9] Y. Hama and F.S. Navarra, Z. Phys. C **53**, 501 (1992); S. Kagiyama, A. Minaka, and A. Nakamura, Progr. Theor. Phys. **81**, 266 (1989).
[10] E. Fermi, Prog. Theor. Phys. **5**, 570 (1950); L.D. Landau, Izv. Akad. Nauk SSSR **17**, 51 (1953); P. Carruthers and Minh Duong-van, Phys. Rev. D **8**, 859 (1973).
[11] H. v. Gersdorff, L. McLerran, M. Kataja, and P.V. Ruuskanen, Phys. Rev. D **34**, 794 (1986); **34**, 2755 (1986).
[12] U. Ornik and R.M. Weiner, Phys. Lett. B **263**, 503 (1991).