Noise Cancellation in Laser Emission

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We have investigated the properties of light emitted into the nearest subthreshold mode on either side of the single lasing mode of an argon-ion laser. This light beats with the laser line and hence contributes to the laser's intensity-fluctuation noise spectrum over a range of frequencies centered close to the longitudinal mode separation. The noise contribution from one subthreshold mode is shown to be strongly anticorrelated with that from the other, leading to optical noise cancellation.

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Noise cancellation is a well-established technique in acoustics [1] and electronic signal processing [2] whereby undesirable fluctuations are suppressed by destructive interference. This is achieved by the addition of an "antinoise" component equal in amplitude, but with opposite phase. In this Letter, we demonstrate that an optical analog of this phenomenon occurs naturally in the intensity-fluctuation noise spectrum of the laser. The fundamental properties of the spontaneous-emission noise that accompanies coherent laser light are only partially understood, despite considerable current research activity [3]. The existence of strong correlations between the gain contributions from opposite sides of the laser line for the above-threshold laser amplifier [4,5] suggests the possibility of similar correlations between the corresponding noise contributions. We report here clear experimental verification of such correlations, for light emitted into the nearest subthreshold mode on either side of the single lasing mode of an argon-ion laser. The correlations produce dramatic cancellation effects between the noise contributions from opposite sides of the laser line.

In previous work [5] we have investigated the gain profiles and noise spectra for a single-mode Ar^+ laser over a range of frequencies centered close to the longitudinal mode spacing. The noise originates from beating between the lasing mode and light emitted into the two adjacent subthreshold modes, one on each side. In the present investigation we observe the noise spectrum resulting from beating between the lasing mode *and only one* of the subthreshold noise sidebands, the other sideband having been eliminated by optical filtering. If the sidebands were to consist of uncorrelated Gaussian noise, then addition of the second sideband would result in a simple doubling of the noise power density, the shape of the spectrum being unaffected. In fact, our data show that adding the second sideband significantly reduces the noise level, demonstrating cancellation of strongly anticorrelated noise sidebands.

Figure 1 shows the experimental arrangement, with a single mode of the double-ended Ar^+ laser (frequency ν_L) selected by means of a temperature-tuned intracavity solid etalon. This introduces a small extra loss for the other modes, keeping them below threshold [Fig. 2(a)]. The etalon is tuned to achieve the symmetrical arrangement by maximizing the laser power. The laser cavity has a mode spacing $\Delta = 156.02$ MHz, and both mirrors have approximately 95% reflectivity. The output power is varied by adjustment of the discharge current. The laser output in the measurement beam (Fig. 1), consisting of the lasing mode with a broadband noise sideband on each side, is incident on a Fabry-Pérot etalon, tuned to transmit nearly all the light in one of the noise sidebands. Thus, the light reflected from the etalon has the spectrum depicted in Fig. 2(b): the lasing mode together with only one noise sideband. This reflected beam is incident on detector A, and the single-sideband intensity-fluctuation noise profiles are obtained by spectral analysis of the detector current. The behavior does not depend upon which of the two sidebands has been filtered.

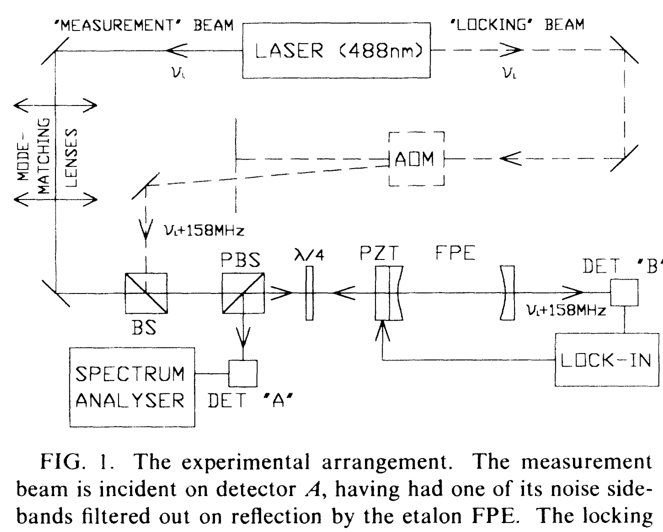

FIG. 1. The experimental arrangement. The measurement beam is incident on detector A, having had one of its noise sidebands filtered out on reflection by the etalon FPE. The locking beam (dashed line) ensures that the etalon is resonant with one noise sideband. AOM: acousto-optic modulator; BS: beam splitter; PBS: polarizing beam splitter; PZT: piezoelectric transducer.

