Muonium to Antimuonium Conversion and the Decay $\mu^+ \to e^+ \bar{\nu}_e \nu_\mu$ in Left-Right Symmetric Models

Peter Herczeg
Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

Rabindra N. Mohapatra
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
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We show that in the minimal left-right symmetric model with triplet Higgs bosons the range of the muon neutrino mass for which ν_μ is required by cosmological considerations to be unstable can be probed by muonium to antimuonium conversion ($M \to \overline{M}$) and/or by the exotic muon decay $\mu^+ \to e^+ \bar{\nu}_e \nu_\mu$. We point out that if all the leptonic mixing matrices are hierarchical and the Dirac masses of the neutrinos are equal to the masses of the corresponding charged leptons, there is a lower bound in this range for the rates of both of these processes. We find $|G_{M\overline{M}}| \gtrsim 7\times 10^{-5} G_F$ and $|G_\mu^{(e)}| \gtrsim 2\times 10^{-4} G_F$ for the strength of the $M \to \overline{M}$ and $\mu^+ \to e^+ \bar{\nu}_e \nu_\mu$ interactions.

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Left-right symmetric models [1] are attractive extensions of the standard electroweak model, which provide a framework for the understanding of parity violation in the weak interactions. The simplest realization of these models is based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ with a discrete left-right symmetry [1,2]. $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ models employing triplet Higgs bosons to induce part of the symmetry breaking [2] also provide a framework for the understanding of the smallness of the masses of the usual neutrinos.

In this Letter we consider the muon neutrino mass in the model of Ref. [2], and show that the range of m_{ν_μ} for which the constraint from the energy density of the present Universe requires ν_μ to be unstable can be probed through searches for muonium to antimuonium conversion ($M \to \overline{M}$) and/or the exotic muon decay $\mu^+ \to e^+ \bar{\nu}_e \nu_\mu$. We point out that if all the leptonic mixing matrices are hierarchical (i.e., if their nondiagonal matrix elements are small relative to the diagonal ones) and the Dirac masses of the neutrinos are equal to the masses of the corresponding charged leptons, there is a lower bound in this range on the rate of both of these processes. The lower bounds correspond to $m_{\nu_\mu} = 270$ keV (the present experimental limit for m_{ν_μ}) and they increase with decreasing m_{ν_μ} [3].

The Higgs sector of the model of Ref. [2] consists of the bidoublet field ϕ (2,2,0) and the triplet fields Δ_L (3,1,2) and Δ_R (1,3,2),

$$\phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix}, \quad \Delta_{L,R} = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}_{L,R}. \qquad (1)$$

The vacuum expectation values of the neutral fields are denoted as $\langle \phi_1^0 \rangle = \kappa$, $\langle \phi_2^0 \rangle = \kappa'$, $\langle \Delta_{L,R}^0 \rangle = v_{L,R}$. $\langle \Delta_R \rangle_0 \neq 0$ breaks $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ to $SU(2)_L \times U(1)_Y$ (with $Y = 2I_{3R} + B - L$), and $\langle \phi \rangle_0 \neq 0$ completes the symmetry breaking to $U(1)_{\text{em}}$. The existence of Δ_L is required by the discrete left-right symmetry.

The Higgs potential allows, for a wide range of parameters, the phenomenologically acceptable hierarchical pattern $v_R \gg \kappa, \kappa' \gg v_L$, with $v_L \simeq \gamma \kappa^2 / v_R$ [2], where γ is a ratio of Higgs potential parameters. Note that one has to have $\kappa' < \kappa$ (or $\kappa < \kappa'$), since otherwise the mass matrices for $Q = \frac{2}{3}$ and $Q = -\frac{1}{3}$ quarks would be equal. Since even an appreciable κ' would have no effect on our conclusions, we shall set for simplicity $\kappa' = 0$ in the following.

