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# Slowing of Decay Processes by Interactions with a Medium

R. F. Sawyer
*Department of Physics, University of California, Santa Barbara, California 93106*
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A particle that decays by the simplest interaction into two other particles is placed in a space-periodic external potential that acts only on the initial particle. For a strong potential, the decay rate is slowed drastically under most initial conditions. Comparing to the second-order perturbation treatment of a model where the decaying particle is imbedded in a medium of heavy, uncorrelated particles, noninteracting except for a potential interaction with the decaying particle, we find similar rate modifications. There exist possible applications in the supernova and early Universe environment, as well as possible laboratory demonstrations of the effect, which bears some relationship to the "quantum Zeno" effect.

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In dense environments, such as neutron star matter, the supernova core, or the early Universe, the rates of comparatively weak reaction processes are sometimes important in determining departures from equilibrium, transport properties, or the rates of production of neutrinos or exotic light particles. The rates needed for the estimation of these properties are usually estimated from the lowest-order calculation involving the minimum number of particles. These rates can be greatly affected by other interactions with the dense environment [1,2].

In a more general context, the effects of interactions with the environment on particle decay have been the subject of a considerable literature directed toward the questions of whether actual decay rates should be perfectly exponential at long times [3,4], and whether there are systematic changes in rates, due to phase disruptions or measurement of the wave function of the decaying particle [5–10]. Much of this discussion has been based on an assumption of multiple collapses of a wave function as the decaying particle interacts with a medium. In recent works that find potential rate reductions from this picture, the effect is called the "quantum Zeno effect." Other recent formulations that do not invoke multiple collapses find related results in particular cases [11–14]. The present work is of the latter category; it examines an idealized model in which the medium induces a slowing of decay processes, one that is not due to wave-function collapse. The physics is apparently related to that of many, but not all, of the above references.

We consider an initial state consisting of a particle that both interacts strongly, and frequently, with its environment and can decay via a prescribed weak interaction into some other particles that (for simplicity) are taken not to interact with the environment. Ideally, we might consider the problem in which the particle propagates through a space populated with randomly situated scatterers; but we here consider instead a distinctly nonrandom potential $V(z)$ in 3D space, periodic in the variable $z$ with period $2L$.

We take the case of a Klein-Gordon particle, and take the potential to be a Lorentz scalar. The case of a Schrödinger particle can be recaptured by taking the nonrelativistic limit of the results. We consider an initial state that is an eigenstate of the strong Hamiltonian with energy $E$, with a wave function that is independent of $x,y$. The wave functions are of the form

$$\Psi(z)=\sum_{n=-\infty}^{n=\infty} a_n e^{i\zeta_n z}, \tag{1}$$

where $\zeta_n=k+n\pi/L$ and $k(E)$ is found by solving the time-independent Klein-Gordon (or Schrödinger) problem within a single cell.

Now we add an interaction leading to the decay of the initial state, described by the scalar field $\Phi$ to particles $(a)$ and $(b)$,

$$H_W=\lambda\Phi\Phi^{(a)}\Phi^{(b)}. \tag{2}$$

To calculate the decay rate to second order in the weak-coupling decay constant $\lambda$ we envision using a standard second-quantized representation of the field of the decaying particle, $\Phi$, in the presence of the potential, taking the energy eigenstates as a basis. Using (1) for the initial state wave function and normalizing such that $\sum|a_n|^2=1$, we obtain

$$\Gamma=\sum_{n=-\infty}^{n=\infty}|a_n|^2\Gamma_n\,, \tag{3}$$

where

$$\begin{aligned}\Gamma_n&=\lambda^2(32\pi^2E)^{-1}\int d^3q^{(a)}d^3q^{(b)}[\omega^{(a)}\omega^{(b)}]^{-1}\delta(\zeta_n-q_3^{(a)}-q_3^{(b)})\delta^{(2)}(\mathbf{q}_\perp^{(a)}-\mathbf{q}_\perp^{(b)})\delta(E-\omega^{(a)}-\omega^{(b)})\\&=\Gamma_{\text{free}}(E,(E^2-\zeta_n^2)^{1/2})\theta[E^2-\zeta_n^2-(\mu_a+\mu_b)^2]\,.\end{aligned} \tag{4}$$

Here $\mu_a$ and $\mu_b$ are the respective masses of the two decay products. The last line in (4) identifies $\Gamma_n$ as the decay rate that would be obtained for a free particle of energy $E$, with the coupling (2), if that free particle had mass $W=(E^2-\zeta_n^2)^{1/2}$,

$$\Gamma_{\text{free}}(E,W)=(16\pi)^{-1}\lambda^2E^{-1}\left[1-\frac{(\mu_a+\mu_b)^2}{W^2}\right]^{1/2}\left[1-\frac{(\mu_a-\mu_b)^2}{W^2}\right]^{1/2}. \tag{5}$$

