On the Precise Formulation of the Equivalence Theorem

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A systematic analysis of renormalization schemes and a general proof of the precise formulation of the equivalence theorem are given in the R_ξ gauge for both the $SU(2)_L$ and the $SU(2)\times U(1)$ theories. The precise formula for the modification factor $C_{\rm mod}$ is obtained, and a convenient particular scheme in which $C_{\rm mod}$ is exactly unity is proposed. $C_{\rm mod}$ in other schemes are discussed up to one loop in the heavy Higgs boson limit.

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Longitudinal weak boson scattering $V_L^a V_L^b \to V_L^c V_L^d$ (V_L^a stands for $W^\pm$ or Z^0) is one of the most important processes to be studied at the Superconducting Super Collider and the CERN Large Hadron Collider. The longitudinal component V_L^a arises from "eating" the would-be Goldstone boson ϕ^a; therefore $V_L^a V_L^b \to V_L^c V_L^d$ is related to the scattering of Goldstone bosons, which probes the mechanism of electroweak symmetry breaking. It is well known that the relation between the two scattering amplitudes at energy $E \gg M_W$ can be described by the equivalence theorem (ET), which states

$$T(V_L^{a_1},\ldots,V_L^{a_n},\Phi)=T(i\phi^{a_1},\ldots,i\phi^{a_n},\Phi)+O(M_W/E)\,, \tag{1}$$

where Φ denotes other possible physical particles. This simple relation was given by many authors [1] and was claimed to hold to all orders in perturbation theory for any value of the Higgs boson mass m_H. Equation (1) is very useful for calculating $T(V_L^{a_1},\ldots,V_L^{a_n},\Phi)$ and has thus been widely used [2]. However, Yao and Yuan [3], and Bagger and Schmidt [4], pointed out recently from more careful examination of loop contributions that, in general, there should be a modification factor C for each external Goldstone boson field ϕ^{a_i}, and $C\neq 1$ beyond the tree level, i.e., (1) should be modified as

$$T(V_L^{a_1},\ldots,V_L^{a_n},\Phi)=C^n T(i\phi^{a_1},\ldots,i\phi^{a_n},\Phi)+O(M_W/E)\,. \tag{2}$$

The formula for C to all orders in perturbation theory given in Ref. [3] is rather complicated, and the renormalization prescription they suggested for making $C=1$ relies on the explicit calculation of C, so that it is cumbersome in practical calculation. Since the ET is so useful, it is of special importance to make this issue clearer and simplify the expression for C. In this Letter, we will give a brief account of our recent work [5], including (i) a systematic analysis of the renormalization schemes in the R_ξ gauge for both the $SU(2)_L$ theory and the $SU(2)\times U(1)$ electroweak theory, (ii) a general proof of the precise formulation of the ET in which a simple formula for C is obtained, and (iii) a proposal for a particular renormalization scheme in which C is exactly unity and which is easy to implement in practical calculations. The details of this study will be presented in a longer paper [6]. We shall also show results, explicit up to one loop, for the heavy Higgs boson decay $H\to W_L^+W_L^-$ in some currently used renormalization schemes which are different from our particular scheme, and we shall see that in those schemes $C-1$ is, in general, *not small* and the ξ-dependent part in $T(i\phi^{a_1},\ldots,i\phi^{a_n},\Phi)$ is generally *not $O(M_W/E)$ suppressed.*

I. Renormalization schemes in the R_ξ gauge.—Consider the standard model. The Higgs and ghost fields are denoted by H, c^a, $\bar{c}^a$, respectively. We take the R_ξ gauge with the gauge fixing term written as

$$\mathcal{L}_{\rm gf}=-\tfrac{1}{2}(F_0^a)^2,\quad F_0^a\equiv(\xi_0^a)^{-1/2}\partial_\mu V_0^{a\mu}-(\xi_0^a)^{1/2}\kappa_0^a\phi_0^a\,, \tag{3}$$

where the subscript 0 denotes the bare quantities, and we have put a free parameter κ_0^a in (3) instead of taking it to be the mass of V_μ^a for generality.

