

## Atomic Interferences and the Topological Phase

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By use of a longitudinal Stern-Gerlach polarization atomic device, we have demonstrated the manifestation of a topological phase for a beam of  $H^*(2s)$  atoms traveling through a conical magnetic field configuration.

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The experimental and theoretical study of quantal phase factors has always been a very fascinating subject, attracting a lot of effort and displaying unexpected features, as the recent and now famous paper by Berry [1] showed to the whole community of physicists (there is considerable literature on the subject now but for a good review see [2]). Recently, these efforts have culminated, in the field of atomic physics, with the realization of six different experiments [3–8]. A short but comprehensive review of these works can be found in [9]. This allows studies which were until now only devoted to electron or neutron interferometers. Because of their great sensitivity (at thermal velocities, the atomic wavelength is  $10^4$  shorter than the optical wavelength in the visible range), the application of these atomic devices, ranging from fundamental tests of physics to more applied ones such as surface imaging, is very attractive. In this Letter, our purpose is to report a simple application of these atomic phase studies, namely, the first direct demonstration of an atomic topological phase. This has been achieved by using a longitudinal Stern-Gerlach polarization atomic device [8,10,11] and a beam of  $H^*(2s)$  atoms traveling through a conical magnetic field configuration. In its basic principle, the experiment is close to the one already performed with neutrons [12] as explained below.

The operation principle of the present longitudinal Stern-Gerlach device (Fig. 1) has been given in [11]. It operates with a thermal beam of  $H^*(2s)$  atoms produced by a 100-eV-electronic bombardment of a thermal beam of  $H_2$  molecules. The resulting time of flight distribution is well fitted by  $f(x) = x^{-5} \exp[-2.5(x^{-2} - 1)]$  where  $x = t/t_0$ ,  $t_0$  being the most probable time of flight (the corresponding velocity is  $v_0 = 10$  km/s) [13]. First an incomplete polarization of the beam in the hyperfine states  $2s_{1/2}$ ,  $F=1$ ,  $M=0,1$  is achieved in  $P$  by passing the atoms through a transverse 600-G polarizing magnetic field  $B_P$  (Lamb and Retherford's method [14]). Then a mixer  $M$  produces a coherent superposition of Zeeman states.  $M$  consists of a pair of half-turn coils, the transverse magnetic field  $B_M(Z)$  of which ( $Z$  is the axis of propagation of the atoms) reverses abruptly along the flight path of the atoms. Hence, the evolution in  $M$  is not adiabatic and transitions among the various Zeeman states are induced, giving the desired result. In region  $\mathcal{R}$ , carefully shielded from outer magnetic fields, the induced coherent

superposition undergoes an evolution through a magnetic phase object. At the output of  $\mathcal{R}$ , a second mixer  $M'$  identical to  $M$  builds another coherent linear superposition of states, the amplitudes of which are linear combinations of the various phase factors. Then an analyzer  $A$  identical to  $P$  (magnetic field  $B_A$ ) filters the outgoing state. Finally, the population of the filtered state is measured by a detector  $D$  specific to  $H^*(2s)$  [13] and the signal contains interference terms characteristic of the chosen phase object.

The external motion of polarized atoms when  $\mathcal{R}$  consists of a magnetic-field profile of constant direction (which automatically ensures adiabaticity), the gradients of which are longitudinal, has been investigated in [11]. The longitudinal forces acting on the atoms in  $\mathcal{R}$  make each individual atomic wave packet split longitudinally. As the net momentum transfer is zero for each Zeeman state, this longitudinal spatial splitting remains permanent beyond  $\mathcal{R}$ . It is proportional to the integral over  $Z$  of the magnetic profile and gives rise to the atomic interferences pattern. Note also that it forbids a classical description of the external motion since, for example, there are as many velocities as there are Zeeman states. In other words, one has to recall that when, as usual, one writes  $Z = vt$ ,  $v$  is in fact an operator [15].

