Glueball Wave Functions in Lattice Gauge Calculations

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The SU(2) glueball wave functions $\langle 0|A_\mu^\dagger(x)A_\nu(y)|G\rangle$ are measured on the lattice in the Coulomb gauge, using a source method. It is found that the tensor glueball is about 4 times as large as the scalar glueball. Phenomenological implications for identifying glueballs are discussed.

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Notwithstanding the fact that quantum chromodynamics (QCD) has been by and large established as the leading fundamental theory of strong interaction, clinching experimental evidence is still needed to prove it beyond doubt. If experiments could confirm the existence of glueballs, which appears to be a crucial prediction of the theory, then QCD would be on much firmer ground. The prediction of these glueball masses has long been attempted in lattice gauge Monte Carlo calculations [1] which show that the lowest scalar, tensor, and pseudoscalar glueballs lie in the mass region of 1–2.5 GeV.

There are several glueball candidates from J/ψ radiative decays and hadronic reactions. The best candidate is perhaps the tensor meson $\theta(1720)$ [$f_2(1720)$] discovered in J/ψ radiative decays with decay modes of $\eta\eta$ [2], $K\overline{K}$ [3], and $\pi\pi$ [3]. Phenomenological analysis suggests that the flavor independence and suppression of peripheral production of θ can be understood in terms of a soft form factor [4].

The need for a soft form factor, at first glance, appears unjustifiable. It is well known that point couplings are sufficient to describe the Okubo-Zweig-Iizuka (OZI) allowed $q\overline{q}$ meson decays in the SU(3) multiplet [5], e.g., $\rho \to \pi\pi$, $\phi \to K\overline{K}$, $f_2 \to \pi\pi$, and $f_2' \to K\overline{K}$. This is the basis of the hadronic models [6] where the sizes of the $q\overline{q}$ mesons are small (0.3–0.4 fm) and their electromagnetic form factors are explained by incorporating the vector dominance coupling to the $q\overline{q}$ core. This picture of the $q\overline{q}$ mesons seems to be consistent with the lattice gauge Monte Carlo calculation of the electric form factor of the pion [7] and the measurement of the $q\overline{q}$ wave function of the pion in the Coulomb gauge [8].

On the other hand, there are indications that the tensor glueball is likely to be much larger than the $q\overline{q}$ mesons as far as the strong decay is concerned. As the tensor-to-scalar-glueball mass ratios T_2/A_1 and E/A_1 [T_2 and E are the two irreducible representations of the cubic group which combine to form the tensor ($J = 2$) glueball in the continuum; A_1 is the scalar representation of the cubic group] are plotted [9] against the scale parameter $z = M_{A_1}L$, where L is the size of the spatial volume, they reveal the following results: T_2/A_1 and E/A_1 differ considerably for small and intermediate volumes ($2 < z \le 10$), and start to join only at large z ($z \ge 10$), apparently restoring rotational invariance in these large volumes ($L \ge 1.6$ fm). This reflects the large finite-size effect for the tensor glueball, whereas no such effect is observed for the scalar glueball.

We think that a straightforward way of demonstrating the glueball size is to calculate the glueball wave function $\langle 0|A_\mu^\dagger(x)A_\nu(y)|G\rangle$, much the same way as the $q\overline{q}$ wave function was measured in the Coulomb gauge [8]. In this Letter, we report on a calculation of the scalar and tensor SU(2) glueball wave functions in the Coulomb gauge.

The gauge-fixing approach can also serve as an intermediate step toward extracting gauge-independent quantities (e.g., mass, charge radius, form factor) with certain numerical advantages (see below and Ref. [10]). The issue of Gribov copies on the lattice, which has drawn quite a bit of attention lately [11,12], is not addressed here. We postpone the study of the impact of such copies on our results.

A sample of SU(2) configurations was generated by the Monte Carlo method, using the usual Wilson action without gauge-fixing terms, but with a static source ($U_x = U_y = 1$) [13] at Euclidean time $t = 0$. The gauge was then fixed by overrelaxation [14].

