Comment on "Wave Field Determination Using Three-Dimensional Intensity Information"

Recently, Nugent has shown [1] that, within the framework of the paraxial approximation, the cross-spectral density function (CSDF) of the light field may be recovered via a direct transformation of the three-dimensional intensity distribution. In this Comment I will show that, actually, the transformation suggested in Ref. [1] does not yield the detailed CSDF but rather an integral of the CSDF along a certain line.

The transformation is given in Eq. (14) of Ref. [1], which reads

$$W(\mathbf{r},\mathbf{x},0)=\frac{x_1x_2}{\lambda_0^2}\int\int I(\mathbf{r}',a)e^{-2\pi i a\mathbf{x}\cdot(\mathbf{r}-\mathbf{r}')/\lambda_0}d\mathbf{r}'\,da\,. \quad (1)$$

In this equation λ_0 is the wavelength, $\mathbf{r}$, $\mathbf{r}'$, and $\mathbf{x}$ are vectors in the plane $z=\text{const}$, x_1,x_2 are the two components of $\mathbf{x}$, and $a=1/z$.

$$W(\mathbf{r},\mathbf{x},z)=\langle E(\mathbf{r}+\mathbf{x}/2,z,\lambda_0)E^*(\mathbf{r}-\mathbf{x}/2,z,\lambda_0)\rangle$$

is the cross-spectral density function, namely, the correlation between the component of the field with wavelength λ_0 at the points $(\mathbf{r}+\mathbf{x}/2,z)$ and $(\mathbf{r}-\mathbf{x}/2,z)$. $I(\mathbf{r},z)\equiv W(\mathbf{r},0,z)$ is the intensity.

Note that according to Eq. (1), by changing the origin of the z coordinate, the CSDF at every z may be recovered. In other words Eq. (1) implies that the CSDF $\langle E(\mathbf{r}+\mathbf{x}/2,z,\lambda_0)E^*(\mathbf{r}-\mathbf{x}/2,z,\lambda_0)\rangle$ at every point $(\mathbf{r},z)$ in the volume and for every difference vector $\mathbf{x}$ may be recovered from the intensity $I(\mathbf{r},z)=\langle E(\mathbf{r},z,\lambda)E^*(\mathbf{r},z,\lambda)\rangle$. This is, of course, a very strong statement. In the rest of this Comment I will show that actually only part of the information about the CSDF may be recovered.

My derivation goes as follows: The 3D distribution of the CSDF is related to the CSDF at the $z=0$ plane through the formula [2]

$$W(\mathbf{r}',\mathbf{x},z)=\int K(\mathbf{r}'+\mathbf{x}/2,\boldsymbol{\rho}+\boldsymbol{\xi}/2,z)K^*(\mathbf{r}'-\mathbf{x}/2,\boldsymbol{\rho}-\boldsymbol{\xi}/2,z)W(\boldsymbol{\rho},\boldsymbol{\xi},0)d\boldsymbol{\rho}\,d\boldsymbol{\xi}\,, \quad (2)$$

where K is the free-space propagator of the paraxial wave equation:

$$K(\mathbf{u},\mathbf{v},z)=(1/\lambda_0 z)e^{2\pi i(\mathbf{u}-\mathbf{v})^2/2\lambda_0 z}\,. \quad (3)$$

In the case $\mathbf{x}=0$ Eq. (2) is a formula for the 3D distribution of the intensity in terms of the CSDF at the $z=0$ plane. In order to get the CSDF at $z=0$ in terms of the 3D intensity distribution one has to invert this formula. A partial inversion is obtained by multiplying this formula by $(1/z^2)\exp[-2\pi i\mathbf{x}\cdot(\mathbf{r}-\mathbf{r}')/\lambda_0 z]$, and then integrating over $\mathbf{r}'$ and z. The result is

$$\frac{x_1x_2}{\lambda_0^2}\int W(\mathbf{r}',\mathbf{x},0)\delta\left(\frac{\mathbf{x}\cdot(\mathbf{r}'-\mathbf{r})}{\lambda_0}\right)d\mathbf{r}'=\frac{x_1x_2}{\lambda_0^2}\int\int I(\mathbf{r}',a)e^{-2\pi i a\mathbf{x}\cdot(\mathbf{r}-\mathbf{r}')/\lambda_0}d\mathbf{r}'\,da\,. \quad (4)$$

The right-hand side of this equation is the same as in the transformation derived in Ref. [1] [Eq. (1) above]. The left-hand side says that this transformation when applied to the intensity does not yield the detailed CSDF but rather an integral of this function along a line (parametrized by l) $\mathbf{r}'(l)$ on which $\mathbf{r}-\mathbf{r}'$ is perpendicular to $\mathbf{x}$.

The difference between the present result [Eq. (4) above] and the result of Ref. [1] [Eq. (1) above] is traced to the improper treatment of the delta function in the paragraph between Eqs. (13) and (14) in Ref. [1], which leads to the incorrect conclusion that $W_m=W$. A careful treatment shows that actually Eq. (13) in Ref. [1] leads to the conclusion that W_m equals a line integral of W.

In summary, the central statement of Ref. [1] should be replaced by a weaker statement saying that *some characteristics* of the cross-spectral density function of the light field may be recovered via a direct transformation of the three-dimensional intensity distribution.

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[1] K. A. Nugent, Phys. Rev. Lett. **68**, 2261 (1992).
[2] See, for example, Eq. (4) in A. Gamliel and G. P. Agrawal, J. Opt. Soc. Am. A 7, 2184 (1990).