The $SU(2)_L$ theory: This is the case of neglecting the Weinberg angle in the $SU(2)\times U(1)$ electroweak theory. In this case $V_\mu^a=W_\mu^a$. We simply take $\xi_0^a=\xi_0$, $\kappa_0^a=\kappa_0$, for $a=1,2,3$. The multiplicative renormalization constants are defined as follows: The renormalization constants for the physical sector are defined in the same way as in Ref. [7], while those for the unphysical sector are defined as $\phi_0^a=Z_\phi^{1/2}\phi^a$, $c_0^a=\tilde{Z}c^a$, $\bar{c}_0^a=\bar{c}^a$, $\xi_0=Z_\xi\xi$, $\kappa_0=Z_\kappa\kappa$.

Taking the functional derivatives of the generating equation for Ward-Takahashi (WT) identities [8], we obtain the following identities:

$$\begin{aligned}&ik^\mu[iD_{0\mu\nu}^{-1}(k)+\xi_0^{-1}k_\mu k_\nu]+M_{W0}\hat{C}_0(k^2)[iD_{0\phi\nu}^{-1}(k)-i\kappa_0k_\nu]=0\,,\\&ik^\mu[-iD_{0\phi\mu}^{-1}(k)+i\kappa_0k_\mu]+M_{W0}\hat{C}_0(k^2)[iD_{0\phi\phi}^{-1}(k)+\xi_0\kappa_0^2]=0\,,\\&iS_{0ab}^{-1}(k)=[1+\Delta_3(k^2)][k^2-\xi_0\kappa_0M_{W0}\hat{C}_0(k^2)]\delta_{ab}\,,\end{aligned} \tag{4}$$

where $D_{0\mu\nu}$, $D_{0\phi\nu}$, $D_{0\phi\phi}$, and S_{0ab} are, respectively, the bare propagators of the W field, W-ϕ, ϕ field, and ghost field, and

$\hat{C}_0(k^2) \equiv [1+\Delta_1(k^2)+\Delta_2(k^2)]/[1+\Delta_3(k^2)]$, with the Δ's defined in Refs. [5,6] (see also the similar definition in Ref. [4]). The identities (4) put constraints on the renormalization constants which can be written as

$$Z_\xi = \Omega_\xi Z_W, \quad Z_\kappa = \Omega_\kappa Z_W^{1/2} Z_\phi^{-1/2} Z_\xi^{-1}, \quad Z_\phi = \Omega_\phi Z_W Z_{M_W}^2 \hat{C}_0^2(\text{sub point}), \quad \tilde{Z} = \Omega_c [1+\Delta_3(\text{sub point})]^{-1}, \tag{5}$$

where Ω_ξ, Ω_κ, Ω_ϕ, and Ω_c are finite constants to be determined by the subtraction conditions.

After renormalization the renormalized $\hat{C}(k^2)$ is

$$\hat{C}(k^2) = (Z_W/Z_\phi)^{1/2} Z_{M_W} \hat{C}_0(k^2). \tag{6}$$

We shall see in Sec. II that this $\hat{C}(k^2)$ is directly related to the modification factor in (2).

The physical propagators can be expressed in terms of the proper self-energies:

$$iD_{\mu\nu}^{-1}(k) = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) [-k^2 + M_W^2 - \Pi_{WW}(k^2)] + \frac{k_\mu k_\nu}{k^2} [-\xi^{-1} k^2 + M_W^2 - \tilde{\Pi}_{WW}(k^2)],$$

$$iD_{\phi\mu}^{-1}(k) = -ik_\mu [M_W - \kappa + \tilde{\Pi}_{W\phi}(k^2)], \quad iD_{\phi\phi}^{-1}(k) = k^2 - \xi\kappa^2 - \tilde{\Pi}_{\phi\phi}(k^2), \quad iS_{ab}^{-1}(k) = k^2 - \xi\kappa M_W - \tilde{\Pi}_{c\bar{c}}(k^2).$$