First we consider a decay into two massless particles, $\mu_a=\mu_b=0$. In this case, the function $\Gamma_{\text{free}}$ of (5) does not depend on the variable $W$, and therefore the function $\Gamma_n$ of (3) depends on $n$ only through the multiplying $\theta$ function from (4). Using the normalization of the coefficients, $a_n$, we obtain

$$\begin{aligned}\Gamma&=(16\pi E)^{-1}\lambda^2\sum_{n=-\infty}^{n=\infty}|a_n|^2\theta[E^2-\zeta_n^2]\\&=\Gamma_{\text{free}}(E)-(16\pi E)^{-1}\lambda^2\sum_{n=-\infty}^{n=\infty}|a_n|^2\theta[\zeta_n^2-E^2]\,.\end{aligned} \tag{6}$$

We see that in this case, for any potential, the effect of the medium is to *reduce* the rate of decay below the free rate, evaluated at the same energy $E$. We cannot expect the medium corrections to be negative under all the kinematical conditions that will occur when the decay products are massive. For example, there are cases in which the free-space decay is kinematically not allowed, but in which the inhomogeneity of the medium induces decay, as in, e.g., the process $\gamma\to e^++e^-$ in matter, where the Coulomb fields of nuclei supply the necessary momentum. However, the result (6) is indicative of a tendency toward reduction of rates, one that becomes universal in a limit considered below.

To discuss the case in which the decay products have mass, and to make quantitative estimates of the domains in which the effect can be significant, we need to consider a specific potential. We choose

$$V(z)=\begin{cases}V_0 & \text{for } 2nL<z<(2n+1)L,\quad n=0,\pm1,\pm2\ldots\\ 0, & \text{otherwise}\,.\end{cases} \tag{7}$$

We define momenta in the two regions, $p_1=[E^2-M^2]^{1/2}$, $p_2=[E_2-M^2-V_0]^{1/2}$, where $m$ is the mass of the initial particle. We consider an initial state that depends only on $z$, with flux in the $+z$ direction, and specialize to the case $p_2=2j\pi/L$, where $j$ is a nonzero integer; the results will be essentially the same as the general case and the algebra considerably simpler. In this case there is a stationary solution for the initial wave function, for any value of $p_1$, in which the motion, in the $V=0$ regions, is in the $+z$ direction only, and where the wave number is given as $k=p_1/2$. The wave function is given in the region $-L<z<L$ by

$$\psi(z)=\begin{cases}e^{ip_1z} & \text{for } -L<z<0,\\ (2p_2)^{-1}[(p_1+p_2)e^{ip_2z}+(p_2-p_1)e^{-ip_2z}] & \text{for } 0<z<L\,.\end{cases} \tag{8}$$

The squares of the coefficients, $a_n$ in (1), are

$$|a_n|^2=L^{-2}\left[\frac{1}{\zeta_n-p_1}-\frac{\zeta_n+p_1}{\zeta_n^2-p_2^2}\right]^2\sin^2\left(\frac{\zeta_nL}{2}\right), \tag{9}$$

where $\zeta_n=p_1/2+n\pi/L$. We specialize to the case of the decay of a particle of mass $M$ into a massless particle plus a particle of mass $(M-Q)$ where $Q\ll M$, taking completely nonrelativistic conditions for the initial particle. Neglecting the $Q^2$ term, the free rate (5) becomes

$$\Gamma_{\text{free}}(E,(E^2-\zeta^2)^{1/2})=4\pi\lambda^2M^{-3}[p_1^2+2MQ-\zeta^2]\,. \tag{10}$$

Using (9) and (10), it is easy to show that in the limit of infinite $L$ the sum (3) now gives a modified decay rate $\Gamma_{L=\infty}$:

$$\begin{aligned}\Gamma_{L=\infty}=&C_1\Gamma_{\text{free}}(E,(E^2-p_1^2)^{1/2})\\&+C_2\Gamma_{\text{free}}(E,(E^2-p_2^2)^{1/2})\,,\end{aligned} \tag{11}$$

where $C_{1,2}$ are the respective probabilities that the initial particle is in the left- ($p_1$) or right-hand half ($p_2$) of the elementary cell, or its periodic repetitions, when the wave function is given by (7). It is clear that from (11) that the effects of the potential are purely kinematic in the limit of large $L$.

