Measurement of Nonlinear Mode Coupling of Tearing Fluctuations

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Three-wave nonlinear coupling of spatial Fourier modes is measured in the MST reversed field pinch by applying bispectral analysis to magnetic fluctuations measured at the plasma edge at 64 toroidal locations and 16 poloidal locations, permitting observation of coupling over 8 poloidal modes and 32 toroidal modes. Comparison to bispectra predicted by resistive MHD computation indicates reasonably good agreement. However, during the crash phase of the sawtooth oscillation the nonlinear coupling is strongly enhanced, concomitant with a broadened (presumably nonlinearly generated) k spectrum.

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A ubiquitous feature of plasmas is the presence of nonthermal fluctuations composed of a spectrum of frequencies and wave numbers. Understanding the spatial and temporal structure of the spontaneously occurring fluctuations remains a daunting goal of nonlinear plasma physics. Nonlinear interactions between spatial Fourier components are a key determinant of the wave vector ($\mathbf{k}$) spectrum of the fluctuations. It is generally presumed that coupling of three spatial components, satisfying the sum rule $\mathbf{k}_1+\mathbf{k}_2=\mathbf{k}_3$, is the dominant nonlinear interaction. Higher-order coupling (four-wave and up) is assumed to be small in most theoretical treatments of fluctuations. In fusion plasmas nonlinear interactions are important since the resulting spectra might determine macroscopic behavior, such as transport, dynamo effects (magnetic field generation), and sawtooth oscillations. Fluctuation spectra have been measured in numerous plasma confinement experiments [1]. However, to our knowledge direct observation of nonlinear interactions in k space, the subject of the present Letter, has not been heretofore obtained in plasmas. Nonlinear interactions have been measured in an air jet by laser scattering [2].

Tearing fluctuations are relatively large-scale oscillations predicted by resistive magnetohydrodynamics (MHD). They are believed to underlie sawtooth oscillations and disruptions in tokamaks [3], and the dynamo effect [4], sawteeth [5], and, perhaps, transport in reversed field pinches. We have directly measured the three-wave coupling of such magnetic fluctuations in the edge of the MST reversed field pinch (RFP) experiment. In the RFP, tearing fluctuations over the wave-number range which we examine appear to be intermediate between fully developed turbulence and isolated coherent modes. Most of the fluctuation power resides in several modes which are coupled to each other, as well as to smaller amplitude higher k modes. Three-wave coupling is characterized by the "bicoherence" $b(k_1,k_2,k_3)$ defined as

$$b(k_1,k_2,k_3)=A\langle B(k_1)B(k_2)B(k_3)\rangle\,,$$

where $A=\{\langle|B(k_1)|^2|B(k_2)|^2\rangle\langle|B(k_3)|^2\rangle\}^{-1/2}$, $k_3=k_1+k_2$, $B(k)$ is the Fourier transform of the magnetic field, and $\langle\,\rangle$ denotes an ensemble average. The expression for the proportionality constant A follows from the representation

$$\frac{B(k_3)}{\langle|B(k_3)|^2\rangle^{1/2}}=B_{\rm unc}+\sum b(k_1',k_2',k_3)\frac{B(k_1')B(k_2')}{\langle|B(k_1')|^2|B(k_2')|^2\rangle^{1/2}}\,,$$

where $B_{\rm unc}$ is the part of $B(k_3)$ which is uncoupled to other modes [i.e., $\langle B_{\rm unc}B(k_1)B(k_2)\rangle=0$], and the summation is over k_1',k_2'. In this expression $B(k_3)$ is decomposed into an uncoupled part and a part which is generated by three-wave coupling. The bicoherence enters as the nonlinear coupling coefficient in the summation. This is easily seen by multiplying by $B(k_1)B(k_2)$ and ensemble averaging, which, using the orthogonality of the modes, yields the expression for A. To obtain A, we have assumed that the fourth-order moment is related to the second-order moment by $\langle B(k_1')B(k_2')B(k_1)B(k_2)\rangle=\langle|B(k_1)|^2|B(k_2)|^2\rangle$. Whereas this identification of the bicoherence with the coupling coefficient is useful (and valid for MHD in which the nonlinearities are quadratic), the use of the bicoherence as a measure of three-wave coupling is independent of any closure assumptions. The bicoherence is nonzero if there is a statistically significant phase relation between the three modes. Such bispectral analysis has been advanced by Ritz, Powers, and Bengston for plasma research, and applied experimentally in the frequency domain to electrostatic fluctuations [6].