We can see that all the unphysical parts of the physical propagators manifest the same tree-level pole at $k^2 = \xi\kappa M_W$. Thus the renormalized WT identities become

$$\begin{aligned}
(\tilde{\Pi}_{WW} - M_W^2)(\tilde{\Pi}_{\phi\phi} - k^2) - k^2(\tilde{\Pi}_{W\phi} + M_W)^2 = {} & \xi^{-1}(1-\Omega_\xi^{-1})[(k^2 - \xi\kappa M_W)^2 - k^2(\tilde{\Pi}_{\phi\phi} + 2\xi\kappa\tilde{\Pi}_{W\phi}) - \xi^2\kappa^2\tilde{\Pi}_{WW}] \\
& + 2\kappa \frac{\Omega_\kappa - 1}{\Omega_\xi} [(k^2 - \xi\kappa M_W) M_W + k^2 \tilde{\Pi}_{W\phi} + \xi\kappa\tilde{\Pi}_{WW}] \\
& + \kappa^2 \frac{(\Omega_\kappa - 1)^2}{\Omega_\xi} [k^2 - \xi M_W^2 + \xi\tilde{\Pi}_{WW}],
\end{aligned} \tag{7}$$

$$\hat{C}(k^2) = [M_W^2 - \tilde{\Pi}_{WW} + k^2 \xi^{-1}(\Omega_\xi^{-1} - 1)]/[M_W^2 + M_W \tilde{\Pi}_{W\phi} + \kappa M_W(\Omega_\xi^{-1}\Omega_\kappa - 1)],$$

$$\tilde{\Pi}_{c\bar{c}} = (k^2 - \xi\kappa M_W) - \tilde{Z}[1+\Delta_3(k^2)][k^2 - \xi\kappa M_W \Omega_\kappa \hat{C}(k^2)].$$

Equations (7) are instructive for finding renormalization schemes simplifying the expression for $\hat{C}(k^2)$.

We first take an on-shell scheme in such a way that for the physical sector we take the usual on-shell scheme [7,9], and for the unphysical sector we take the following on-shell conditions:

$$\tilde{\Pi}_{WW}(\xi\kappa M_W) = \tilde{\Pi}_{W\phi}(\xi\kappa M_W) = \tilde{\Pi}_{\phi\phi}(\xi\kappa M_W) = \tilde{\Pi}_{c\bar{c}}(\xi\kappa M_W) = 0.$$

We call this scheme I. When $\xi = 1$, $\kappa = M_W$, and at one-loop level, these on-shell conditions reduce to that adopted in Ref. [7]. In the general R_ξ gauge, it can be shown that $\tilde{\Pi}_{WW}(\xi\kappa M_W) = 0$ can be satisfied by adjusting the parameter $\Omega_\xi - 1$; $\tilde{\Pi}_{\phi\phi}(\xi\kappa M_W) = 0$ can be satisfied by adjusting $\Omega_\kappa - 1$. Then the conditions $\tilde{\Pi}_{W\phi}(\xi\kappa M_W) = \tilde{\Pi}_{c\bar{c}}(\xi\kappa M_W) = 0$ are guaranteed by the WT identities provided $\kappa = M_W$. [For $\tilde{\Pi}_{W\phi}(\xi\kappa M_W) = 0$, the condition $\kappa = M_W$ is needed only beyond one loop.] The remaining $Z_\phi, \tilde{Z}$ are then determined by the conventional normalization of residues at the pole $k^2 = \xi\kappa M_W$. After doing this, the expression for $\hat{C}(\xi\kappa M_W)$ can be greatly simplified,

$$\hat{C}(\xi\kappa M_W)|_{\kappa = M_W} = \left. \frac{M_W^2 + \kappa M_W(\Omega_\xi^{-1} - 1)}{M_W^2 + \kappa M_W(\Omega_\xi^{-1}\Omega_\kappa - 1)} \right|_{\kappa = M_W} = \Omega_\kappa^{-1}. \tag{8}$$

Now $\hat{C}(\xi\kappa M_W)|_{\kappa = M_W}$ is exactly given by a *single quantity* Ω_κ *which has already been determined in our renormalization scheme I itself.*

Furthermore, from (7), we can propose a particular renormalization scheme which makes $\hat{C}(\xi\kappa M_W)$ *exactly unity*. We shall call it scheme II. In this scheme we simply take $\Omega_\kappa = 1$. Then the on-shell condition $\tilde{\Pi}_{\phi\phi}(\xi\kappa M_W) = 0$ can be satisfied by adjusting Z_ϕ. (In this case the residue of $\tilde{D}_{\phi\phi}$ at $k^2 = \xi\kappa M_W$ is not normalized in the conventional way, but this does not affect the physics.) This determination of Z_ϕ also concerns only the renormalization of $\tilde{\Pi}_{\phi\phi}$, so that it is easy and natural to implement.