The neutrino mass matrix of the model yields an expression for the mass matrix of the light neutrinos in which the masses of the light neutrinos are inversely proportional to v_R [2]. We shall work in the basis where the $\Delta_{L,R}$-lepton couplings are diagonal. Assuming, as we shall do, that in this basis all the leptonic mixing matrices are hierarchical [4], and that the Dirac masses of the neutrinos are equal to the masses of the corresponding charged leptons, this expression takes the form

$$m_{\nu_l} = 2 f_{ll} \gamma \frac{\kappa^2}{v_R} - \frac{m_l^2}{2 f_{ll} v_R} \quad (l = e, \mu, \tau), \qquad (2)$$

where f_{ll} is the coupling constant of the interaction of Δ_L with the l family [see Eq. (4)].

The masses and the lifetimes of the neutrinos are constrained by the requirement that the energy density of the neutrinos in the present Universe does not exceed the upper limit on the present total energy density of the Universe [5]. This implies that neutrinos of masses between 35 eV [6] and ~ 3 GeV [7] have to be unstable. Our interest here is in the muon neutrino with a mass in the range excluded for stable neutrinos. Note that Eq. (2) allows m_{ν_μ} to be in that range. The lifetime τ_{ν_μ} of such a neutrino has to satisfy the bound [8]

$$\tau_{\nu_\mu} \lesssim (5.4 \times 10^{10}\,\text{sec})[(100\,\text{keV})/m_{\nu_\mu}]^2. \qquad (3)$$

The ν_μ's in the model can decay either radiatively ($\nu_\mu \to \nu_e \gamma$, $\nu_\mu \to \nu_e \gamma\gamma, \ldots$) or into three neutrinos ($\nu_\mu \to \nu_e \nu_e \bar{\nu}_e$) via Z_1 exchange [9] and Δ_L exchange [10]. The radiative lifetimes do not satisfy (3) for any m_{ν_μ} [11]; the same is true also for the Z_1-exchange contribution to $\nu_\mu \to \nu_e \nu_e \bar{\nu}_e$ [9]. The only decay mode which has a chance to have a sufficiently short lifetime is the Δ_L^0-

mediated $\nu_\mu \to \nu_e \nu_e \bar{\nu}_e$ decay [12].

The coupling of the Δ_L to the leptons in the mass-eigenstate basis is given by

$$\mathcal{L} = (\bar{\nu}_L^c F' \nu_L)\Delta_L^0 - (\bar{E}_L^c F \nu_L + \bar{\nu}_L^c F^T E_L)\Delta_L^+/\sqrt{2} - (\bar{E}_L^c f' E_L)\Delta_L^{++} + \text{H.c.}, \quad (4)$$

where $\nu^T = (\nu_e, \nu_\mu, \nu_\tau)$, $E^T = (e, \mu, \tau)$, $\psi_L = \frac{1}{2}(1-\gamma_5)\psi$ ($\psi = \nu, E$), and $f' = U^T f U$, with f the diagonal matrix of the Δ_L-lepton couplings, and U defined by $E' = UE$; the primed fields denote the gauge group eigenstates. The matrices F and F' in Eq. (4) are $F = f'K$, $F' = K^T f' K^T$, where $K = U^+ V$ and V is the matrix which diagonalizes the light neutrino mass matrix, so that $\nu' = V\nu$. Note that the matrix K is observable.

The Hamiltonian for $\nu_\mu \to \nu_e \nu_e \bar{\nu}_e$ resulting from (4) has the form

$$H = (G_0/\sqrt{2})\bar{\nu}_e \gamma^\lambda (1-\gamma_5)\nu_e \bar{\nu}_e \gamma_\lambda (1-\gamma_5)\nu_\mu + \text{H.c.}, \quad (5)$$

where $G_0 = \sqrt{2} F_{ee}'^* F_{e\mu}'/4m_0^2$, and m_0 is the mass of the Δ_L^0. Noting that $\Gamma_{\nu_\mu} = 2G_0^2 m_{\nu_\mu}^5/192\pi^3$, the bound (3) requires

$$|G_0| \gtrsim (1.9 \times 10^{-12}\,\text{GeV}^{-1/2}) m_{\nu_\mu}^{-3/2}. \quad (6)$$

Keeping only terms not higher than first order in nondiagonal mixing matrix elements, we have $F_{ee}'^* F_{e\mu}' \simeq f_{ee}^*(f_{ee}K_{e\mu} + f_{\mu\mu}K_{\mu e})$. Here we have omitted the term $f_{ee}'^* f_{e\mu}'$ containing first-order contributions, since it is too small to play a role in G_0 due to the experimental limit on the $\mu \to 3e$ branching ratio [13].