The Bethe-Salpeter wave function of a glueball in the continuum can be represented as

$$\langle 0|\sum_{\mu,\nu}\alpha(\mu,\nu)\sum_{\hat{r}}Y_{lm}(\hat{r})A_\mu^\dagger(\vec{x})A_\nu(\vec{x}+\vec{r})|G\rangle, \qquad (1)$$

where the two sums enforce the appropriate symmetries of the polarizations and orbital angular momentum, respectively. As a lattice version of the 2-glue operator $A_\mu^\dagger(\vec{x})A_\nu(\vec{x}+\vec{r})$, we choose

$$C_{\mu\nu}(\vec{r},t) \equiv \sum_{\vec{x}}\{\mathrm{ReTr}[U_\mu^\dagger(\vec{x},t)U_\nu(\vec{x}+\vec{r},]-\tfrac{1}{2}\langle\mathrm{ReTr}[U_\mu(\vec{x},t)]\rangle\langle\mathrm{ReTr}[U_\nu(\vec{x},t)]\rangle\} \qquad (2)$$

$$= a^2\sum_{\vec{x}}A_\mu^\dagger(\vec{x})A_\nu(\vec{x}+\vec{r})\{1-\tfrac{1}{3}a^2[A_\mu^{\dagger 2}(\vec{x})+A_\nu^2(\vec{x}+\vec{r})]+O(a^4)\}, \qquad (3)$$

where $\langle\cdots\rangle$ denotes a Monte Carlo average. The summation runs over all sites of a given time slice in order to project onto a $\vec{0}$-momentum state. The second term is needed to subtract out contact terms like $a^2A_\mu^2(\vec{x})$, and to enforce $\lim_{\vec{r}\to\infty}C_{\mu\nu}(\vec{r},t)=0$.

The a^2 dependence of our glueball operator is made possible by gauge fixing. It is to be contrasted with the a^4 dependence of the gauge-invariant Wilson loops previously used in glueball studies. This lower dimension operator yields a linear a dependence in the correlation function as opposed to the a^5 dependence for Wilson loops. This considerably improves the glueball signal as a is reduced [9, 15].

One pays a price here: our 2-link observable sums over many paths connecting the two links; some paths wind around the lattice and others do not. Therefore we risk a mixing of our glueball states with the flux states [16] (sometimes referred to as torelons in the literature). However, we believe that the mixing is minimized by the subtraction in Eq. (2) in view of the fact that flux states are the most extended objects on the lattice [15].

We now need to select the desired quantum numbers S and L for the spin and orbital angular momentum. In this study we only consider $S=L=0$ (0^+ or $^1S_0$) and $S=2$, $L=0$ (2^+ or $^5S_2$). Furthermore we only consider the E representation of the cubic group as a discrete version of $S=2$, leaving the T_2 representation for a later study [15]. The two lattice observables measured are therefore

$$W_0(|\vec{r}|,t)=\sum_\mu\sum_{\hat{r}}C_{\mu\mu}(\vec{r},t), \tag{4}$$

$$W_2(|\vec{r}|,t)=\sum_{\hat{r}}[2C_{33}(\vec{r},t)-C_{11}(\vec{r},t)-C_{22}(\vec{r},t)]. \tag{5}$$

In the presence of the source at $t=0$, we observe a deviation ΔW of these observables from their vacuum expectation values. For large t, the dependence of ΔW on r and t factorizes:

$$\Delta W(r,t)\sim e^{-mt}\psi(r), \tag{6}$$

where m is the glueball mass and $\psi(r)$ the Bethe-Salpeter wave function. Contamination by excited states, for t insufficiently large, shows up both as a deviation of ΔW from a single exponential in t, and as a modification of the spatial wave function with t. We have checked that our results are consistent with the above factorization.