We address the case of small $L$ by direct evaluation of the sum, (3), using the $a_n$ of (9) and the free rate (10). We compare the calculated rates to $\Gamma_{L=\infty}$ in order to separate the effects of altered momentum, in the $p_2$ region, from the specific effects of $L$. Results for $\Gamma/\Gamma_{L=\infty}$ for various values of $p_1$, $p_2$, and $Q$ are shown in the third column in Table I. The last column of Table I gives the result of the very same calculation, except in a "reversed" case in which $p_1$ and $p_2$ are interchanged in Eq. (8); for given $p_{1,2}$ the values of $E$ and $V$ are different from in the previous case, and $p_1$ is replaced by $p_2$ in (10). If we consider the $V=0$ regions to be free space, then all the entries in the third column reflect the effects of passage through an array of attractive wells, and the entries in the last column reflect the effects of passage through an array of repulsive barriers.

We note that, depending sensitively on the choice of parameters, the results range from major reduction to no effect or a slight increase, confirming our intuition from the massless case that the general trend will be negative. The most important parameter is $Q$; for sufficiently small values of $(MQ)^{1/2}L$ the suppression is great in almost all cases. However, for there to be a significant effect there must be some minimum strength of potential; that is to say, $(p_2-p_1)L >$ (some small number dependent on $Q$).

Our conclusion is that there is a fairly consistent suppression of the decay rate, for small values of $L$, due to the interactions with the medium. The results are, we believe, closely related to the rate reductions found in Refs. [5-14]. These Zeno effects have usually been described in terms of time-dependent interruptions, by measurement, of the evolution of the wave function of an unstable system; we have formulated a problem in space, but with similar elements. We note, however, that in the present treatment there are no repeated collapses of a wave function [15]. There are some recent papers that bear on the possibilities of a Zeno effect without wave-function collapse [16].

In the example based on (9) and (10), giving the numbers in Table I, we see roughly the same criterion governing the size of the effect that is stated in Ref. [2]; the combination $pL \approx 1$ and $(MQ)^{1/2} \approx 1$ that we need for a big effect leads to $QT \approx 1$, where $T$ is taken as the transit time across the cell. The same criterion has been cited in describing the Zeno effect, where $T$ now is taken as the time between measurements [7]. Our description can lead to sharper results, e.g., the minimum strength requirement cited above.

The physical limitations on the applicability of the results to any physical problem are limited by a qualitative consideration: It is very hard to contrive an example in which the texture of a real medium is fine enough to affect a weak decay process; momentum releases are of order MeV/$c$ at the least, inverse atomic spacings of order keV/$c$. [We do not consider here possible effects on *atomic* radiative rates due to electrical interactions with surrounding atoms, which are considered in much of the Zeno literature. Here the $Q$ values will be much less, making the effect possibly much more promising; on the other hand, the effects on the internal structure of the initial atom, which are not simulated in our zero-range decay interaction (2) must come into play.] The neutron star or supernova core environments do supply spacings of scatterers that are of the order of, or smaller than, the inverse momenta of particles participating in reactions that are important to the physics.

It is interesting to see whether or not the use of the potential model can give any guide to the effects of a medium of uncorrelated individual scatterers, say a Boltzmann gas. To pursue this question, we compare perturbation expansions for the two cases. Let us consider the way that perturbation theory works with the potential (7), taking the example of decay into massless particles for simplicity. We ask how it can be that the second-order result gives a result of the form of (6), in which the only contributing terms to the correction to the zero-order result come in a limited kinematical region, namely, from $n$ *outside* the domain defined by $\zeta_n < E$.

The answer is the following: The second-order terms come about in two ways, the first of which is through a medium-dependent wave-function renormalization constant $Z_2 < 1$, coming from the graph in which the potential acts twice on the incoming line, giving no resultant

TABLE I. Values of the suppression factor $\Gamma/\Gamma_\infty$ for various values of energy release $Q$ and free momentum $p_1$. The momentum in the cells where the potential acts, $p_2$, is fixed at $2\pi L^{-1}$, implicitly defining the potential, which is attractive for the case of the third column. In the last column the result $\Gamma(\text{rev})$ is given for the same set of parameters, but reversed, in the sense that $p_2$ is the free momentum, and $p_1$ is the momentum in the presence of the potential, redefining both the energy and the potential, which is now repulsive.

| $p_1L/\pi$ | $(2MQ)^{1/2}L/\pi$ | $\Gamma/\Gamma_\infty$ | $\Gamma(\text{rev})/\Gamma_\infty$ |
|---|---|---|---|
| 0.5 | 0.5 | 0.254 | 0.436 |
| 0.5 | 1.0 | 0.602 | 0.402 |
| 0.5 | 2.0 | 1.05 | 0.346 |
| 0.5 | 10.0 | 1.00 | 0.984 |
| 1.0 | 0.5 | 1.15 | 0.338 |
| 1.0 | 1.0 | 0.50 | 0.295 |
| 1.0 | 2.0 | 1.03 | 0.235 |
| 1.0 | 5.0 | 1.00 | 0.995 |
| 1.5 | 0.5 | 0.68 | 0.152 |
| 1.5 | 1.0 | 0.55 | 0.114 |
| 1.5 | 2.0 | 1.00 | 0.995 |
| 1.5 | 4.0 | 1.00 | 1.00 |
| 1.95 | 0.1 | 0.28 | 0.30 |
| 1.95 | 0.2 | 0.07 | 0.08 |
| 1.95 | 0.5 | 1.00 | 1.00 |
| 1.95 | 1.0 | 1.00 | 1.00 |

