How Efficient is the Langacker-Pi Mechanism of Monopole Annihilation?

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We investigate the dynamics of monopole annihilation by the Langacker-Pi mechanism. We find that considerations of causality, flux-tube energetics, and the friction from Aharonov-Bohm scattering suggest that the monopole annihilation is most efficient if electromagnetism is spontaneously broken at the lowest temperature ($T_{\rm em} \approx 10^7$ GeV) consistent with not having the monopoles dominate the energy density of the Universe.

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As is well known, all grand unified theories (GUT's) must of necessity give rise to 't Hooft-Polyakov magnetic monopole solitons [1]. As a practical matter, these will arise whenever a U(1) subgroup appears after spontaneous symmetry breaking (a more general criterion involves the second homotopy group of the vacuum manifold [2]).

From a cosmological viewpoint, these monopoles are disastrous. They have a mass $m_M \sim M_{\rm GUT} \sim 10^{16}$ GeV, and since they are created via the misalignment of the Higgs fields in different horizon volumes [3], we expect to have at least one monopole per horizon at the time of the GUT phase transition giving rise to the monopoles. These two facts then lead us to the conclusion that the Universe would have become monopole dominated long ago, contrary to evidence from nucleosynthesis [4].

Historically, the monopole problem was an important factor in arriving at the inflationary universe scenario. Indeed, with an appropriate amount of supercooling (as in the case of a first-order phase transition), the monopole number density could be diluted away. However, there are other solutions to the monopole problem. In particular, Langacker and Pi [5] proposed such a solution some time ago. They argued that if the electromagnetic gauge group $U(1)_{\rm em}$ were broken for a period of time and then restored, then monopole-antimonopole pairs would become bound by flux tubes and then annihilate each other. Recently, there has been a revival of interest in this work from a variety of standpoints [6–11].

Our aim in this Letter is to elucidate some points concerning the efficiency of the Langacker-Pi mechanism and, in particular, discuss the issue of when $U(1)_{\rm em}$ should be broken. The results of our analysis are rather surprising (at least to us): For the Langacker-Pi mechanism to be most efficient, the time $t_{\rm em}$ at which $U(1)_{\rm em}$ is broken should be postponed as long as possible, i.e., until just before (or even after) the monopoles begin to dominate the energy density of the Universe.

This is rather counterintuitive; the natural expectation given the energetics of the monopole–flux-tube system is that the temperature $T_{\rm em}$ corresponding to the time $t_{\rm em}$ should be as close to the GUT phase transition temperature T_M as possible. The reason for this is that the tension in the flux tube is $\sim T_{\rm em}^2$. Thus the force between monopoles is stronger for larger $T_{\rm em}$. However, this cursory analysis neglects some important factors, such as the role of Aharonov-Bohm scattering by the flux tube, in determining the annihilation efficiency. It is to these issues we now turn.

Causality efficiency.— Let us suppose that $U(1)_{\rm em}$ is broken spontaneously at a temperature $T_{\rm em}$ well below the monopole production scale T_M. The magnetic monopoles were produced with an initial density $n_M(T_M) \sim \xi^{-3}(T_M)$, where $\xi(T)$ is the correlation length of the Higgs field at temperature T. While the actual value of $\xi(T)$ depends sensitively on the nature of the GUT phase transition, we can use causality to bound it above by the horizon size $2t(T_M)$, where $t(T) \approx 0.03 M_{\rm Pl}/T^2$ during the radiation dominated era. This yields the following lower bound on the monopole number density at creation:

$$n_M(T_M) \gtrsim 10^4 T_M^6 / M_{\rm Pl}^3 . \tag{1}$$

If $U(1)_{\rm em}$ were broken immediately after the GUT phase transition, there would not be enough monopoles available to be connected by the flux tubes within a Hubble time scale. On the other hand, at later times when the Universe cools down to a temperature T, the *total monopole number inside the horizon* grows as

$$N_M(T) \sim (T_M/T)^3 . \tag{2}$$

The ever increasing total monopole number inside the horizon at temperature $T \ll T_M$ implies that the flux-tube network is easily formed within a Hubble time scale. For example, when the temperature $T \approx 10^7$ GeV, at which the Universe starts to become monopole dominated, the total monopole number inside a horizon is $\approx 10^{30}$.

