# Cosmic String with a Light Massive Neutrino

Andreas Albrecht[a] and Albert Stebbins

*NASA/Fermilab Astrophysics Center, Fermi National Accelerator Laboratory, Box 500, Batavia, Illinois 60510-0500*

(Received 13 April 1992)

We have estimated the power spectra of density fluctuations produced by cosmic strings with neutrino hot dark matter (HDM). Normalizing at $8h^{-1}$ Mpc [where $h = H_0/(100$ km/sec Mpc), and $H_0$ is the Hubble constant] we find that the spectrum has more power on small ($\lesssim 10h^{-1}$ Mpc) scales than HDM + inflation, less than cold dark matter (CDM) + inflation, and significantly less the CDM + strings. With HDM, large wakes ($\sim 20h^{-2}$ Mpc) give significant contribution to the power on the galaxy scale and may give rise to large sheets of galaxies.

PACS numbers: 98.80.Cq, 14.60.Gh, 98.60.Eg

In a previous Letter (Ref. [1], hereafter AS92) we estimated the power spectrum of density inhomogeneities produced by cosmic strings in a universe consisting mostly of cold dark matter (CDM). There we had found that spectrum had too much power on small scales compared to what is thought to be required. We also found that the superposition of many cosmic string wakes is liable to make individual "stringy" features in the density field less apparent. Here we use the same techniques to estimate the power spectrum of density fluctuation in a universe containing hot dark matter (HDM) and cosmic strings. The free streaming of the HDM will damp some of the small-scale perturbations produced at early time and this may lead to a better behaved power spectrum. The damping of the early wakes may also cause the wakes produced at later times to become more prominent, thus leading to a more stringy and non-Gaussian density field.

Here we will consider the standard HDM model where there is one species of massive neutrino with a sufficiently large mass to make $\Omega = 1$, i.e., $m_\nu = 93h^2$ (Ref. [2]). We use the notation $h = H_0/(100$ km/sec Mpc) (where $H_0$ is the Hubble constant), cosmic scale factor $= a$, time $= t$, conformal time $= \eta$, and $c = G = 1$ unless otherwise stated. For all figures we take $h = 1$ and use the standard $8h^{-1}$ Mpc normalization. The actual perturbations produced by the strings will depend on the evolution of the network of cosmic strings. Because of the extremely large dynamic range needed to model the cosmic string networks (see Ref. [3]), there remains some uncertainty in the exact nature of this evolution. As in AS92 we will consider three different models for this evolution, which we think span the plausible range of possibilities.

The three models for the network are denoted by AT, I, and X and are described in detail in AS92. The AT model is fitted directly to the simulations of Albrecht and Turok (Ref. [4]). Here the strings have no significant small-scale structures and the coherence length of the network is much smaller than the horizon. The I (intermediate) model is patterned after the numerical simulations of Bennett and Bouchet (Ref. [5]) and Allen and Shellard (Ref. [6]), although we have no direct comparison to their simulation. Their simulations show a somewhat smaller density of the long strings, and the long strings have significant small-scale structure. The X (extreme) model is meant to be close to the original picture of the string network in which coherence length of strings is close to the horizon and there is no small-scale structure on the strings.

The number and mass of loops emitted from a cosmic string network are quantities which are most uncertain in this scenario. The Albrecht and Turok simulations (Ref. [4]) find loops produced with size $\xi$ which then fragment, while the other simulations (Refs. [5,6]) find loop production at a much smaller scale. We have noted in AS92 that even with the larger-size loops their contribution to the CDM power spectrum is small and unimportant. With many smaller loops the effects will be even smaller (Refs. [7,8]). The importance of loops in the HDM case is not so clear. It is clear that earlier work (Refs. [9–11]) based entirely on string loops represents a poor approximation to the full effects of cosmic strings on neutrinos. If loops are as small as the gravity wave cutoff then they are probably not important for density perturbations; however, if the loop size is $\sim \xi$ then they may give the dominant perturbation on galaxy scales. In this paper we have chosen to follow AS92 and not include the effects of loops. Thus the power on small scales may be greater than we indicate.

