Muon Decay: Measurement of the Energy Spectrum of the ν_e as a Novel Precision Test for the Standard Model

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The energy distribution of the ν_e from the decay of unpolarized μ^+ has been calculated on the basis of the most general four-fermion interaction. It depends on a parameter called ω which equals zero in the standard model. It is shown that the recent measurement of the reaction $^{12}C(\nu_e,e^-)^{12}N$(g.s.) with ν_e from μ^+ decay at rest not only yields the absorption cross section, but also independently determines ω with a precision of $\Delta\omega=0.026$ for 100 measured events. This corresponds to $|g^S_{LL}|\leq 0.37$, where g^S_{LL} describes a scalar interaction with left-handed μ and e and right-handed neutrinos.

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Since the discovery of the muon more than fifty years ago, its main decay mode $\mu^+\to e^+\nu_e\bar{\nu}_\mu$ has been of special interest both to experiment and theory. The decay is purely leptonic and therefore allows study of the charged weak interaction at low energies without disturbance by the strong interaction. It can be described by the most general, local, derivative-free and lepton-number-conserving four-fermion point interaction Hamiltonian [1]. It contains ten complex coupling constants corresponding to nineteen independent parameters to be determined by experiment. The observables are described most conveniently in terms of a chiral Hamiltonian in charge-changing form [2,3] which is characterized by fields of definite handedness. The matrix element is given by [4,5]

$$M=4\frac{G_F}{\sqrt{2}}\sum_{\substack{\gamma=S,V,T\\ \varepsilon,\mu=R,L}} g^\gamma_{\varepsilon\mu}\langle\bar{e}_\varepsilon|\Gamma^\gamma|(\nu_e)_n\rangle\langle(\bar{\nu}_\mu)_m|\Gamma_\gamma|\mu_\mu\rangle . \quad (1)$$

Here G_F is the Fermi coupling constant, while γ labels the type of interaction: Γ^S, Γ^V, Γ^T (4-scalar, 4-vector, 4-tensor). The indices ε and μ indicate the chirality (left- or right-handed) of the spinors of the charged leptons; ε denotes electron, μ the muon. The chiralities n and m of the ν_e and the ν_μ spinors, respectively, are uniquely determined for given γ, ε, and μ. In this picture, the standard model corresponds to $g^V_{LL}=1$, all other couplings being zero.

All the experimental results from μ decay, which were obtained from the study of the μ and e (positron spectrum, decay asymmetry, and positron polarization), were consistent with the $V-A$ hypothesis ($g^V_{LL}=1$); it was, however, impossible to deduce the Lorentz structure from these experiments alone. This was observed in 1966 by Jarlskog [6] who proposed to measure neutrino-electron correlations to resolve the remaining ambiguity. In 1986 it was finally recognized [4] that the measurement of the cross section of the inverse muon decay reaction $\nu_\mu e^-\to\mu^-\nu_e$ using ν_μ of known helicity supplied the missing information. Together with the results from normal muon decay it was possible to obtain a lower limit to g^V_{LL} for the first time. This, in turn, allowed one to obtain an upper limit for the hitherto undetermined coupling constant $|g^S_{LL}|$ by exclusion. This scalar coupling is characterized by left-handed μ and e and right-handed ν_μ and ν_e and could be mediated, for example, by a charged Higgs boson [7].

In the following we describe a novel method by which g^S_{LL} is determined directly using the experimental results of a recent neutrino measurement [8]. We have calculated the energy distribution $d\Gamma/dy$ of ν_e from the decay of unpolarized μ^+, where $y=2E_\nu/m_\mu$ is the reduced neutrino energy ($0\leq y\leq 1-x_0^2$), $x_0=m_e/m_\mu$, and m_e and m_μ are the masses of electron and muon, respectively,

$$\frac{d\Gamma}{dy}=\frac{m_\mu^5G_F^2}{16\pi^3}\{F_1(y)+\omega F_2(y)+\eta x_0F_3(y)\}, \quad (2)$$

with

$$F_1(y)=\frac{(1-x_0^2-y)^2y^2}{1-y}, \quad (2a)$$

$$F_2(y)=\frac{2}{9}\frac{(1-x_0^2-y)^2y^2}{(1-y)^3}\times[-4y^2+y(7-x_0^2)-3+3x_0^2], \quad (2b)$$

$$F_3(y)=\frac{(1-x_0^2-y)^2y^2}{(1-y)^2}. \quad (2c)$$

We note that Eq. (2) closely resembles the distribution of electrons from μ decay: ω is the neutrino analog of the famous Michel parameter ρ. The integral of $F_2(y)$ over the allowed region of y vanishes, and the total decay probability Γ depends only on G_F and η. The so-called low-energy parameter η is already known from the electron spectrum. Although ω and η are strongly correlated, we can neglect the influence of η on ω because η is known very precisely to be small, $\eta=(-7\pm13)\times10^{-3}$ [9], and because $F_3(y)$ is additionally suppressed by the factor x_0. The new decay parameter ω may be obtained from ρ by the substitutions $g^V_{LL}\to g^V_{LR}$, $g^V_{RR}\to g^V_{RL}$, $g^T_{LR}\to -g^T_{LR}$, and $g^T_{RL}\to -g^T_{RL}$. Is it given in terms of our coupling constants by