In the present study, the phase object in  $\mathcal{R}$  is simply a conical magnetic-field configuration. It is realized by the

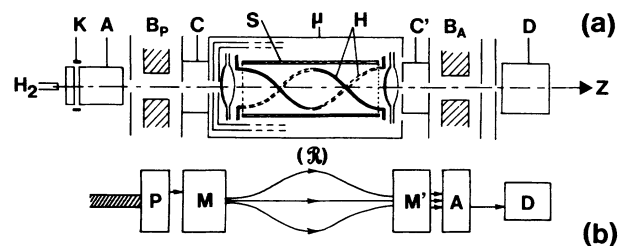


FIG. 1. (a) Scheme of the experiment.  $K, A$ : electron gun;  $B_{P,A}$ : magnetic fields of the polarizer and the analyzer;  $C, C'$ : quasizero field chambers;  $H$ : double helix;  $S$ : solenoid;  $\mu$ : mu-metal shielding;  $D$ : detector. (b) Principle of the experiment: An unpolarized beam (hatched area) is polarized in  $P$ ;  $M$ : mixer, building a superposition of Zeeman states (one assumes that spin  $F=1$ );  $\mathcal{R}$ : phase object;  $M'$ : second mixer (analog to  $M$ );  $A$ : analyzer (analog to  $P$ );  $D$ : detector.

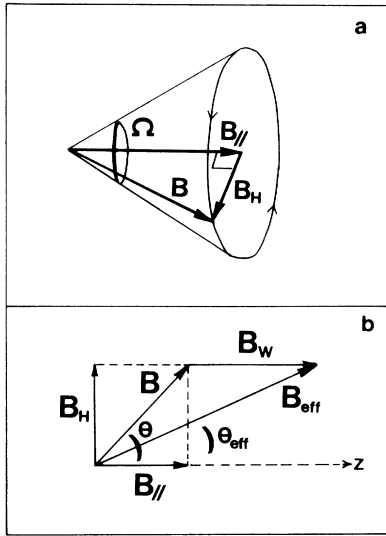


FIG. 2. (a) Cone followed by the total magnetic field  $\mathbf{B}$ , sum of  $\mathbf{B}_H$  (helix) and  $\mathbf{B}_||$  (solenoid), in region  $\mathcal{R}$ .  $\Omega$  is the solid angle of the cone. (b) The effective field  $\mathbf{B}_{\text{eff}}$  (see text).

superposition of two magnetic fields: that of a solenoid of axis  $Z$  (radius 6 mm, length 50 mm)  $\mathbf{B}_|| (Z)$  and that of a double right-handed helix of axis  $Z$  (radius 6 mm, length 60 mm)  $\mathbf{B}_H (Z)$ . Hence the  $H^*(2s)$  atoms on their flight along  $Z$  see a conical magnetic field  $\mathbf{B}(Z) = \mathbf{B}_|| (Z) + \mathbf{B}_H (Z)$  rotating by  $2\pi$  over  $L = 60$  mm [Fig. 2(a)]. The production of the  $H^*(2s)$  beam being continuous, all atomic velocities participate in the signal. The resulting population in the detector  $D$  versus the current  $i_H$  in the helix has been measured for various fixed values of the current  $i_||$  in the solenoid. The polarizing and analyzing fields have opposite directions but the same magnitude ( $\geq 600$  G), and the current in the mixers is 150 mA. Figure 3 shows our results. Apart from a permanent central peak essentially due to the poor mixing obtained with only longitudinal fields [8] (when  $i_H = 0$  only  $\mathbf{B}_||$  is present), it is seen that the central bright fringe, originally located at  $i_H = 0$  when  $i_|| = 0$ , appears more and more symmetrically split as  $i_||$  takes on more and more *negative* values, whereas it disappears for *positive* values. Correlatively, the two first symmetric minima of the pattern undergo a similar evolution insofar as their mutual distance continuously decreases as  $i_||$  increases from  $-6$  mA up to 6 mA. The present behavior is characteristic of the *conical* configuration of the field. This is obvious from the data represented by triangles in Fig. 3 which are obtained with  $i_|| = -4$  mA by using a vertical frame instead of the helix. As one can see, no splitting of the central peak occurs.