Equation (6) implies the upper bound

$$m_0 \lesssim (9.4 \times 10^4\,\text{GeV}^{1/4}) m_{\nu_\mu}^{3/4} \quad (7)$$

for m_0 ($m_0 \lesssim 200$ GeV for $m_{\nu_\mu} = 270$ keV). To obtain (7) we used $|f_{ee}| \lesssim 1.2$, $|f_{\mu\mu}| < 0.16$ [14], and $|K_{e\mu}|, |K_{\mu e}| \lesssim 2.9 \times 10^{-2}$ [15].

The mass of the Δ_L^0 also has a lower bound. This follows from the experimental value of the invisible width Γ_{inv}^Z of the Z_1. If $m_0 < \frac{1}{2} m_{Z_1}$, the Z_1 decays into $\Delta_L^0 \bar{\Delta}_L^0$ with a rate $\Gamma_\Delta = 2\Gamma_\nu (1 - 4m_0^2/m_{Z_1}^2)^{3/2}$, where Γ_ν is the $Z_1 \to \nu\bar{\nu}$ decay width, and we have neglected the small effect of Z_1-Z_2 mixing. Γ_Δ has to be included in Γ_{inv}^Z [16], so that $\Gamma_{\text{inv}}^Z/\Gamma_\nu = N_\nu + 2(1 - 4m_0^2/m_{Z_1}^2)^{3/2}$, where N_ν is the number of light neutrino generations. Using the experimental value $\Gamma_{\text{inv}}^Z/\Gamma_\nu \simeq 2.99 \pm 0.05$ [17] and $N_\nu = 3$, we find $m_0 > 42.9$ GeV (90% C.L.). Combining this with the upper bound (7), we obtain $m_{\nu_\mu} \gtrsim 35$ keV.

We can conclude therefore that in addition to muon neutrinos of masses $m_{\nu_\mu} \le 35$ eV, the model of Ref. [2] is viable also for m_{ν_μ} in the range

$$35\,\text{keV} \lesssim m_{\nu_\mu} < 270\,\text{keV}. \quad (8)$$

Muonium to antimuonium conversion and $\mu^+ \to e^+ \bar{\nu}_e \nu_\mu$ are processes forbidden in the standard model since they violate the conservation of lepton family numbers.

$M \to \bar{M}$ [18] can occur in the model of Ref. [2] at the tree level via Δ_L^{++} and Δ_R^{++} exchange [19]. The Δ_L^{++} contribution is described by the Hamiltonian [cf. Eq. (4)]

$$H = (G_{M\bar{M}}/\sqrt{2})\bar{\mu}\gamma_\lambda (1-\gamma_5) e \bar{\mu}\gamma^\lambda (1-\gamma_5) e + \text{H.c.}, \quad (9)$$

where

$$G_{M\bar{M}} = G_{++} \equiv \sqrt{2} f_{ee}' f_{\mu\mu}'^*/8m_{++}^2 \simeq \sqrt{2} f_{ee} f_{\mu\mu}^*/8m_{++}^2.$$

We note that for m_{ν_μ} in the range (8) the contribution of (9) cannot be arbitrarily small. This can be seen as follows. G_{++} is related to G_0 as

$$G_{++}^* = (G_0/2) f_{\mu\mu} (f_{ee}K_{e\mu} + f_{\mu\mu}K_{\mu e})^{-1} m_0^2/m_{++}^2. \quad (10)$$