The SU(2)×U(1) electroweak theory: In the charged sector, the WT identities for the bare propagators are also of the form (4) but with much more complicated Δ's [6]. The renormalized $\hat{C}^W(k^2)$ is $\hat{C}^W(k^2) = (Z_W/Z_\phi)^{1/2} \times Z_{M_W} \hat{C}_0^W(k^2)$. We can still have renormalization scheme I and scheme II parallel to the two schemes in the $SU(2)_L$ theory, in which

$$\hat{C}^W(\xi^W \kappa^W M_W) = \begin{cases} (\Omega_\kappa^W)^{-1} \ (\kappa^W = M_W), & \text{scheme I}, \\ 1, & \text{scheme II}, \end{cases} \tag{9}$$

where the Ω_κ^W is the finite constant in Z_κ^W [similar to Z_κ in (5)].

In the neutral sector, the corresponding formulas are much more complicated due to mixings. Quite lengthy analysis and derivation show that [6]

$$\hat{C}^Z(\xi^Z \kappa^Z M_Z) = \begin{cases} (\Omega_\kappa^{ZZ})^{-1} \ (\kappa^Z = M_Z), & \text{scheme I}, \\ 1, & \text{scheme II}, \end{cases} \tag{10}$$

where Ω_κ^{ZZ} is the finite constant in Z_κ^{ZZ}.

II. Precise formulation of equivalence theorem.—The general proof of ET consists of two parts: (i) Deriving a Slavnov-Taylor (ST) identity for the gauge fixing function F_0^a, $\langle 0|F_0^{a_1}(k_1)\cdots F_0^{a_n}(k_n)\Phi|0\rangle=0$; (ii) doing renormalization and amputating the external F_0^a lines to obtain the scattering amplitude. Since F_0^a contains both a $k^\mu V_{\mu 0}^a(k)$ term and a $\phi_0^a(k)$ term, this amplitude can then give the relation between $T(V_L^{a_1},\ldots,V_L^{a_n},\Phi)$ and $T(i\phi^{a_1},\ldots,i\phi^{a_n},\Phi)$, which is the desired ET. The above identity has already been proved by Gounaris, Kögerler, and Neufeld [1]. Therefore the crucial thing for obtaining the precise formulation of ET is to do the renormalization and amputation properly. The technique for doing amputation in the $SU(2)_L$ theory has been developed in Ref. [4]. We have generalized it to the more complicated realistic $SU(2)\times U(1)$ theory.

The $SU(2)_L$ theory: Directly applying the technique in Ref. [4] we obtain

$$T(V_L^{a_1},\ldots,V_L^{a_n},\Phi)=(C_{\rm mod})^n T(i\phi^{a_1},\ldots,i\phi^{a_n},\Phi)+O(M_W/E)\,, \quad (11)$$

where the modification factor is

$$C_{\rm mod}=(M_W/M_W^{\rm phys})\hat{C}(M_W^2)\,. \quad (12)$$

Here we have considered the possible difference between the multiplicatively renormalized M_W and the physical mass (pole of the full physical propagator) $M_W^{\rm phys}$ in some renormalization schemes. For on-shell schemes, $M_W^{\rm phys}=M_W$. Then with Eq. (6), (12) reduces to the modification factor given in Ref. [3]. However, in our renormalization schemes I and II, the formulas for $\hat{C}(M_W^2)$ are greatly simplified, and we obtain

$$C_{\rm mod}=\begin{cases}(\Omega_\kappa)^{-1}\ (\kappa=M_W,\ \xi=1),\ \text{scheme I}\,,\\ 1\ (\kappa=M_W/\xi),\ \text{scheme II}\,.\end{cases} \quad (13)$$

It can be shown that scheme II is the only renormalization scheme that makes $C_{\rm mod}$ exactly unity [10].