We compare the measurements with the bicoherence for tearing fluctuations calculated by a three-dimensional, nonlinear, resistive MHD code. MHD predicts an essentially stationary steady-state spectrum; thus, the k-space analysis in this paper provides better contact with existing theory than bispectra in frequency. We show that the strong dominant coupling of poloidal modes $m=1$ and 2 in the experiment is consistent with theory. The toroidal mode coupling is similar in experiment and theory, although the experiment indicates a broader coupling of modes (perhaps due to the different Lundquist numbers in the code and experiment). During a sawtooth crash, the experiment displays strongly enhanced mode coupling with a substantial increase in the modes involved in the nonlinear interactions.

MST [7,8] is a large RFP ($a=0.5$ m, $R=1.5$ m) with typical plasma parameters $I_p<600$ kA, $n_e\approx10^{13}$ cm^{-3},

$T_e < 500$ eV, and pulse length < 80 ms. The magnetic field was measured with pickup coils attached to the inside surface of the vacuum chamber. A toroidal array contains coils at 64 equally spaced locations. A poloidal array contains coils at 16 locations. Hence, we can resolve toroidal and poloidal mode numbers up to $n=32$ and $m=8$. At each location is a triplet of coils which can measure all three components of the field. The coils are separated from the plasma by graphite covers for electrostatic shielding and protection. Signals are digitized with a 1-MHz sampling rate and the electronics limits the frequency response to 250 kHz. Ensembles for the bispectral analysis consist of 256 time slices of 256 μsec duration assembled from 256 reproducible discharges (which were selected from a data run of 400 discharges).

The initial value resistive MHD code employed for comparison to experiment solves the full compressible nonlinear MHD equations for a periodic cylinder with an aspect ratio of 3, as in the experiment. The code includes 8 modes poloidally (azimuthally, $-2 \leq m \leq 2$, after aliasing effects are taken into account) and 128 modes toroidally (axially, $-42 \leq n \leq 42$). The profiles and spectra are self-consistently evolved. The zero-pressure code has been described extensively, and is commonly used to treat current-driven tearing fluctuations in the RFP [9]. The main difference between the code assumptions and the experimental conditions, given a resistive MHD model, is the value of Lundquist number, the ratio of the resistive diffusion time to the poloidal Alfvén transit time (10^4 for the code and order 10^6 for the experiment). Also, since the code is cylindrical it does not include mode coupling arising from toroidicity. The code was run 256 times with random initial conditions to generate an ensemble of computational data. For initialization of each individual run the phases between the various components of the magnetic field and between the various components of the velocity were separately randomized (with the constraint $\nabla \cdot \mathbf{B}=0$). The computational data (the magnetic field values at the plasma boundary) are then analyzed with the same bispectral analysis software used for the experimental data.

The k spectra of the edge magnetic fluctuations have been measured earlier in MST [10]. A large fraction of the fluctuation energy is contained in several modes with mode numbers $m=1$, $n=5$–7 (and frequency below 30 kHz), which are resonant inside the reversal surface. The fluctuation amplitudes (to within a factor of 2) and dominant k values are in good agreement with prediction of tearing mode theory [11]. Typically, these modes are observed to be phase locked in the experiment; i.e., they constructively interfere [10]. Thus, it is already clear that coupling likely exists, although these prior observations provide no information on the coupling mechanism.