$$\omega=\tfrac{3}{16}\{|g^S_{LL}|^2+|g^S_{RR}|^2+4|g^V_{LR}|^2+4|g^V_{RL}|^2+|g^S_{LR}+2g^T_{LR}|^2+|g^S_{RL}+2g^T_{RL}|^2\} \tag{3}$$

with $0\leq\omega\leq 1$ for arbitrary $g^{\gamma}_{\varepsilon\mu}$ and $0\leq\omega\leq\frac{3}{4}$ if the standard decay parameters are restricted to their $V-A$ values $\rho=\delta=\frac{3}{4}$ and $\xi=\xi'=1$.

In order to compare the physical significances of measurements of ω and ρ we write ρ in the form [10]

$$\rho-\tfrac{3}{4}=-\tfrac{3}{4}\{|g^V_{LR}|^2+|g^V_{RL}|^2+2|g^T_{LR}|^2+2|g^T_{RL}|^2+\mathrm{Re}(g^S_{LR}g^{T*}_{LR}+g^S_{RL}g^{T*}_{RL})\}\,. \tag{4}$$

This has intriguing consequences: A precise measurement yielding $\rho=\frac{3}{4}$ which is the $V-A$ value cannot exclude all those interactions which are built as arbitrary combinations of g^S_{LL}, g^S_{LR}, g^S_{RL}, g^S_{RR}, g^V_{RR}, and g^V_{LL} [11]. For ω, on the other hand, the standard model predicts $\omega=0$. By obtaining an experimental upper limit of $\Delta\omega\gtrsim 0$, one immediately gets upper limits for $|g^S_{LR}+2g^T_{LR}|$, $|g^S_{RL}+2g^T_{RL}|$, $|g^V_{LR}|$, $|g^V_{RL}|$, $|g^S_{RR}|$, and, for the first time, an upper limit for $|g^S_{LL}|$ in a direct measurement.

The ν_e spectrum and therefore the new parameter ω can be measured with a neutrino spectrometer of sufficient energy resolution and known energy dependence of the detection efficiency. One possible process is the reaction $^{12}\mathrm{C}(\nu_e,e^-)^{12}\mathrm{N}$(g.s.) which has been first explored by a UC Irvine, LAMPF, and University of Maryland Collaboration [12]. There, π^+ stop in a beam dump and decay into an unpolarized sample of μ^+. The μ^+ decay with a mean lifetime of 2.2 μs, and ν_e are converted in a scintillator to e^-. The subsequent β^+ decay of ^{12}N(g.s.) with a mean lifetime of 15.9 ms greatly reduces background. The authors assumed the μ^+ to decay according to the pure $V-A$ law and obtained a total absorption cross section. This method has been improved by the KARMEN Collaboration [8] which was able to measure the energy of the e^- with the high precision $\sigma(E_e)/E_e=11.5\%/\sqrt{E}$ (E in MeV).

The neutrino transfers its energy via a two-particle reaction to the e^- with negligible recoil energy of the nucleus. Therefore we have a one-to-one correspondence between the energy E_ν of the ν_e and E_e of the e^- with $E_e=E_\nu-Q$, where $Q=17.3$ MeV is the Q value of the inverse β decay. Thus the KARMEN work constitutes the first low-energy, high-resolution neutrino spectrometer.

The KARMEN Collaboration has also assumed $V-A$ for μ decay in order to deduce the energy dependence of the absorption cross section. We note, however, that only the *product* of the neutrino spectrum $d\Gamma/dy$ and the absorption cross section $\sigma_A(y)$ has been measured. Is it possible to disentangle $d\Gamma/dy$ and σ_A? Let us return to Eq. (2). As already mentioned we can safely neglect $F_3(y)$. $F_1(y)$ is independent of the Lorentz structure of the interaction and happens to correspond to the $V-A$ distribution. It is peaked at the intermediate energy of $E_\nu=35.2$ MeV and goes to zero at the maximal energy of 52.8 MeV. This can be easily understood for example for $V-A$ from the left-handedness of all particles and angular momentum conservation. The couplings which contribute to ω and therewith to $F_2(y)$, however, produce particles with different chiralities which do not suppress, but even enhance, the rate at the maximal neutrino energy (see Fig. 1). Thus a value $\omega\neq 0$ adds events at the spectrum end point where otherwise none are expected: This makes the method a very sensitive precision test for ω.