Following AS92 we find the power spectra of the overdensity fluctuations may be approximated by

$$P(k) = 16\pi^2(1+z_{\mathrm{eq}})^2\mu^2 \int_{\eta_i}^{\infty} |T(k;\eta')|^2 \mathcal{F}(k\xi/a)\,d\eta' , \tag{1}$$

where $\mathcal{F}$ is an integral over the temporal correlation function of the "$\rho+3p$" part of the string stress-energy tensor. It essentially gives the power spectrum of the overdensity which is laid down at each time. The function $T(k;\eta')$ is the transfer function which gives how this initial perturbation spectrum is evolved to the present. Of course $T$ for an HDM cosmology will be different than for the CDM cosmology considered in AS92, but the perturbations initiated by the strings, given by $\mathcal{F}$, will be the same (the difference in the expansion laws for the two cases is extremely small). We take $T$ to be

$$T(\mathbf{k},\eta) = \left[\frac{1}{1+[0.435kD(\eta)]^{2.03}}\right]^{4.43} \times \frac{1}{1+(k_c/k)^2}\tilde{T}_2^c(\mathbf{k},\eta)\,. \quad (2)$$

The last two terms are exactly as in AS92. The $\tilde{T}_2^c(\mathbf{k})$ is the approximation to the CDM transfer function from Ref. [12], and its immediate prefactor approximates the effects of "compensation," $k_c^{-1}$ giving the compensation scale. The first prefactor represents the damping of perturbations by neutrino free streaming, and is a fit to numerical calculations of the transfer function of a Fermi-Dirac distribution of nonrelativistic neutrinos. The function $D(\eta)$, which gives the comoving damping length, is the comoving distance traveled by a neutrino with momentum $T_\nu/m$ from time $\eta$ to time $\infty$. The reason for the simplicity of this damping factor derives from the fact that the neutrinos are very nonrelativistic for most of the time when the perturbations of interest are produced. Perturbations produced when the neutrinos are relativistic are either highly damped or highly compensated, and thus contribute little to the final power spectrum. Our damping factor is inaccurate in this regime, but it hardly matters to the final power spectrum.

The input of the string comes from the "form factor"

$$\mathcal{F}(k\xi/a) = \frac{1}{\pi^2}\beta^2\Sigma\frac{\chi^2}{\xi^2}\left[\frac{1}{1+2(k\chi/a)^2}\right], \quad (3)$$

which has been used in AS92. The length $\xi \equiv (\rho_L/\mu)^{1/2}$ gives the density of long strings, the length $\chi$ indicates a typical curvature radius or coherence of the long strings, $\beta$ gives the rms velocity of the strings, and $\Sigma$ gives the increase in the surface density of the wakes due to wiggliness (Refs. [13,14]) over that of smooth strings with $\gamma\beta = 1$. The parameters for the different models are exactly as in AS92 and are listed in Table I.

In the universes with either CDM or HDM we now compare the power spectra of the string model with that of primordial adiabatic Harrison-Zel'dovich spectrum (HZ) of perturbations. In Fig. 1 we plot $4\pi k^3 P(k)$ for (1) the I model of strings in a universe with CDM as taken from AS92, (2) a CDM universe with a primordial adiabatic HZ spectrum, (3) the I model of strings in a universe with HDM as calculated here, and (4) an HDM universe with a primordial adiabatic HZ spectrum. Both (2) and (4) were taken from Ref. [15]. We see that strings + HDM give significantly less power on small scales than either HZ + CDM but also much more power than HZ + HDM. Note also that the peak of the strings + HDM power spectrum is at a somewhat smaller scale than HZ + HDM. The origin of these differences between the two HDM models is that the strings are able to seed some of the perturbations after the neutrinos do most of their free streaming, thus avoiding some of the damping that neutrinos do. In fact the short wavelength tail of the strings + HDM picture come from the wakes at the late epochs in the matter era when the neutrino free-streaming length becomes smaller and smaller.

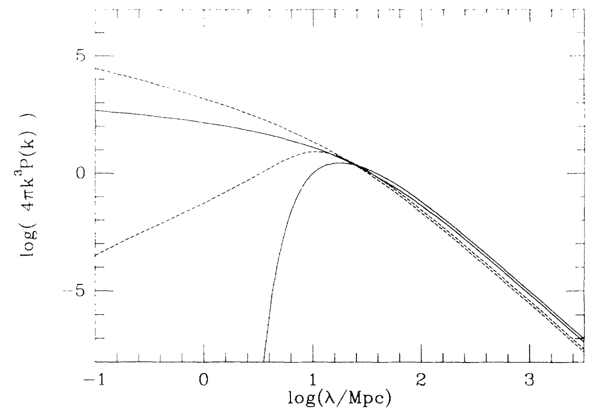


FIG. 1. $\log_{10}[4\pi(2\pi/\lambda)^3 P(\lambda)]$ vs $\log_{10}(\lambda/\mathrm{Mpc})$ for I strings (dashed), and the HZ spectrum (solid). The curves with smaller power are for CDM, and the others for HDM.