Energetic efficiency.— When $U(1)_{\rm em}$ is spontaneously broken, the flux tube connecting a monopole-antimonopole pair provides a linearly increasing confining potential. The string tension μ is

$$\mu \approx T_{\rm em}^2 . \tag{3}$$

If T_{em} is much less than T_M, the motion of the monopole pair is described by Newton's equation of motion

$$m_M \frac{d^2 l(t)}{dt^2} = F_{conf} \approx -T_{em}^2 . \tag{4}$$

Here $l(t)$ denotes the monopole-antimonopole separation (which is the same as the flux-tube length). The initial separation $l(t_{em})$ should be of the same order of magnitude as the mean separation distance among the monopoles:

$$\langle l(t_{em})\rangle \approx [n_M(T_{em})]^{-1/3} \approx (T_M/T_{em})\xi(T_M) \approx M_{Pl}/20T_{em}T_M . \tag{5}$$

The energy stored inside the flux tube is

$$E_{flux} \equiv \mu(t_{em})\langle l(t_{em})\rangle \approx (M_{Pl}/20T_M)T_{em} . \tag{6}$$

We should mention that if the length in Eq. (5) is long enough so that the energy contained in the flux tube is larger than $2m_M$, it becomes energetically possible for the tube to break via monopole pair creation. We see from Eq. (5) that this happens when $T_{em} > 400T_M^2/M_{Pl} \approx T_M/25$. Although enough energy is available, the monopoles never acquire relativistic speeds because of the effect of damping by the plasma. The mean separation of a monopole-antimonopole pair after monopole pair creation by the tube is

$$\langle l(t_{em})\rangle_r \approx \frac{20T_M}{T_{em}^2} \approx \left(\frac{20T_M}{T_{em}}\right)\left(\frac{1}{T_{em}}\right) . \tag{7}$$

We should emphasize that this only happens if T_{em} is rather close to T_M.

From Eq. (4), we find that the characteristic time scale τ_a for monopoles and antimonopoles to annihilate (assuming an efficient flux-tube energy dissipation mechanism, but ignoring damping of the monopoles; see below) is

$$\tau_a \approx \left(\frac{m_M\langle l(t_{em})\rangle}{T_{em}^2}\right)^{1/2} \approx \left(\frac{M_{Pl}}{T_{em}^3}\right)^{1/2} . \tag{8}$$

Comparing this with the Hubble time scale, $\tau_H \approx 2t_{em}$, we find

$$\tau_a/\tau_H \approx 30(T_{em}/M_{Pl})^{1/2} . \tag{9}$$

Hence, the monopole annihilation rate becomes larger as T_{em} becomes lower.

Intuitively, this can be understood as follows. The energetics argument based on the flux-tube string tension effect favors having T_{em} as close to T_M as possible. On the other hand, the formation of a network of monopoles connected by flux tubes favors lower values of T_{em}, as can be seen from Eq. (2). This is a direct consequence of the slowing expansion rate of the Universe. The two effects compete with each other, but the latter dominates at lower temperatures. Indeed, using Eq. (2), one can rewrite Eq. (9) as

$$\left(\frac{\tau_a}{\tau_H}\right)^3 \approx 3\times 10^4 \left(\frac{T_M}{M_{Pl}}\right)^{3/2} \frac{1}{[N(t_{em})]^{1/2}} . \tag{10}$$

This clearly shows that the monopole annihilation rate depends only upon the instantaneous total monopole number within the horizon.

Thermal fluctuations.— So far, we have not taken into account the effects of the thermal bath on the monopoles. These are important since the thermal energy of monopoles provides transverse velocity to the flux tubes, and thus nonzero angular momentum to the monopole pair connected by the flux tube. First of all, monopoles at a temperature T_{em} are expected to be in good thermal contact with the background photons and the ambient plasma. Indeed, the strength of monopole-photon interaction is of order unity, and the cross section for charged plasma-monopole interactions is correspondingly $O(\alpha_{em}^{-1})$ larger than that among charged particles.