The above results can be interpreted within the frame-

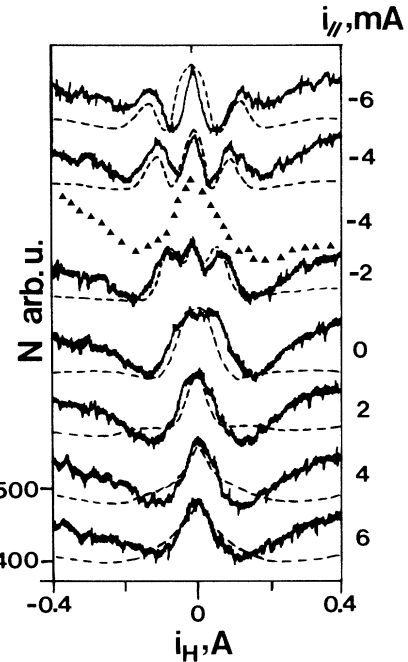


FIG. 3.  $H^*(2s)$  counting rate as a function of  $i_H$  scanned from  $-0.4$  A up to  $+0.4$  A, for fixed values of  $i_||$  ranging from  $-6$  mA up to  $+6$  mA. No velocity selection is made. For each spectrum the acquisition time is 1600 s. (When  $i_H = i_||$ , the magnetic field at the center of the solenoid is about 15 times larger than that of the helix.) Triangles correspond to a spectrum obtained with a vertical frame in place of the helix, with  $i_|| = -4$  mA. Broken curves: calculated interference patterns.

work of the vector model of a spin  $F$  ( $F = 1$ ) for two basic reasons: (i) The magnitude of the applied magnetic fields in the mixing zones and in  $\mathcal{R}$  is weak ( $\lesssim 0.5$  G); (ii) the  $2s_{1/2}, F = 0$  level, initially quenched by the polarizer, will never be repopulated because of its sufficiently large energetic separation from the  $2s_{1/2}, F = 1$  level. As the angular frequency of  $\mathbf{B}_H$  seen by the atoms and the Larmor frequency of the atoms in  $\mathbf{B}$  have the same order of magnitude ( $\nu_H \sim \nu_L \sim 0.15$  MHz at  $v \sim 10$  km/s and  $B \sim 0.1$  G), the adiabatic approximation is *not* valid. We then have to determine the external motion of an atom in this time-independent conical magnetic field. We shall use as an internal basis set, the Zeeman states  $|M\rangle$  referred to a  $Z$ -dependent axis  $\hat{e}(Z)$  making a constant angle with the  $Z$  axis, the azimuthal angle being that of  $\mathbf{B}(Z)$ , namely,  $\Psi(Z) = WZ$  where  $W$  is the spatial frequency of the helix. Expanding the complete atomic state over this basis set and leaving aside the unaffected  $X$  and  $Y$  motions, one gets, from the stationary Schrödinger equation, a set of coupled equations for the external amplitudes  $\mathcal{F}_M$  associated with  $|M\rangle$ :

$$E\mathcal{F}_M = -\frac{\hbar^2}{2\mathcal{M}}\partial_Z^2\mathcal{F}_M + \sum_{M'} \left[ g\mu_B \langle M | \mathbf{F} \cdot \mathbf{B} | M' \rangle - \frac{\hbar^2}{2\mathcal{M}} [2\langle M | \partial_Z | M' \rangle \partial_Z + \langle M | \partial_Z^2 | M' \rangle] \right] \mathcal{F}_{M'}, \quad (1)$$