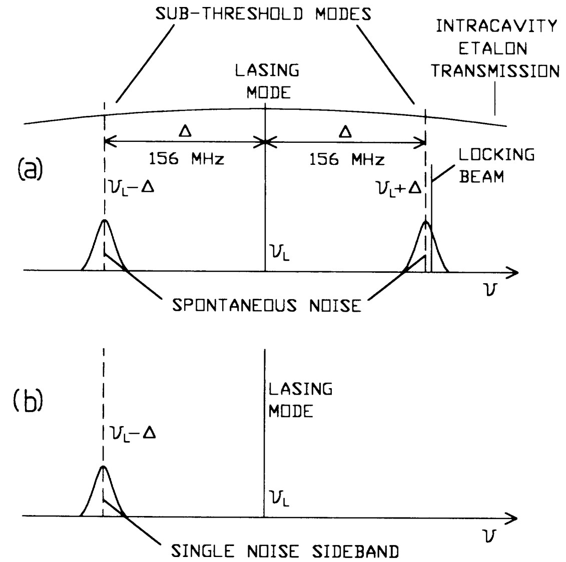

FIG. 2. (a) Position of the lasing mode, subthreshold modes with locking beam, and intracavity etalon transmission function centered on the lasing mode. (b) Spectrum of the beam incident on detector A, for single-sideband intensity-fluctuation noise measurements.

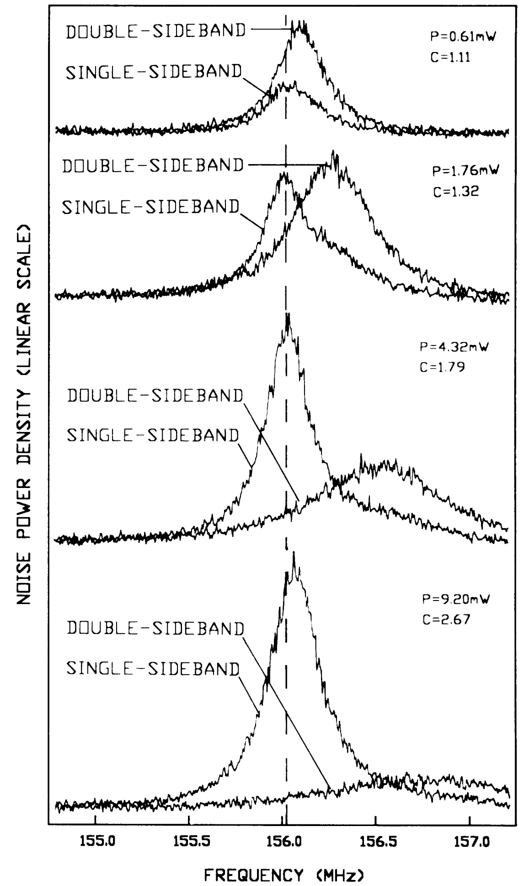

FIG. 3. Intensity-fluctuation noise spectra for single- and double-sideband detection at several laser powers. In the absence of correlations, the double-sideband noise power would simply be twice that for the single sideband.

A standard combination of polarizing beam splitter and quarter-wave plate allows efficient separation of reflected and incident beams, and eliminates feedback of light into the laser. The filtering etalon (FPE) consists of a pair of identical mirrors, with reflectivity 89% and radius of curvature 15 m, set 21.5 cm apart in a thermally isolated cavity. Its mode spacing is thus 698 MHz with a mode width of 26 MHz. The etalon is accurately piezo tuned to transmit one of the noise sidebands. This is achieved by shifting the frequency of the other laser output (the locking beam in Fig. 1) to lie close to that of the sideband. The etalon is locked into resonance with this beam by maximizing its transmission, via feedback to the piezo. It is essential to match the shape of the incident measurement beam to the resonant mode of the Fabry-Pérot cavity [6]. Any mismatch reduces the etalon transmission leading to significant reflection of the rejected sideband in the measurement beam. By careful alignment, this residual reflection was reduced to less than 1% of the incident sideband intensity [7].