Thus, the initial kinetic and potential energies of the magnetic monopoles at temperature $T_{em} \ll \frac{1}{25}T_M$ are

$$K \approx T_{em} , \quad V \approx T_{em}^2\langle l(t_{em})\rangle \approx 500T_{em} . \tag{11}$$

The typical transverse momentum of the monopoles due to thermal motion is $P_\perp(T_{em}) \approx (20T_MT_{em})^{1/2}$. Thus, the initial angular momentum of the flux-tube–monopole pair reads

$$L \approx \langle l(t_{em})\rangle P_\perp(t_{em}) \approx \left(\frac{M_{Pl}^2}{20T_MT_{em}}\right)^{1/2} . \tag{12}$$

In the absence of friction, energy and angular momentum conservation lead to a final mean separation

$$\langle l(T_{em})\rangle_{fin} \approx \frac{1}{20}\left(\frac{M_{Pl}}{T_M}\right)^{1/2} \frac{1}{T_{em}} . \tag{13}$$

It is seen that the final mean separation of the monopole pair is larger by a factor of 100 than the flux-tube thickness $1/eT_{em}$. At the same time, the final transverse momentum of monopoles at the above separation is of order $\frac{1}{10}(M_{Pl}T_{em})^{1/2} \ll T_M$, showing that the monopoles are always nonrelativistic.

For potentially relativistic monopoles (i.e., if $\frac{1}{25}T_M \le T_{em} \le T_M$), the maximum transverse momentum is $P_\perp \approx E \approx M_{Pl}T_{em}/T_M$. The flux tubes whose original length was given by Eq. (7) shrink to a mean separation which cannot be less than

$$\langle l(T_{em})\rangle_{r,fin} \approx \left(\frac{10T_M}{T_{em}}\right)^{1/2} \frac{1}{T_{em}} . \tag{14}$$

They are longer than the flux-tube thickness by a factor of $\gtrsim 3$.

In both the relativistic and the nonrelativistic cases, it is seen that the final monopole pair is separated by a centrifugal barrier due to the angular momentum. Thus the wave-function overlap and the annihilation cross section are exponentially suppressed.

This leads us to a crucial point: In order for the monopole pair to be confined by the flux tube and annihilate efficiently, the *initial angular momentum* must be dissipated by friction.

Friction from Aharonov-Bohm scattering.— There are several mechanisms for dissipating the initial angular momentum: (1) radiation of long-range gluons and/or weak gauge bosons, (2) interactions between the magnetic monopole and the ambient plasma, and (3) the interaction between the flux tube and the plasma through Aharonov-Bohm scattering. We now estimate the dissipation rate $\Gamma \equiv -d\ln L/dt$ in each of these cases.

The interaction between magnetic monopole and the plasma gives rise to a friction force $F_M(T) \approx \rho(T)\sigma_{\mathrm{CR}}v \approx T_{\mathrm{em}}^2 v$, where ρ is the background plasma energy density, σ_{CR} the Callan-Rubakov [12,13] cross section of the monopole, and v the monopole terminal velocity. Thus, the monopole dissipation rate is

$$\Gamma_{\mathrm{mon}} \approx \rho\sigma_{\mathrm{CR}}/m_M \approx \left(T_{\mathrm{em}}/m_M\right) T_{\mathrm{em}} . \tag{15}$$

The monopole energy dissipation rate from radiation of gluons and weak gauge bosons is found to be

$$\Gamma_{\mathrm{rad}} \approx \frac{1}{\alpha}\left(\frac{T_{\mathrm{em}}^2}{T_M}\right)^2 \frac{1}{m_M v^2} \approx 40\left(\frac{T_{\mathrm{em}}}{T_M}\right)^2 T_{\mathrm{em}} . \tag{16}$$