where  $E = \hbar^2 K^2 / 2M$  is the initial kinetic energy,  $K$  the initial wave number,  $g$  the Landé factor,  $M$  the mass of the atom, and  $\mu_B$  the Bohr magneton. The two latter terms in the brackets of Eq. (1) are dynamical couplings coming from the  $Z$  dependence of the basis set, whereas the first term is a static coupling due to the fact that  $\hat{\mathbf{e}}$  and  $\mathbf{B}$  are not collinear. It is readily seen that  $\partial_Z = -iWF_Z$  and that the matrix elements of  $\partial_Z^2$  are completely negligible when compared to those of  $\partial_Z$  since  $W \sim 1/L$  and  $|\partial_Z \mathcal{F}_M| \sim K$ . As all terms in the sum are small compared to the free kinetic energy, one is allowed to replace  $\partial_Z \mathcal{F}_M$  by  $iK \mathcal{F}_M$ , which gives for the sum in (1)

$$\sum_M g \mu_B \langle M | \mathbf{F} \cdot \mathbf{B}_{\text{eff}} | M' \rangle \mathcal{F}_{M'} \quad (2)$$

The effective magnetic field  $\mathbf{B}_{\text{eff}} = \mathbf{B} + B_W \hat{\mathbf{u}}_Z$ , with  $B_W = 2EW/gK\mu_B$  and  $\hat{\mathbf{u}}_Z$  the unit vector of the  $Z$  axis, is exactly the one introduced by Schwinger in [16] for the case of an atom at rest in a time-dependent field. If  $\hat{\mathbf{e}}$  is taken along  $\mathbf{B}_{\text{eff}}$ , all coupling terms cancel. The  $|M\rangle$  states are then identified with the cyclic states defined by Aharonov and Anandan in [17] and the accumulated phase shifts, calculated for a rectangular  $\mathbf{B}$  profile, read

$$\Phi_M = M\Phi = 2\pi M(B_{\text{eff}}/B_W - 1); \quad (3)$$

$\Phi$  is an even function of  $i_H$  but it has no definite parity property with respect to  $i_{\parallel}$ . This behavior may be physically understood from the fact that the sign of  $B_W$  is imposed by the sense of the helix with respect to the external motion independently of that of  $B_{\parallel}$ . It is also important to note that  $B_W$  and then also  $\mathbf{B}_{\text{eff}}$  are velocity dependent. In Fig. 4, the interference order  $p = \Phi/2\pi$  is plotted as a function of  $y = B_H/B_W$  for some fixed values of  $k = B_{\parallel}/B_W$ . A single minimum  $p_{\min}$  is obtained at  $B_H = 0$ . For  $k \in [-2, 0]$ ,  $p_{\min} \leq 0$  whereas for  $k \notin [-2, 0]$ ,  $p_{\min} > 0$ . As a consequence, the zero interference order  $p = 0$  is never realized for  $B_{\parallel} > 0$  and the interference pattern "starts" at some positive value of  $p$ . For  $B_{\parallel} = 0$ ,  $p = 0$  is obtained at  $i_H = 0$  and the central corresponding bright fringe looks broadened compared to [10] because here  $p_{\min} = 0$  is a stationary value of  $p$ . At last, for  $k \in [-2, 0]$ ,  $p = 0$  is obtained for two opposite values of  $i_H$ . Hence the central bright fringe, together with the whole interference pattern, is symmetrically split into two bright fringes. This is indeed the behavior observed from the data in Fig. 3 even if the situation is complicated by (i) the velocity spread of the beam, and (ii) the mixing mechanisms. This latter point has been treated in [10]: To summarize, each mixer induces a rotation of the spin which can be described by a Wigner matrix  $D^{(1)}(a, b, c)$  where the Eulerian angles  $a, b, c$  are specific to the mixers. For two identical mixers, it can be shown that  $A = D^{\dagger} \mathcal{U} D a$ , where  $\mathcal{U}$  is the external evolution diagonal matrix of phase factors  $\exp(iM\Phi)$ ,  $A(a)$  is the final (initial) column vector of external amplitudes on  $|F=1, M\rangle$ . The detected signal  $\mathcal{S}$  is now identified with the sum of