We first display our results on the bicoherence of poloidal modes, $b(m_1,m_2,m_3)$. We present our results in a 3D plot (Fig. 1) in which the two horizontal axes represent m_1 and m_2 and the vertical axis represents bicoherence corresponding to a value of m_3 given by $m_3=m_1+m_2$. We note that the bicoherence shown in Fig. 1(a) has two dominant peaks, of comparable magnitude, corresponding to $m_1=m_2=1$, $m_3=2$, and $m_1=-1$, $m_2=2$, $m_3=1$. The sign of the mode number indicates the sense of the rotation. There is also a small peak at slightly higher m values. The poloidal bicoherence predicted by the MHD code is shown in Fig. 1(b) for direct comparison with the experiment. We note that the dominant peaks are identical with the experiment, although peaks at higher mode numbers than $m=2$ would not be resolved. In both the experiment and the resistive MHD model there is strong mode coupling between two $|m|=1$ modes and an $|m|=2$ mode. The direction of energy flow in k space is not discernible from this analysis, although it has been predicted theoretically that unstable $m=1$ modes would drive a stable $m=2$ mode, which transfers energy to the small scale where it is dissipated [12].

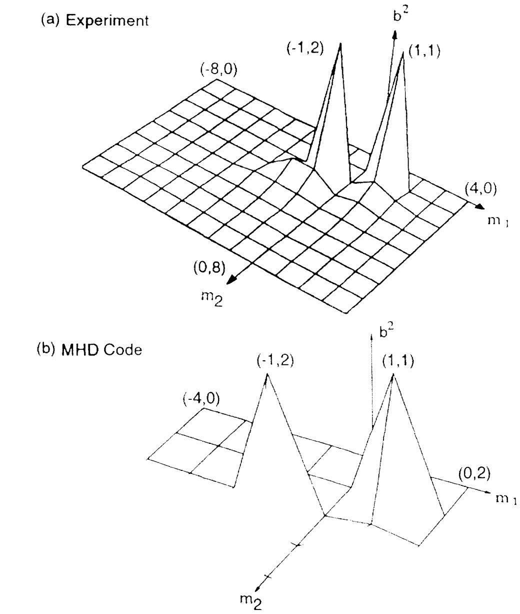

FIG. 1. Bicoherence, $b(m_1,m_2,m_3)$, of poloidal modes obtained from the poloidal magnetic field component from (a) experiment and (b) MHD code. The horizontal axes denote m_1 and m_2. The vertical axis denotes the bicoherence corresponding to $m_3=m_1+m_2$. The peak amplitude has a value 0.35 in (a) and 0.6 in (b).

The poloidal bicoherence obtained from an array at one toroidal location includes contributions from all toroidal modes. Coupling of toroidal modes, obtained from the toroidal coil array, is displayed in Fig. 2 for the experiment [Fig. 2(a)] and code [Fig. 2(b)]. Toroidal mode coupling involves a large number of modes. However, the experimental coupling is broader in n than theoretically predicted (the n spectra of the fluctuations is also broader in the experiment). This distinction is perhaps simply a feature of the different values of Lundquist number, S. One might expect that at the higher S value of the experiment, nonlinear energy transfer is

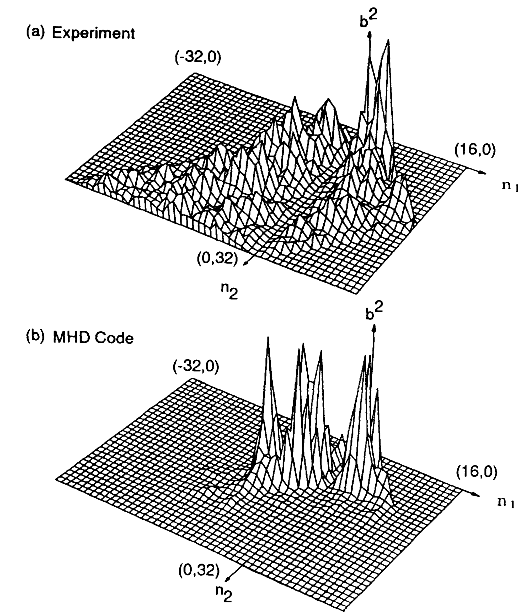

FIG. 2. Bicoherence, $b(n_1,n_2,n_3)$, of toroidal modes obtained from the toroidal magnetic field component from (a) experiment and (b) MHD code. The horizontal axes denote n_1 and n_2. The vertical axis denotes the bicoherence for $n_3=n_1+n_2$. The peak amplitude has a value 0.45 in (a) and 0.61 in (b).