The Aharonov-Bohm (AB) scattering [14] arises because the magnetic field is confined inside the flux tube while the color and the weak gauge field are not. As a result of the fractional electric charges $Q_u = 2e/3$ and $Q_d = -e/3$ carried by the quarks, the flux tube connecting the monopoles experiences nontrivial AB scattering with a cross section

$$\frac{d\sigma_{\mathrm{AB}}}{d\theta} = \frac{\sin^2[(Q_{u,d}/e)\pi]}{2\pi k \sin^2(\theta/2)} . \tag{17}$$

This result does not contradict the Dirac quantization condition as the latter applies to the *total sum* of color, weak isospin, and electromagnetic quantum numbers [15]. The AB dissipation rate is

$$\Gamma_{\mathrm{AB}} \approx \rho\sigma_{\mathrm{AB}}\bar{l}/m_M \approx (T_{\mathrm{em}}/m_M) T_{\mathrm{em}} , \tag{18}$$

where $\bar{l}$ is the distance over which the motion of the flux tube is correlated with that of the monopole. Thus, we find that radiation dissipation is negligible while monopole-plasma dissipation and AB scattering give comparable contributions.

From Eq. (18), we find that

$$\tau_{\mathrm{AB}}/\tau_a \approx 10^{-3}(M_{\mathrm{Pl}}/T_{\mathrm{em}})^{1/2} . \tag{19}$$

Similarly, comparing τ_{AB} with the Hubble expansion time, we find

$$\tau_{\mathrm{AB}}/\tau_H \approx 1/60 . \tag{20}$$

From Eqs. (19) and (20), we thus come to our main conclusion: Monopole annihilation by the Langacker-Pi mechanism is most efficient for the lowest possible T_{em}, i.e., for $T_M \gg T_{\mathrm{em}} > 3\times 10^4$ GeV.

Recall that the Hubble time scale increases as $t \propto T^{-2}$, which is faster than the monopole annihilation time. This was responsible for the efficiency of the annihilation at the lower temperature of EM breaking. We have now found that the friction due to the AB scattering not only dissipates the angular momentum efficiently but also helps monopole annihilation at lower temperature scales. For temperatures in the range $T_M \gg T_{\mathrm{em}} \gtrsim 3\times 10^4$ GeV, the dominant time scale is that for dissipation of angular momentum, namely, τ_{AB} ($\approx \tau_{\mathrm{CR}}$) which always satisfies

$$\tau_{\mathrm{AB}} \ll \tau_H . \tag{21}$$

The highest efficiency for monopole annihilation occurs at the lowest possible temperature. Of course, there is another reason the scale T_{em} cannot be too low: The monopoles will eventually dominate the energy density of the Universe. With the initial monopole density given by Eq. (1), we find that the temperature at which monopoles dominate the energy density of the Universe (i.e., $\rho_M/\rho_{\mathrm{total}} \approx 1$) is $T_c \approx 10^7$ GeV. However, we should note that we could allow the monopoles to dominate the energy density of the Universe all the way down to the electroweak scale. The only constraint in this case is that the baryon to entropy density not be diluted beyond 10^{-9} by the monopole annihilation process.

In this Letter, we have examined the detailed dynamics of the Langacker-Pi mechanism. As a result of the unusual temperature dependence of the characteristic time scales as summarized in Eq. (19), we find the counterintuitive result that the most efficient scenario of monopole annihilation occurs when $U(1)_{\mathrm{em}}$ is broken just before the monopoles dominate the energy density of the Universe. The fact that the photon is massive and electric charge is spontaneously broken leads us to expect that charge nonconserving processes may provide novel signatures of the phase, which should be left over until today. In addition, the Callan-Rubakov effect [12] may provide additional baryon-asymmetry generation at a relatively low energy scale [9,10], and we expect sizable entropy generation from the monopole and antimonopole annihilation. We are currently investigating these issues, and will report them in a separate publication. After this work was completed we were informed that Gates, Krauss, and Terning [16] have recently studied the monopole annihilation efficiency using W-condensate flux tubes.

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