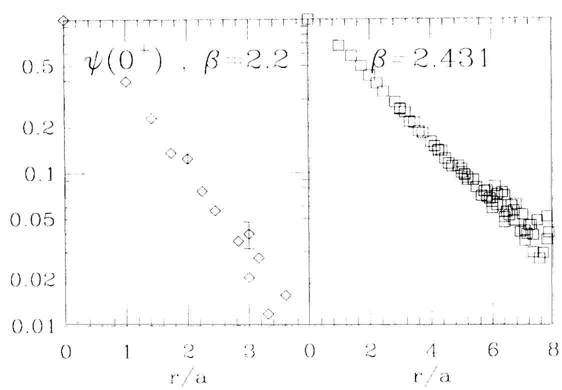

FIG. 1. The SU(2) scalar glueball wave function measured on the lattice in Coulomb gauge at $\beta=2.2$ and 2.431. The normalization is arbitrary. Error bars shown are of typical size for these data.

Two sets of measurements were taken to check the scaling of our results at $\beta=2.2$ and 2.431 (700 configurations on a 12^4 lattice at each β, separated by 4 Metropolis updates; an average of ca. 12 gauge-fixing iterations for each time slice was sufficient). This latter β value is chosen to be the same as that in Ref. [8] in order to allow a direct comparison with the meson wave functions.

Our results for the wave functions are contained in Figs. 1 and 2 and in Table I. The wave functions, normalized to 1 at the origin, are shown for the scalar and the tensor glueballs, at $\beta=2.2$ and 2.431, 2 and 3 Euclidean time units away from the source, respectively. We observe, as was the case for mesons, an exponential dependence $\psi(r)\sim e^{-r/r_0}$. The values of r_0, for the two glueball states and the two β values, are listed in Table I with the former meson results. One can see that the lattice spacing changes by a factor of ~ 2 between $\beta=2.2$ and $\beta=2.431$, and that the tensor glueball is much more extended than the scalar, by a factor of ~ 4. The tensor is therefore more sensitive to the finite-size effect, which is quite visible in the distortion of the wave function for large r at $\beta=2.431$.

Effective masses $\log[\psi(r=0,t=1)/\psi(r=0,t=2)]$ are reported in Table I. They are consistent with those obtained with much higher statistics [16]. If one assigns a physical value $a\sim 0.22$ fm at $\beta=2.2$, by setting the string tension to 420 MeV, one obtains physical radii and masses as in Table I. The tensor glueball has a radius ~ 0.8 fm, i.e., 2–3 times that of a pion [8].

The color charge distribution $r^2|\psi^2(r)|$ is shown in Fig. 3, for the two glueball states at $\beta=2.2$. One can roughly estimate the charge radius to be ~ 1 and $\sim 4-5$ lattice units, or ~ 0.2 and ~ 0.8 fm, for the scalar and tensor glueballs, respectively. The scalar glueball radius we obtain agrees with earlier calculations from $\langle G|O|G\rangle$ where O is a plaquette of varying size, which yielded a radius of ~ 0.25 fm [17]. On the other hand, the radius ob-

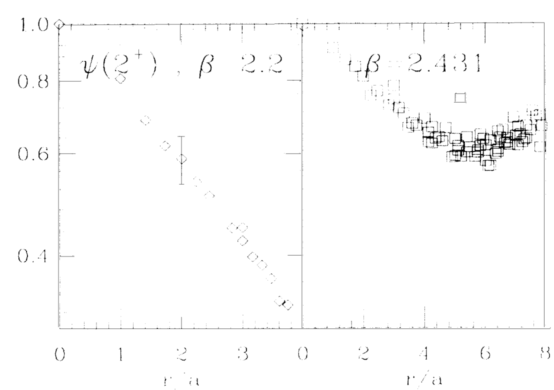

FIG. 2. Same as Fig. 1, for the tensor glueball.

TABLE I. Sizes and masses (with statistical errors) of SU(2) glueballs and mesons, in lattice and physical units.