The absorption cross section, on the other hand, has never been measured independently. It has been calculated, however [13], and all the calculations show a strong rise with energy with no small-scale structure in an energy region of a few MeV around the maximal energy of the neutrino where the experiment is most sensitive to ω. Here only the slope of σ_A matters. We have therefore parametrized σ_A as

$$\sigma_A=\alpha(y-y_0)+(y-y_0)^2 \tag{5}$$

with $\alpha=0.14$ as best value and $y_0=2Q/m_0$ as threshold energy. The slope can be changed by changing α. Figure 2 shows the resulting normalized electron energy spectrum for $\alpha=0.14$ and for $\omega=0$ and $\omega=0.2$. Because of the sharp rise of σ with energy the upper end of the e^- spectrum, which is the region of interest, is magnified as with a looking glass. This greatly enhances the sensitivity.

With the knowledge that ω is close to zero we can get an estimate of the statistical error $\Delta\omega$ which can be achieved for a given number N of events. For the hy-

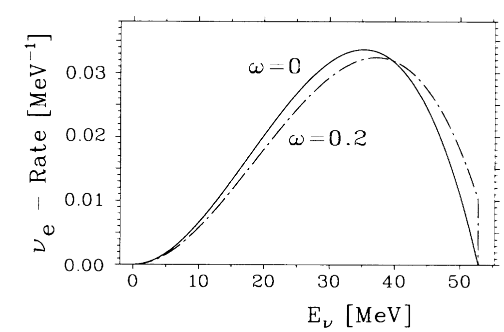

FIG. 1. Normalized energy distribution of ν_e from the decay of unpolarized μ^+. ω is the spectrum shape parameter of the ν_e and is the analog of the Michel parameter ρ of the e^+. For a pure $V-A$ interaction ω is equal to zero.

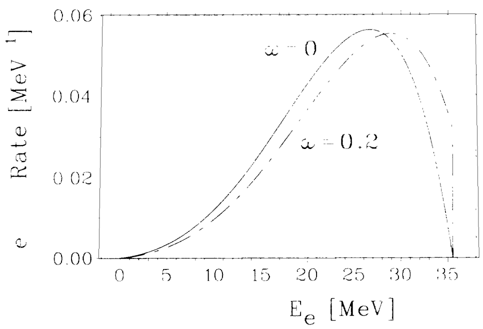

FIG. 2. Sensitivity of the normalized energy distribution of e^- from the reaction $^{12}C(\nu_e,e^-)^{12}N(g.s.)$ to ω. The curves represent the product of the ν_e distributions of Fig. 1 with an absorption cross section σ_A parametrized by a polynomial of second order in the electron energy E_e. ω is the spectrum shape parameter of the ν_e.

pothetical case of a constant absorption cross section (σ_A=const) we find

$$\Delta\omega = 1.02/\sqrt{N}\,. \qquad (6)$$

If we use the cross section of Eq. (5) instead, setting $\alpha=0.14$, we find

$$\Delta\omega = 0.26/\sqrt{N}\,, \qquad (7a)$$

$$\Delta\alpha = 1.83\sqrt{N}\,, \qquad (7b)$$

$$\rho_{\omega\alpha} = +0.22\,, \qquad (7c)$$

where $\rho_{\omega\alpha}$ is the error correlation between ω and α. $\Delta\omega$ is almost independent of the value of α chosen, as can be seen from the small value of $\rho_{\omega\alpha}$. Equations (7) show that ω can be determined with high precision, and that the information about the absorption cross section (parametrized by α) is almost independent of the information about the shape of the ν_e spectrum parametrized by ω.

We have also investigated the effect of a finite energy resolution; with the value of the KARMEN experiment it is negligible. The quoted error (7a) is thus quite realistic. It should be mentioned, however, that radiative corrections should be included in the data analysis.

An experimental result of $\omega=0$ and $\Delta\omega=2.6\times10^{-2}$ does not improve the existing upper limits for most of the coupling constants except for $|g^S_{LL}|$, for which we obtain

$$|g^S_{LL}| \leq \sqrt{\tfrac{16}{3}\Delta\omega}\,, \qquad (8)$$

which yields $|g^S_{LL}| \leq 0.37$ (68% C.L.). This is already better in precision than the value of $|g^S_{LL}|$ as determined from inverse muon decay, which relies on the relation $|g^S_{LL}|^2 \leq 4(1-S)$ [4], where S is obtained by dividing the measured cross section with the cross section calculated for $V-A$. Present experiments achieved a ΔS between 6% and 8% [14]. It should be mentioned, however, that the measurement of an absolute cross section is much more prone to systematic errors than the measurement of the shape of the neutrino spectrum. We therefore expect that it will be the measurement of ω which will determine the absence of a scalar interaction with left-handed e and μ most precisely.

In summary we have shown that the measurement of ν_e from stopped μ^+ decay by the reaction $^{12}C(\nu_e,e^-)$-$^{12}N(g.s.)$ yields independently information about the absorption cross section and about the neutrino spectrum. This constitutes a totally new kind of muon decay experiment in which the new spectrum shape parameter ω of the ν_e can be determined with a statistical error of 0.026 for 100 events. The parameter ω is the analog to the Michel parameter ρ but for the neutrino spectrum. In the standard model its value is zero. A precise measurement of ω allows us to give improved upper limits on the scalar coupling $|g^S_{LL}|$ with left-handed e and μ which has been determined previously only in an indirect way.

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