enhanced and resistive energy dissipation is diminished. One feature which experiment and code share is that the dominant mode coupling occurs at intermediate n values. That is, n values of order unity are only weakly coupled to other modes and the coupling at n greater than about 20 is also weak. Thus, the modes with $n=5$–7, which we know are coupled from the observed constructive interference, do not couple through an $n=1$ mode (except during a sawtooth crash, as discussed below). For example, the $n=5$ and $n=6$ modes couple through the $n=11$ mode with higher probability than through the $n=1$ mode. The $n=11$ mode can then couple to other modes, and so on. However, the coupling does not proceed strongly to n values as high as 32.

MST displays sawtooth oscillations [13], which have also been observed in other RFP experiments [5] (but not exhibited in the resistive MHD code runs used here). Magnetically, these sawtooth oscillations suddenly deepen reversal, by decreasing the boundary toroidal magnetic field and increasing the total toroidal flux. We find that during a sawtooth crash the nonlinear coupling changes dramatically. The data shown above were obtained at the beginning of the rise phase of the sawtooth oscillation (time 1 in Fig. 3). We have also obtained bicoherence at the onset of the crash phase (time 2 in Fig. 3). The bicoherence at the sawtooth crash (obtained from an ensemble of sawtooth crashes) is shown in Fig. 4. There is a dramatic broadening in the number of modes involved in the nonlinear coupling. The dominant poloidal mode interaction [Fig. 4(a)] is now coupling of two $|m|=2$ modes to an $m=|4|$ mode. The toroidal modes [Fig.

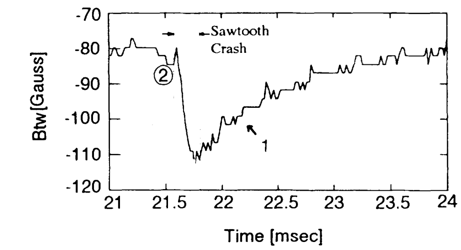

FIG. 3. Toroidal magnetic field measured at the wall for a 3-msec duration, illustrating a sawtooth oscillation. Time 1 (21.5 msec) is the time at which the bicoherence was measured (from an ensemble of sawteeth) for Figs. 1 and 2. Time 2 (22.2 msec) is the time of measurement for Fig. 4. The horizontal length of the time markers indicates the duration of the time record of each sample of the ensemble.

4(b)] now display strong nonlinear coupling over the entire range of resolvable n values, from 1 to 32. Concurrent with the enhanced nonlinear interactions, we observe that the n spectrum of the fluctuation power is substantially broadened, as shown in Fig. 5 (similar broadening has been observed in TPE-1RM15 [14]). These results are consistent with the view that the broadened n spectrum arises from the nonlinear generation of high and low n modes, as opposed to generation from linear instabilities (for example, arising from profile modification during the crash).

In summary, we have measured directly three-wave nonlinear coupling of magnetic fluctuations which are identified as tearing fluctuations. There is good agreement between the measurements of the k-space couplings and predictions of nonlinear resistive MHD computation.

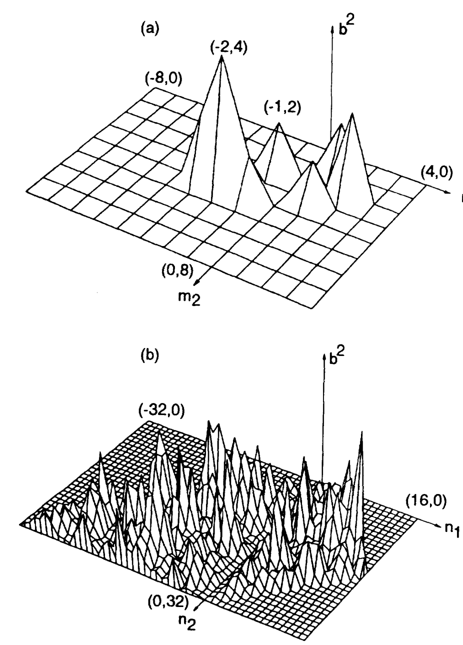

FIG. 4. Experimental bicoherence at the onset of a sawtooth crash for (a) poloidal modes and (b) toroidal modes. The peak amplitude has a value of 0.52 in (a) and 0.42 in (b).