	0^+	2^+	π	ρ
Size				
$\beta = 2.2$	~ 0.9	~ 3.5	-	-
$\beta = 2.431$	~ 2	~ 8	~ 3.2	~ 4.5
With $a(\beta = 2.2) = 0.22$ fm	~ 0.2 fm	~ 0.8 fm	~ 0.32 fm	~ 0.45 fm
Masses				
$\beta = 2.2$	1.33(3)	1.85(7)	-	-
$\beta = 2.431$	0.86(4)	1.04(23)	-	-
With $a(\beta = 2.2) = 0.22$ fm	~ 1.2 GeV	~ 1.7 GeV		

tained from the form factor calculation is much larger [18]. This seems to be reminiscent of the pion situation: The radius from the Coulomb wave function [8] is much smaller than the charge radius determined from the form factor [7]. The difference there is interpreted as due to the vector dominance in the pion charge form factor.

For the scalar glueball, its S-wave decay to pseudoscalar pairs may proceed via a pointlike coupling [Fig. 4(a)]. In this case, the form factor will merely reflect the small $q\overline{q}$ size of the pseudoscalar mesons, and we expect it to resemble that in the ordinary OZI-allowed meson decays. In other words, if the form factor is parametrized by the form e^{-q^2/λ^2}, λ^2 will be in the range of 4–9 GeV^2 [19, 20].

The form factor of the tensor glueball is a different story. Its pointlike coupling to a pseudoscalar pair is presumably suppressed due to its D-wave decay, so that the form factor will be dominated by the nonlocal coupling like in Fig. 4(b) where the pseudoscalar mesons couple to the glueball at different positions, thus reflecting the size of the glueball. To evaluate this form factor, we employ the following approximation. We consider that the coupling between the pion and the glue field A_μ in the glueball proceeds via the fundamental coupling $\overline{\Psi}\gamma_\mu\frac{\lambda^a}{2}\Psi A^a_\mu$ and an OZI-allowed $\overline{q}q$ creation in the $^3P_0$ channel. Since the $q\overline{q}$ separation in the pion is small ($\sim$ 0.32 fm) from the Coulomb wave-function calculation [8], we can treat the pion as a point field at low energies as in the chiral perturbation theory and approximate the vertices in Fig. 4(b) by the effective coupling $A^a_\mu\eta^a\partial_\mu\phi$, where ϕ is the pion field and η^a is an effective $q\overline{q}$ field in the adjoint representation to be exchanged between the gauge fields A_μ. Since all the approximation to the effective vertex is done locally, we have not compromised the gauge invariance. Then the $G \to \pi\pi$ decay amplitude at the tree level involves a second-order process of the effective coupling and the η^a propagator. As far as the momentum transfer is concerned, the $G \to \pi\pi$ decay form factor is proportional to the Fourier transform of the product of the glueball wave function $\langle 0|A^\dagger_\mu(0)A_\nu(r)|G\rangle$, the adjoint propagator $G_\eta(0,r)$ which mediates the glueball to $\pi\pi$ decay with the t-channel exchange, and the relative D-wave between the pions, i.e.,

$$F_{GPP}(q^2) \sim \int d^3r\, e^{-i\vec{q}\cdot\vec{r}/2}\langle 0|A^\dagger_\mu(0)A_\mu(r)|G\rangle \times G_\eta(0,r)Y_2(\hat{r}), \tag{7}$$

where q is the relative momentum between the pions and the Y_2 is due to the D-wave decay. We believe this is a good approximation for the q^2 dependence which reflects the glueball size. Since the expression in Eq. (7) is gauge invariant to the leading order of the effective coupling, we may evaluate it in any gauge. Since we have calculated the glueball wave function in the Coulomb gauge, we need to find the adjoint propagator in the same gauge. In the spirit of the Born-Oppenheimer approximation, we further replace the adjoint propagator $G_\eta(0,r)$ by the static potential between the sources in the adjoint representation. The latter is measured through the correlation of