While strings + HDM may have increased power on small scales when compared with HZ + HDM, it may at first seem that the two scenarios would suffer from the same problems, namely, that galaxies would have to fragment from much larger "neutrino pancakes" and all the problems that entails (see Ref. [16]). These problems will be less severe for two reasons: (1) because the strings + HDM power spectrum is peaked at somewhat smaller scales than for HZ + HDM, and (2) because the linear perturbations can be non-Gaussian on the galaxy sale (see Fig. 4) so that the number of galaxy-sized objects which collapse and virialize may be much greater than for a Gaussian distribution with the same power spectrum. In fact the wakes produced by the individual strings at matter radiation equality and later will lead to perturbations with a width $\lesssim h^{-2}$ Mpc (Refs. [17,18]),

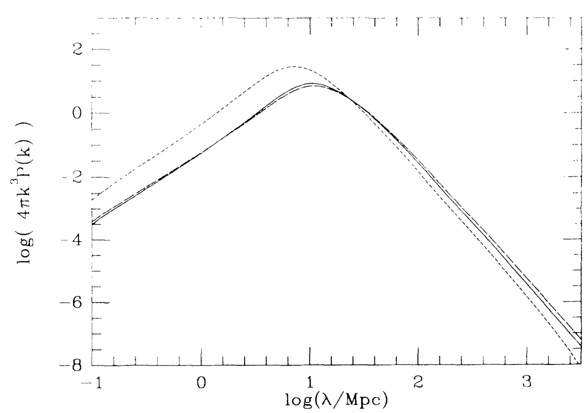


FIG. 2. Same as Fig. 1 except comparing AT strings (short-dashed), X strings (solid), and I strings (long-dashed).

TABLE I. Values of (a) the model parameters of Eq. (3), (b) the fit parameters of Eq. (4) when $k$ is in units of $h^2$/Mpc, and (c) the values of $\mu$ which give unity for the rms mass fluctuation in a ball of radius $8h^{-1}$ Mpc. Here "$c_r = \cdots$" indicates that this value of $c_r$ was used in Eq. (3.7) of Ref. [4] to determine $\xi$.

| Model | $\beta^2\Sigma$ | $\chi/\xi$ | $\xi$ | $k_c\eta$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\theta_5$ | $\theta_6$ | $\theta_7$ | $\theta_8$ | $\mu(h=0.5)$ | $\mu(h=1)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| AT | 0.5 | 0.58 | $c_r=0.08$ | $4\pi$ | 1.77 | 2.3 | 1.8 | 0.52 | 787 | 1.51 | 6.7 | 2.0 | $8.1\times10^{-6}$ | $5.0\times10^{-6}$ |
| I | 1.2 | 2 | $c_r=0.16$ | $2\pi$ | 6.8 | 4.7 | 4.4 | 1.55 | 2198 | 2.46 | 6.6 | 3.2 | $4.0\times10^{-6}$ | $2.0\times10^{-6}$ |
| X | 0.5 | 1 | $a\eta$ | $2\pi$ | 0.98 | 12.0 | 6.2 | 2.11 | 2899 | 2.82 | 6.5 | 3.5 | $39\times10^{-6}$ | $18\times10^{-6}$ |

corresponding to the galaxy scale and smaller. The power spectrum comes from a superposition of these narrow features.