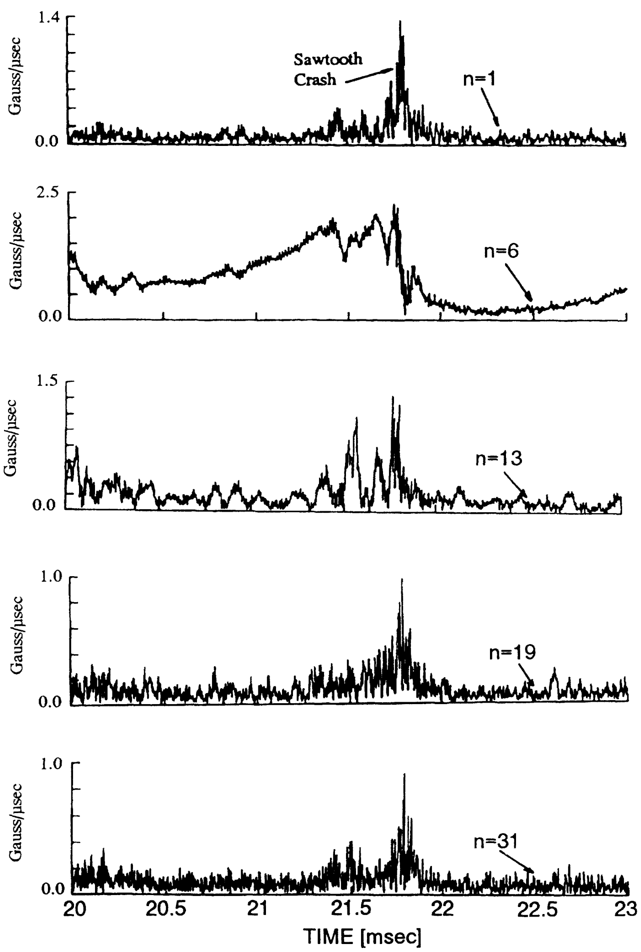

FIG. 5. Time dependence of various toroidal modes for a 3-msec slice during which a sawtooth crash occurs.

We measure that the dominant poloidal mode coupling in the experiment involves two $|m|=1$ modes coupling to an $|m|=2$ mode. Toroidal mode number coupling is much broader, involving tens of modes. These measurements add credibility to the nonlinear MHD theory used to describe magnetic reconnection in tokamaks and RFPs.

During the crash phase of a sawtooth oscillation (which is not modeled by the MHD computation employed here) the nonlinear couplings are strongly enhanced, simultaneous with a broadening of the mode spectra. This is consistent with nonlinear coupling being the source of generation of spectral broadening during a crash (although bispectral analysis does not indicate causality).

The data reported here are the first example of measurements of k-space coupling of fluctuation spectra in a plasma. An important question is whether three-wave coupling is the dominant nonlinear interaction. For MHD, the nonlinearities are quadratic and thus three-wave coupling is a complete description of the nonlinear behavior. The semiquantitative agreement between the experiment and resistive MHD theory provides encouragement that we have detected the dominant nonlinearity at the wavelengths resolved. This work lends support to the resistive MHD description of the dynamo effect, which arises from nonlinearly interacting modes. The experimental realization of nonlinear coupling as described by MHD also indirectly encourages the application of MHD to other phenomena such as tearing modes in tokamaks. To establish experimentally the completeness of the three-wave description would require a measurement of higher-order wave coupling, a substantial experimental task. Measurement of nonlinear interactions, by the techniques such as those realized here and extensions thereof, are essential for an adequate understanding of the nonlinear plasma physics intrinsic to confined plasmas.

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