For G_0 in Eq. (10) we have the lower bound (6) from cosmology. Also, the ratio m_0^2/m_{++}^2 for a given m_0 [satisfying Eq. (7)] has a lower bound from the experimental value of the neutral current parameter ρ_1 (defined in Ref. [20]). In the model we are considering $\rho_1 \simeq 1 + \rho_\theta + \rho_\Delta$, where ρ_θ is a correction of the order of κ^2/v_R^2 due to Z_1-Z_2 mixing, and ρ_Δ (which involves m_0^2/m_{++}^2) comes from the Δ_L-loop contribution to the Z_1 and W_1 mass [21]. Lastly, for a given m_{ν_μ} the coupling constant $f_{\mu\mu}$ is also bounded from below. Since the γ term in Eq. (2) can be neglected (see Ref. [14]), we have $f_{\mu\mu} \simeq m_\mu^2/2m_{\nu_\mu} v_R$. Neglecting mixing among the neutral Higgs fields, the mass of the Δ_L^0 is $m_0^2 = R v_R^2$, where R is a combination of Higgs potential parameters [22]. The parameter R cannot be smaller than 10^{-3}-10^{-4}, which is the size of an R induced by radiative corrections. Thus $|v_R| \lesssim (30\text{-}100)(m_0)_{\max}$, where $(m_0)_{\max}$ is the largest m_0 allowed by (7).

The contribution of Δ_R^{++} also has a lower bound, but it can be smaller than that for the Δ_L^{++} contribution, since the mass of the Δ_R^{++} is constrained only by the limit on v_R from (7).

The decay $\mu^+ \to e^+ \bar{\nu}_e \nu_\mu$ [23] occurs in the model via Δ_L^+ exchange [24]. The corresponding Hamiltonian is

$$H = (G_\mu^{(e)}/\sqrt{2})\bar{\mu}\gamma_\lambda (1-\gamma_5) e \bar{\nu}_\mu \gamma^\lambda (1-\gamma_5)\nu_e + \text{H.c.}, \quad (11)$$

where

$$G_\mu^{(e)} = 2G_+ \equiv \sqrt{2} F_{ee} F_{\mu\mu}^*/4m_+^2 \simeq \sqrt{2} f_{ee} f_{\mu\mu}^*/4m_+^2.$$

For the same reason as G_{++}, the constant G_+ has a lower bound for m_{ν_μ} in the range (8) [note that $m_+^2 = \frac{1}{2}(m_0^2 + m_{++}^2)$].

We find for m_{ν_μ} in the range (8) $|G_{M\bar{M}}| \gtrsim 7 \times 10^{-5} G_F$ and $|G_\mu^{(e)}| \gtrsim 2 \times 10^{-4} G_F$. For a given m_{ν_μ} the bounds increase with decreasing v_R. The lower bounds on $|G_{M\bar{M}}|$ and $|G_\mu^{(e)}|$ in the ranges 35 keV $\le m_{\nu_\mu} \le A$ keV for several values of A are shown in Table I. As seen, the lower bounds are the smallest for $m_{\nu_\mu} = 270$ keV, and

TABLE I. The lower bounds on $|G_{M\bar{M}}|$ and $|G_\mu^{(e)}|$ in the range 35 keV $\le m_{\nu_\mu} \le A$ keV for some values of A.

A	35	100	150	200	270
$\lvert G_{M\bar{M}}\rvert_{\min}/G_F$	4×10^{-3}	6×10^{-4}	2×10^{-4}	1×10^{-4}	7×10^{-5}
$\lvert G_\mu^{(e)}\rvert_{\min}/G_F$	2×10^{-2}	2×10^{-3}	9×10^{-4}	5×10^{-4}	2×10^{-4}