It is interesting to see how the spectrum changes with the different models of strings. In Fig. 2 we plot $4\pi k^3P(k)$ for the three models of strings. All the curves have the standard normalization of unit variance of $\delta m/m$ in an $R=8h^{-1}$ Mpc sphere, chosen to reproduce galaxy clustering. Thus the string mass per unit length $\mu$ differs between each of the curves, as reflected in Table I. The slope of all the curves is $\lambda^{-4}$ for large $\lambda$, and $\lambda^2$ for small $\lambda$. For wavelengths with significant power, $4\pi k^3P(k)>0.1$, one can fit the curves in Fig. 1 to about 10% accuracy with the function

$$4\pi k^3P(k)=\frac{4\pi h^4\theta_1^2\mu_6^2k^4}{1+(\theta_2k)+(\theta_3k)^2+(\theta_4k)^3+(\theta_8k)^4+(\theta_6k)^{\theta_7}}\left(\frac{1}{1+1/(\theta_5k)^2}\right)^2 \tag{4}$$

when one takes the $\theta$'s given in Table I. The last factor in Eq. (4) represents a deviation from $\lambda^{-4}$ behavior at very large wavelengths, which occurs because the very largest scales have just entered the horizon and have yet to receive their full complement of perturbations.

To determine how apparent individual wakes are we can compare the rms $\Delta$ ($\equiv\delta m/m$) in a ball of radius $R$ to the $\Delta$ in the ball if centered on a single isolated wake seeded at time $\eta_i$. These are given by

$$\Delta^2_{\rm rms}(R)=\int|w(kR)|^2P(k)4\pi k^2dk\,,$$
$$\Delta_W(R,\eta_i)=8\mu\Sigma\int_0^\infty T(k;\eta_i)w(kR)dk\,, \tag{5}$$

where $w(x)=3(\sin x-x\cos x)/x^3$. We now consider the degree to which single string wakes stand out. To estimate this, we consider the galaxy scale, roughly $R=1h^{-2}$ Mpc. In Fig. 3 we plot the ratio $\Delta_W/\Delta_{\rm rms}$ as a function of the string coherence length $\chi$ at time $\eta_i$ when the wake was seeded. For an individual wake to stand out, it must successfully compete with all the other wakes produced at other times. For HDM, wakes produced at early times are washed out by neutrino free streaming, so there is less competition, and hence we find that the maximum value of the ratio in Fig. 3 is higher for HDM than for CDM. Also, neutrino free streaming suppresses the

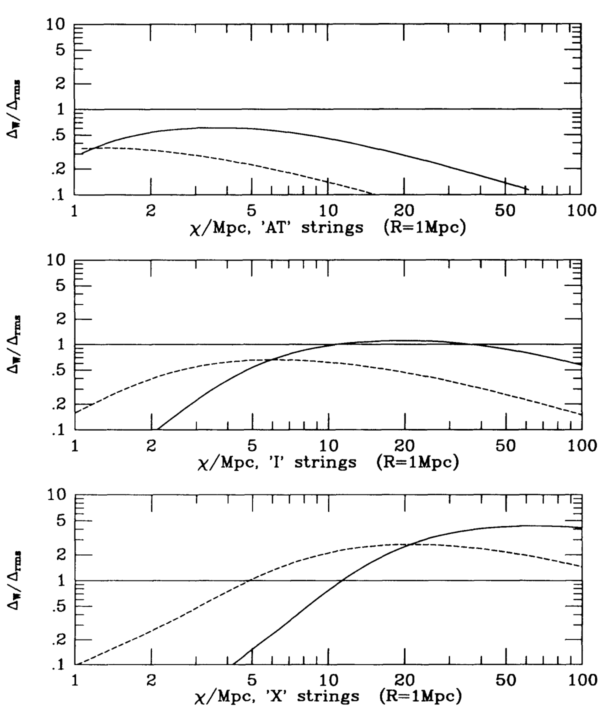


FIG. 3. $\Delta_W/\Delta_{\rm rms}$ for the three string models evaluated at $R=1$ Mpc. The solid curves correspond to HDM and the dashed curves to CDM.

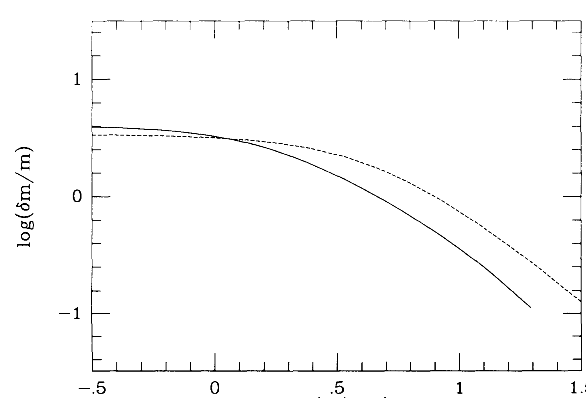


FIG. 4. Comparing $\Delta_W(R)$ for the $\chi=20$ Mpc wake (solid) with $\Delta_{\rm rms}(R)$ (dashed) using I strings.

earlier (small $\chi$) wakes, which shifts the maximum $\Delta_W$ to larger $\chi$ for HDM.