momentum transfer to the medium. Multiplying the free rate by $Z_2$ and expanding $Z_2$ to second order in $V$ gives a negative correction to the rate, in the form of a sum over momenta, $\zeta_n$. The other terms are those in which we calculate a first-order amplitude for transferring some momentum to the medium, square, and then sum over the momenta. These terms cancel the $Z_2$ terms exactly *within* the domain defined by $\zeta_n < E$, leaving only the piece of the $Z_2$ correction from the terms outside of this domain.

We turn briefly from the fixed-background-potential model to the case of scattering off a Boltzmann gas of nonrelativistic heavy particles, with density $n_B$ and mass $M \gg m$, where $m$ is the mass of the decaying particle. The gas particles do not interact with each other, or with the decay products, but they interact individually with the decaying particle through a potential $V$. To second order in $V$, we find

$$\Gamma - \Gamma_{\rm free} = -\lambda^2 (64\pi^4 E)^{-1} n_B m^2 \times \int\int \frac{d^3q\, d^3k\, |V(\mathbf{k})|^2}{\omega^a(\mathbf{q})[(\mathbf{p}+\mathbf{k})^2 - \mathbf{p}^2]^2} \left[ \frac{\delta(E - \omega^a(\mathbf{q}) - \omega^b(\mathbf{p}-\mathbf{q}))}{\omega^b(\mathbf{p}-\mathbf{q})} - \frac{\delta(E - \omega^a(\mathbf{q}) - \omega^b(\mathbf{p}+\mathbf{k}-\mathbf{q}))}{\omega^b(\mathbf{p}+\mathbf{k}-\mathbf{q})} \right]. \quad (12)$$

In deriving (12) we have taken $M$ sufficiently large so that the energy differences of the decay products ($a$) and ($b$) are large compared to the energy differences of the bath particles. The first term under the integral is the (negative) $Z_2$ contribution; the second is the (positive) contribution in which a background particle is scattered to a new state. Again, there is a large cancellation, with a negative remainder. The structure of (12) is interesting in another respect: If we had neglected the $Z_2$ correction, the second-order calculation would have given a divergent integral, because of the singularity at $(\mathbf{p}+\mathbf{k})^2 = \mathbf{p}^2$. An attempted fix of this pathology by putting in a finite imaginary part of the propagator would be incorrect many-body physics. We believe that the estimation of effects of a medium on reaction rates, such as the axion and neutrino rates treated in Ref. [2], should (a) take into account processes on the external lines, including the medium-dependent $Z_2$ factor, as well as effects on internal lines; and (b) avoid the introduction of *ad hoc* imaginary parts in propagators, letting the formalism provide the needed sums of graphs in the needed places.

We have framed our examples as effects on decay rates, but it is clear that the same kind of analysis can be applied to reaction cross sections in the medium. The effect should be to reduce cross sections, for sufficiently small $L$ and strong potentials, under the majority of kinematical circumstances.

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[15] Our model allows a limit in which the decay rate is slowed arbitrarily; therefore the results could be construed to be evidence against the conclusions of Ref. [6], based on the repeated collapse description, to the effect that there cannot be a genuine Zeno effect that is potentially observable under any circumstances.

[16] For example, Refs. [11–14] each offer analyses of the experiments similar to that discussed in Ref. [10], but avoid the invocation of repeated collapse of the wave function. In Refs. [11] and [12] especially, the key is identified as the phase decoherence among different outcomes when the decaying particle, whose wave function we are following, is imbedded in a medium whose wave function we are not interested in following. In recent literature on quantum measurement [e.g., E. Joos and H. D. Zeh, Z. Phys. B **59**, 223 (1985); A. Caldeira and A. Leggett, Physica (Amsterdam) **121A**, 587 (1983); W. Zurek, in *Non-Equilibrium Quantum Statistical Physics*, edited by G. Moore and M. Scully (Plenum, New York, 1984)] such decoherence is established, on some course-grained scale of time, for a number of different specific examples. It seems likely that a well-formulated decoherence assumption can replace the repeated wave-function collapse invoked in Refs. [5–10] in order to make real decays perfectly exponential, except at very small times. Turning to effects on the lifetime, on the other hand, note that decoherence is not a quality of our model, in which the background medium does not possess degrees of freedom; we find the slowing of decay under a wide category of conditions nonetheless.