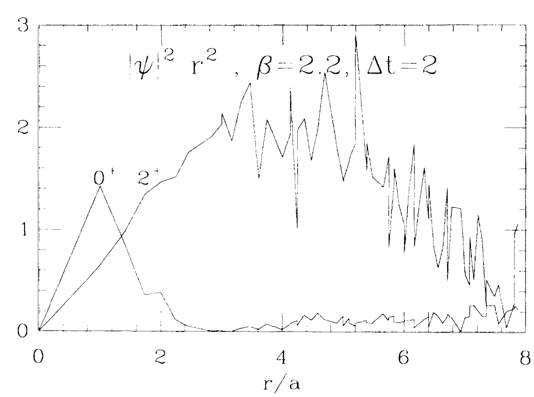

FIG. 3. The SU(2) color charge distribution in a scalar or tensor glueball, measured on the lattice at $\beta = 2.2$. The normalization is arbitrary.

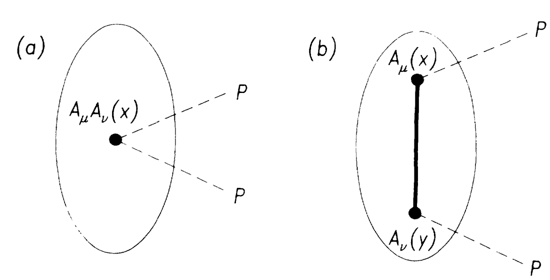

FIG. 4. Dominant decay process for (a) the scalar glueball and for (b) the tensor glueball.

two timelike Wilson lines in the Coulomb gauge. To compare with the phenomenological fit of the form factor at relatively low momentum transfers (q^2 in the range of 1–3 GeV^2), we use the long-range part of the potential which is linear in the Coulomb gauge. As a result, Eq. (7) becomes

$$F_{GPP} \sim \int dr\, r^3 j_l(qr/2) e^{-\frac{r}{r_0}}, \tag{8}$$

where we have used the lattice result $e^{-\frac{r}{r_0}}$ for the glueball wave function, with $r_0 \sim 0.8$ fm. Equation (8) gives the following prediction for the tensor form factor F^T_{GPP}:

$$F^T_{GPP} \sim q^2(1 + q^2 r_0^2/4)^{-3}. \tag{9}$$

We can fit the phenomenological form $q^2 \exp(-q^2/\lambda^2)$ to Eq. (9) for the $q^2 = 1.75$, 1.98, and 2.88 GeV^2 from the corresponding decays of θ to $\eta\eta, K\overline{K}$, and $\pi\pi$. The fitted λ^2 in this q^2 range is 0.89 GeV^2. This is very close to $\lambda^2 = 1$ GeV^2 which is needed to explain the flavor-independent decay pattern of θ [4].

To conclude, we have calculated the SU(2) glueball wave functions in the Coulomb gauge at $\beta = 2.2$ and 2.431 with the source method. Except for the finite-size effect at the edge of the lattice for the tensor glueball, the results at these two β values seem to scale. It is learned that the tensor glueball size is about 4 times as large as that of the scalar glueball. This is consistent with the large finite-size effect for the tensor (E) glueball mass found in the previous lattice calculations. It also confirms the phenomenological speculation of a soft form factor for the tensor glueball in order to identify θ [$f_2(1720)$] as a fairly pure glueball from the available experiments.

We show, albeit in an approximate manner, how to construct physical quantities like the form factors from the gauge-dependent Bethe-Salpeter wave functions. This opens up the possibility of calculating physical processes utilizing the hadronic structure obtained from the lattice calculation as an intermediate input.

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Note added.—After this work was completed, we learned that a new spin-parity analysis of J/ψ radiative decays reports a large spin zero component in the $\theta(1720)$ region [21]. The phenomenological implication of the present work could be modified pending better data for the branching ratios for the spin 2 component.

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