The $SU(2)\times U(1)$ electroweak theory: A lengthy derivation shows that [6] ET in the $SU(2)\times U(1)$ theory is also of the form (11) with

$$C_{\rm mod}^W=(M_W/M_W^{\rm phys})\hat{C}^W(M_W^2)\,,\qquad C_{\rm mod}^Z=(M_Z/M_Z^{\rm phys})\hat{C}^Z(M_Z^2)\,. \quad (14)$$

In our renormalization schemes I and II, $C_{\rm mod}^W$ and $C_{\rm mod}^Z$ take the simple forms

$$C_{\rm mod}^W=\begin{cases}(\Omega_\kappa^W)^{-1}\ (\kappa^W=M_W,\xi^W=1),\ \text{scheme I}\,,\\ 1\ (\kappa^W=M_W/\xi^W),\ \text{scheme II}\,,\end{cases}$$
$$C_{\rm mod}^Z=\begin{cases}(\Omega_\kappa^{ZZ})^{-1}\ (\kappa^Z=M_Z,\xi^Z=1),\ \text{scheme I}\,,\\ 1\ (\kappa^Z=M_Z/\xi^Z),\ \text{scheme II}\,.\end{cases} \quad (15)$$

To summarize, the precise formulation of ET for $SU(2)_L$ and $SU(2)\times U(1)$ theories is Eq. (11) which holds to all orders in the gauge couplings with arbitrary m_H. It reduces to (1) only in scheme II. In general $C_{\rm mod}$ may be significantly different from unity and it may even contain a large [not of $O(M_W/E)$] ξ-dependent piece. As an example, we consider the amplitude up to one loop of $H\to W_L^+W_L^-$ in various currently used renormalization schemes other than scheme II. For simplicity, we only present the results in the heavy Higgs boson limit. In this case, the $H\to W_L^+W_L^-$ decay amplitude for $m_H=1$ TeV and $g^2=0.422$ [11] is

$$T(H\to W_L^+W_L^-)=\left[1+\frac{g^2}{16\pi^2}\frac{m_H^2}{M_W^2}\left(\frac{19}{16}-\frac{3\sqrt{3}}{8}\pi+\frac{5\pi^2}{48}\right)\right]T_0=[1+0.0731]T_0\,, \quad (16)$$

where T_0 is the tree-level amplitude and only $O(g^2m_H^2/M_W^2)$ terms are kept in (16).

(i) In the on-shell scheme by Böhm, Hollik, and Spiesberger and by Hollik [7], Z_ϕ is chosen to be $Z_\phi=Z_H$. For $m_H^2\gg M_W^2$,

$$C_{\rm mod}^W=1+\frac{g^2}{16\pi^2}\left[\frac{m_H^2}{M_W^2}\left(-\frac{13}{16}+\frac{\sqrt{3}\pi}{8}\right)-\frac{3}{8}\ln\frac{m_H^2}{M_W^2}+\frac{\xi^W}{4}\ln\frac{m_H^2}{M_W^2}\right]$$

and

$$T(H\to\phi^+\phi^-)=-\left\{1+\frac{g^2}{16\pi^2}\left[\frac{m_H^2}{M_W^2}\left(\frac{45}{16}-\frac{5\sqrt{3}}{8}\pi+\frac{5\pi^2}{48}\right)-\frac{\xi^W}{2}\ln\frac{m_H^2}{M_W^2}\right]\right\}T_0\,,$$

where T_0 is the same as that in (16). We see that both $C_{\rm mod}^W$ and $T(H\to\phi^+\phi^-)$ have large ξ-dependent pieces and they cancel in the product $(C_{\rm mod}^W)^2T(H\to\phi^+\phi^-)$ up to one loop. The ξ-independent part in $i^2(C_{\rm mod}^W)^2T(H\to\phi^+\phi^-)$ just coincides with that in (16) as it should according to (11). Numerically, $(C_{\rm mod}^W)^2=1-0.111+0.02\xi^W$, $T(H\to\phi^+\phi^-)=-[1+0.184-0.02\xi^W]T_0$. We see that $(C_{\rm mod}^W)^2-1$ *is not small.* Therefore, improper use of Eq. (1) in this scheme is apparently inadequate. Note that in the longitudinal-W^a-boson scattering, $W_L^aW_L^b\to W_L^aW_L^b$, the total modification factor in (11) is $(C_{\rm mod}^W)^4=1-0.222+0.04\xi^W$, which is *quite different from unity.*

(ii) In the on-shell scheme in Landau gauge by Marciano and Willenbrock [11], Z_ϕ is determined by $d\Pi_{\Phi\Phi}/dk^2|_{k^2=0}=0$. Up to one loop, $C_{\rm mod}=1+O(g^2)$, i.e., there is neither $O(g^2m_H^2/M_W^2)$ [4] nor $O(g^2\ln(m_H^2/M_W^2))$ terms in $C_{\rm mod}$. On the other hand, the value of the obtained $i^2T(H\to\phi^+\phi^-)$ with $m_H=1$ TeV coincides with the right-hand side of (16). Thus this scheme is convenient in the heavy Higgs boson limit up to one loop.