Figure 4 shows $\Delta_{\rm rms}$ (dashed) and $\Delta_W$ (solid) for the maximal HDM wake ($\chi = 20h^{-2}$ Mpc) using the I string model. It is interesting that the single wake begins to compete with the rms only on small scales. This suggests a rather radical "biasing" scheme: An individual wake can produce large-scale ($\sim\chi$) features in the distribution of galaxies (as first suggested by Vachaspati [19]) by being a dominant source of perturbations on *galactic* scales. This may occur without the wake having a major impact on the overall matter distribution on the scale $\chi$.

In summary, strings with HDM give more power on small scales than HZ with HDM and less than HZ with CDM. This is roughly what is thought to be required. There is also a greater tendency for individual wakes to stand out than in the CDM case, and the wakes which stand out the most tend to be larger. Whether this scenario can be developed into a realistic theory of galaxy formation will require more detailed calculations. Recent Cosmic Background Explorer (COBE) observations of microwave background anisotropy indicate that the amplitude of anisotropy is *roughly* consistent with this scenario (see Ref. [20]).

This work was supported in part by DOE and NASA (Grant No. NAGW-2381) at Fermilab.

---

[a]Permanent address: Theoretical Physics, The Blackett Laboratory, Imperial College, Prince Consort Road, London SW7 2BZ, England.

[1] A. Albrecht and A. Stebbins, Phys. Rev. Lett. **68**, 2121 (1992).

[2] R. Cowsik and J. McClelland, Phys. Rev. Lett. **29**, 669 (1972).

[3] D. Bennett, F. Bouchet, N. Turok, A. Albrecht, P. Shellard, and B. Allen, in *The Formation and Evolution of Cosmic Strings*, edited by G. Gibbons, S. Hawking, and T. Vachaspati (Cambridge Univ. Press, Cambridge, 1990), p. 321.

[4] A. Albrecht and N. Turok, Phys. Rev. D **40**, 973 (1989).

[5] D. Bennett and F. Bouchet, Phys. Rev. D **41**, 2408 (1990).

[6] B. Allen and P. Shellard, Phys. Rev. Lett. **64**, 119 (1990).

[7] A. Stebbins, in *The Formation and Evolution of Cosmic Strings*, edited by G. Gibbons, S. Hawking, and T. Vachaspati (Cambridge Univ. Press, Cambridge, 1990), p. 503.

[8] F. Bouchet and D. Bennett, Astrophys. J. Lett. **354**, L41 (1990).

[9] R. Brandenberger, N. Kaiser, and N. Turok, Phys. Rev. D **36**, 2242 (1987).

[10] R. Brandenberger, N. Kaiser, D. Schramm, and N. Turok, Phys. Rev. Lett. **59**, 2371 (1987).

[11] E. Bertschinger and P. Watts, Astrophys. J. **328**, 23 (1987).

[12] S. Veeraraghavan and A. Stebbins, Astrophys. J. **365**, 37 (1990).

[13] T. Vachaspati and A. Vilenkin, Phys. Rev. Lett. **67**, 1057 (1991).

[14] D. Vollick, Phys. Rev. D **45**, 1884 (1992).

[15] J. Bardeen, J. Bond, N. Kaiser, and A. Szalay, Astrophys. J. **304**, 15 (1986).

[16] S. White, in *The Early Universe*, edited by W. Unruh and G. Semenoff (Reidel, Dordrecht, 1988), p. 239.

[17] A. Stebbins, S. Veeraraghavan, R. Brandenberger, J. Silk, and N. Turok, Astrophys. J. **322**, 1 (1987).

[18] L. Perivolaropoulos, R. Brandenberger, and A. Stebbins, Phys. Rev. D **41**, 1764 (1990).

[19] T. Vachaspati, Phys. Rev. Lett. **57**, 1655 (1986).

[20] D. Bennett, A. Stebbins, and F. Bouchet (to be published).