they increase with decreasing m_{ν_μ}. Thus, as the experimental limits on $|G_{M\bar{M}}|$ and/or $|G_\mu^{(e)}|$ become more and more stringent, the allowed range of m_{ν_μ} becomes increasingly smaller. To obtain these results we used $|K_{e\mu}|=2.9\times10^{-2}$, and the lower bound in (6) for G_0. For $|f_{ee}|$, m_0, and v_R we took $|f_{ee}|=|f_{ee}|_{\max}\simeq1.2$, $m_0=(m_0)_{\max}$, and $v_R=100(m_0)_{\max}$. Inspection shows that this choice of the parameters yields the smallest lower bound. For ρ_1 we used the value of the parameter ρ_0 ($\rho_0=0.0996^{+0.009}_{-0.014}$) obtained in a three-parameter $(\sin^2\theta_W,\rho_0,m_t)$ fit to electroweak data in the standard model with an arbitrary Higgs sector [17,25]. This gives $(16\pi^2m_W^2/g^2)\rho_\Delta<4.7\times10^4$ GeV^2 (90% C.L.). The identification of the experimental value of ρ_1 with that for ρ_0 neglects the small (of the order of κ^2/v_R^2) contributions of Z_1-Z_2 mixing and of the Z_2 to the observables (other than m_{Z_1}) used in the fit.

The present experimental limits are $|G_{M\bar{M}}|_{\rm expt}<0.16G_F$ (90% C.L.) [26] and $|G_\mu^{(e)}|<0.14G_F$ (90% C.L.) [27]. In the model we are considering $|G_{M\bar{M}}|$ and $|G_\mu^{(e)}|$ can be as large as these limits [28]. An experiment under way at PSI [29] is expected to lower the upper limit on $|G_{M\bar{M}}|$ to $10^{-3}G_F$, and an experiment in preparation at LAMPF [30] plans to search for $\mu^+\to e^+\bar{\nu}_e\nu_\mu$ with a sensitivity corresponding to $|G_\mu^{(e)}|\simeq10^{-2}G_F$.

The lower bounds on $|G_{M\bar{M}}|$ and $|G_\mu^{(e)}|$ could also be improved by setting more stringent experimental upper limits on m_{ν_μ}, $K_{e\mu}$, f_{ee}, and $f_{\mu\mu}$. The LAMPF experiment will improve simultaneously the limit on $K_{e\mu}$ by a factor of ~3.5. This will increase the lower bound on $|G_{M\bar{M}}|$ and $|G_\mu^{(e)}|$ by a factor of ~2. An upper limit on $|f_{ee}|$ below 0.2 for m_0 in the range defined by Eqs. (7) and (8) would exclude the range (8), since the bound (6) could not be satisfied for any m_{ν_μ} in (8). The present limit is $|f_{ee}|\lesssim1.6$ from data on Bhabha scattering [31].

Our lower bounds would not hold under some scenarios. First, we derived the bounds assuming that all the leptonic mixing matrices are hierarchical, and have taken $f'_{\mu\mu}\simeq f_{\mu\mu}$. The full expression for $f'_{\mu\mu}$ is given by $f'_{\mu\mu}=U^2_{\mu\mu}f_{\mu\mu}+U^2_{e\mu}f_{ee}+U^2_{\tau\mu}f_{\tau\tau}$. If the nondiagonal elements of U are large, the corresponding terms in $f'_{\mu\mu}$ could be important, and moreover m_{ν_μ} would no longer be given by the simple formula (2). If the nondiagonal elements are small, $f'_{\mu\mu}$ could still be smaller than $f_{\mu\mu}$ in the unlikely event that there is a cancellation between $f_{\mu\mu}$ and the $U^2_{\tau\mu}f_{\tau\tau}$ term [32]. To obtain the lower bounds we have also assumed that the Dirac masses $m^D_{\nu_l}$ of the neutrinos are equal to the masses of the corresponding charged leptons. If $m^D_{\nu_\mu}<m_\mu$, our lower bounds would be reduced by the (unknown) factor $(h_{\mu\mu}/\tilde{h}_{\mu\mu})^2$, where $h_{\mu\mu}$ and $\tilde{h}_{\mu\mu}$ are the couplings of the muon family to ϕ and $\tilde{\phi}$, respectively. Further, the relation $m_0^2=Rv_R^2$ is modified when Δ_L^2-ϕ_2^0 mixing is included. The effect of mixing could be large if the mixing term is large compared to $\tilde{m}_\phi^2-\tilde{m}_0^2$, where $\tilde{m}_\phi$ and $\tilde{m}_0$ are the masses of the ϕ^0 and Δ_L^0 before mixing. Assuming for simplicity $\tilde{m}_0=\tilde{m}_\phi$, one would have $m_0^2=Rv_R^2-\beta'\kappa v_R$, where β' is a Higgs potential parameter. This relation can allow for a given m_0 considerably larger upper limits on v_R than we had before, and therefore correspondingly smaller lower bounds on $f'_{\mu\mu}$ [33]. We do not regard this, however, as a likely scenario. We note yet that if both κ and κ' are non-negligible, the mixing term would be small, since then all the associated Higgs potential parameters would contribute to the parameter γ in Eq. (2) [34]. Finally, in the unlikely event that due to accidental cancellations the value of the parameter R is below 10^{-4}, i.e., if $R=10^{-4}k$ $(k<1)$, our lower bounds would have to be multiplied by $\sim\sqrt{k}$.