(iii) In the on-shell scheme by Aoki *et al.* [9], the physical particles are renormalized on-shell, while the unphysical sector is renormalized by the minimal subtraction scheme. In the heavy Higgs boson limit, up to one loop, with $m_H=1$ TeV, $(C^W_{\rm mod})^2=1-0.0522+0.02\xi^W$ and $T(H\to\phi^+\phi^-)=-[1+0.1253-0.02\xi^W]T_0$, where $(C^W_{\rm mod})^2-1$ is also not negligible, and $i^2(C^W_{\rm mod})^2T(H\to\phi^+\phi^-)$ coincides with (16) as it should since (16) is scheme independent.

(iv) In the complete minimal subtraction scheme, the result is the same as that in the on-shell scheme by Aoki *et al.* [9]. This is easy to understand since $C^W_{\rm mod}$ is related only to the renormalization of the unphysical sector.

(v) The intermediate scheme [7] is a widely used scheme with G_μ taken as input instead of M_W. In this scheme $M_W^{\rm phys}\neq M_W$. The renormalization scheme for the unphysical sector is not specified. If we take the scheme in Ref. [7] or in Ref. [9] for the unphysical sector, we get a large $C_{\rm mod}-1$. If we take our scheme I for the unphysical sector, we get $C_{\rm mod}=1+O(g^2)$.

Conclusions.— We have proved that the precise formulation of ET in the R_ξ gauge in both the $SU(2)_L$ and the $SU(2)\times U(1)$ theories is of the form (11). The modification factor $C_{\rm mod}$ is given by (12) and (14) which is both renormalization scheme and ξ dependent. In scheme I, the expression for $C_{\rm mod}$ is *already determined by the renormalization scheme I itself.* We have also proposed a particular scheme II in which $C_{\rm mod}$ is *exactly unity*, so that ET is described by the simplest form (1). Scheme II is easy to implement in practical calculations.

We have also calculated $C_{\rm mod}$ in other currently used schemes other than scheme II up to one loop in the heavy Higgs boson limit. It is shown that the intermediate scheme with scheme I for the unphysical sector is a convenient scheme in which (1) holds approximately in the heavy Higgs boson limit. In other schemes, such as the Böhm-Hollik-Spiesberger scheme [7], etc., $C_{\rm mod}$ is *significantly different from unity even in the heavy Higgs boson limit and it even contains a large ξ-dependent piece.* So, one should be very careful when using ET with these schemes.

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[10] In a recent paper [Phys. Rev. D **41**, 2294 (1990)], H. Veltman studied the ET for the $SU(2)_L$ theory in the 't Hooft-Feynman gauge in a renormalization scheme keeping the gauge fixing function F^a unchanged which corresponds to $\Omega_\xi=\Omega_\kappa=1$ in our formalism. This scheme is different from our scheme II. Veltman's conclusion is that $C_{\rm mod}$ is also exactly unity in that scheme. We would like to point out that in Veltman's amputation procedure two assumptions were actually made: (i) The poles of the unphysical parts of the full propagators coincide with M_W. (ii) There is no mixing between the W and ϕ propagators. However, unfortunately, these two assumptions do not really hold in the renormalization scheme keeping F^a unchanged beyond tree level [D. A. Ross and J. C. Taylor, Nucl. Phys. **B51**, 125 (1973)]. For instance, in a U(1) Higgs theory wherein the gauge boson mass is M, an explicit one-loop calculation shows that in Veltman's scheme

$$C_{\rm mod}=1+\frac{g^2}{16\pi^2}\left[\left(\frac{13}{6}-x^2\right)+\left(\frac{5}{2}-\frac{7}{2}x^2+x^4\right)\ln x -\frac{1}{2}\frac{2x^2-5}{x^2-4}x(x^2-3)(x^2-4)^{1/2}\times\ln\frac{x+(x^2-4)^{1/2}}{2}\right],$$

where $x\equiv m_H/M>2$. Therefore $C_{\rm mod}\neq1$.

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