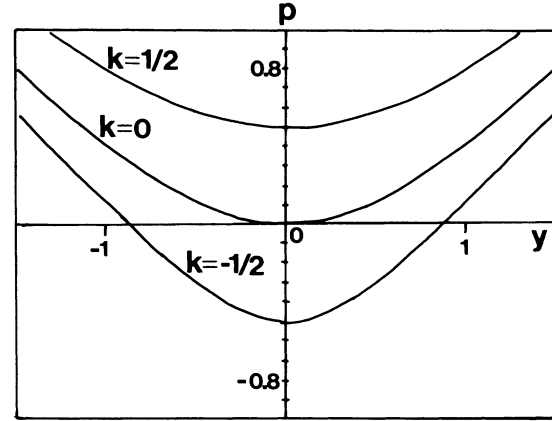


FIG. 4. Interference order  $p$  as a function of  $y = B_H/B_W$  for fixed values of  $k = B_{\parallel}/B_W$ . For an atomic velocity of 10 km/s,  $B_W \approx 100$  mG.

the populations of the analyzed states  $M = 1, 0$  for two initial conditions, namely,  $a = (1, 0, 0)$  and  $a = (0, 1, 0)$ , because of the incomplete initial polarization of the beam. Taking into account now the velocity spread of the beam, and after some tedious calculations, one gets

$$\mathcal{S} = \int_0^{\infty} dx f(x) \frac{4 + [\lambda \cos 2\Phi + 2 - \lambda]^2}{8}, \quad (4)$$

where  $\lambda = \sin^2 b$  is the only mixing parameter. The treatment of the mixers, as given in [18], led us to the following form for  $\lambda$ :

$$\lambda = 4 \exp(-\gamma x) [1 - \exp(-\gamma x)], \quad (5)$$

where  $\gamma$  is some function of the magnetic field in the mixing region, i.e., a function of  $i_H$ ,  $i_{\parallel}$ , etc. In our calculations we have simply put  $\gamma = 2i_H$ . Figure 3 shows the results. As one can see the global behavior is well predicted and the quantitative agreement, if not perfect, is rather good. This could be surely improved if one knew more precisely the mixing mechanism (field lines in the mixers, etc.) but this problem, if important, is not the central point of our paper.

Since the work of Aharonov and Anandan [17] and that of Samuel and Bhandari [19], it is known that the quantum evolution displays topological effects even if it is neither adiabatic nor cyclic. This latter point has been nicely demonstrated with neutrons [20]. For our part, we are only concerned with [17] since the evolution of the atoms in the conical field can be considered as a nonadiabatic cyclic one. As clearly outlined in [17], the phase  $\Phi$  contains a geometrical part  $\beta = \Phi - \alpha$  where  $\alpha = 2\pi\mu_B BT/h \cos(\theta - \theta_{\text{eff}})$  is the dynamical part ( $T$  is the transit time in the conical field). The geometrical character of  $\beta$  essentially deals with the Hilbert space of states in the sense that it only depends on the closed curve  $\hat{C}$  followed by the state in the projection map [17]. In the

present case,  $\beta$  takes the remarkable form  $\beta = -2\pi(1 - \cos\theta_{\text{eff}})$  which is just the opposite of the solid angle  $\Omega_{\text{eff}}$  sustained by  $\mathbf{B}_{\text{eff}}$ . This is reminiscent of the formula  $(-m\Omega)$  with  $m=1$  given in [1] (which is recovered from  $\beta$  in the adiabatic limit  $v \rightarrow 0$ ) since in the spatially rotating frame defined by  $\hat{\mathbf{e}}$  the evolution is adiabatic (all coupling terms cancel). The topological effect is demonstrated here via the symmetrical splitting of the patterns (note again that this splitting does not occur when a rectangular frame is used in place of the helix) and it is only in the adiabatic limit that  $\beta$  is connected to a geometrical quantity in parameter space (here the solid angle  $\Omega$  is sustained by the tip of  $\mathbf{B}$ ). Another point that could be checked with atoms is the one demonstrated in [20], namely, a topological effect occurring in a noncyclic evolution.

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