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come smaller only by a factor of ~ 2. The cosmological bound holds in the form (3) provided that at the time of the decay the neutrinos are nonrelativistic and the Universe is matter dominated; for $m_{\nu_\mu} \simeq 1$ keV this is still satisfied for $\tau_{\nu_\mu} \gtrsim 10^6$ sec [E. W. Kolb (private communication)], while the lifetime of $\nu_\mu \to \nu_e \nu_e \bar{\nu}_e$ via Δ_L^0 exchange (see the text further on) is longer than the age of the Universe for $m_{\nu_\mu} \lesssim 2$ keV. In the extreme approximation, when the Universe is taken to be radiation dominated at all times, the only change in the bound (3) is that the factor 5.4 gets replaced by ~ 2 [see P. B. Pal, Nucl. Phys **B227**, 337 (1983)].

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[28] Concerning indirect empirical limits on $G_{M\overline{M}}$ see Gunion *et al.* (Ref. [14]), Chang and Keung (Ref. [14]), and Schwartz (Ref. [14]). For $G_\mu^{(e)}$ we have evaluated constraints from data on μ decay and inverse μ decay [S. R. Mishra *et al.*, Phys. Lett. B **252**, 170 (1990); see also P. Langacker and D. London, Phys. Rev. D **39**, 266 (1989)] from the W mass, and from charged current universality [see D. H. Wilkinson, TRIUMF Report No. TRI-PP-91-9 (unpublished)]. We find at 90% C.L., $|G_\mu^{(e)}| < 0.28 G_F$, $|G_\mu^{(e)}| < 0.24 G_F$, and $|G_\mu^{(e)}| < 0.052 G_F$, respectively. The last bound may be affected by theoretical uncertainties.

[29] K. Jungman and W. Bertl *et al.*, PSI Experiment No. R-89-06.1.

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[31] See Schwartz (Ref. [14]); the constraint from ρ_1 gives $m_{++} < 568$ GeV.

[32] Note that the term $U^2_{e\mu} f_{ee}$ in $f'_{\mu\mu}$ cannot cancel $f_{\mu\mu}$, since the experimental limit on the $\mu \to 3e$ branching ratio requires (for the range of f_{ee} and $f_{\mu\mu}$ we are dealing with here) $|U^2_{e\mu} f_{ee}| \ll f_{\mu\mu}$ (see Refs. [13] and [14]).

[33] If the mixing term is small relative to $\tilde{m}_\phi^2 - \tilde{m}_0^2$, one has for $R_\phi - R \simeq 1$ (where R_ϕ is the Higgs potential parameter in $\tilde{m}_\phi^2 \simeq R_\phi v_R^2$) $m_0^2 = R v_R^2 - (\beta')^2 \kappa^2$. In this case even for $\beta' = 1$, $(v_R)_{\rm max}$ would increase for $m_{\nu_\mu} = 270$ keV and $m_{\nu_\mu} = 35$ keV only by a factor of ~ 1.3 and ~ 4, respectively.

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