Measurements with both noise sidebands present are also easily obtained by tuning etalon FPE so that the laser frequency lies halfway between two of the etalon modes. All three spectral components are then detected by A, since they are reflected with close to 100% efficiency, and negligible phase change [6]. A direct quantitative comparison between the single-sideband and double-sideband noise spectra can be made, to assess the strength of correlation between sidebands. Figure 3 shows a series of single- and double-sideband spectra at increasing values of laser power P. The normalized pumping rate, or cooperation parameter C, is also indicated [4,5]. The double-sideband noise spectra are symmetrical Lorentzians, which display the shift and broadening analyzed in detail in Ref. [5]. In contrast, however, the single-sideband noise is always centered close to the mode spacing, and shows a varying degree of asymmetry, while its width is relatively insensitive to changes in C. The noise cancellation clearly manifests itself in the behavior of the peak noise power levels: While the single-sideband noise shows a gradual increase with increasing pump rate, the double-sideband noise rapidly declines. The degree of cancellation is greatest close to the mode spacing Δ. It may be quantified by comparing the double-sideband noise level with the single-sideband noise level at frequency Δ. Their ratio is plotted in Fig. 4; the noise reduction which results from addition of the second sideband is already over an order of magnitude at $P=5.5$ mW ($C=2$).

Closer inspection of the experimental noise spectra in Fig. 3 reveals that the asymmetric single-sideband profiles can be accurately represented by a sum of two

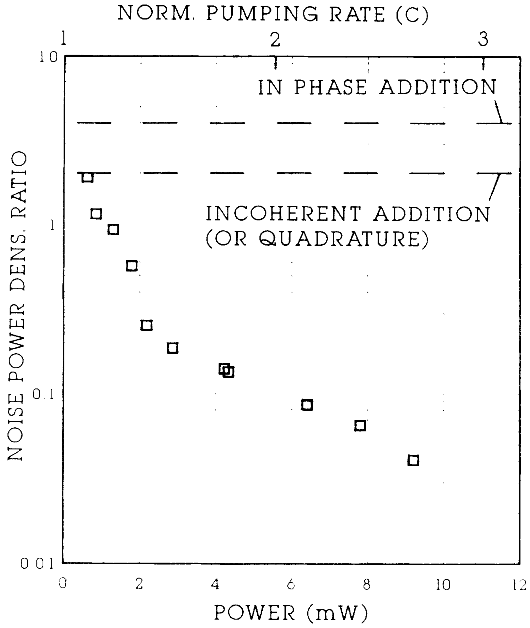

FIG. 4. Ratios of double-sideband noise to single-sideband noise, measured at the mode spacing Δ.

Lorentzians [8]. One of these has a width of about 250 kHz and is centered at the mode spacing Δ. The other Lorentzian has the same width and shift as the double-sideband noise spectrum, reduced in height by approximately a factor of 4. To explain these results we have developed a theory of laser noise [8], which derives spectra for double- and single-sideband detection, using a standard approach based on the Maxwell-Bloch equations with Langevin noise forces [9]. However, for the purposes of this Letter, our observations can be interpreted in terms of a very general classical model. Consider a carrier wave (representing the lasing mode) of angular frequency ω_L modulated by two weak sidebands (representing emission into the subthreshold modes) at $\omega_L+\omega$ (which we call the signal) and $\omega_L-\omega$ (the image). The total complex field is therefore

$$E_T=E_L+E_S+E_I\,, \tag{1}$$

where

$$E_L=E\exp(-i\omega_L t)\,,$$
$$E_S=\varepsilon\exp[-i(\omega_L+\omega)t+i\phi]\,, \tag{2}$$
$$E_I=\varepsilon\exp[-i(\omega_L-\omega)t]\,.$$

The signal and image fields are assigned the same amplitude $\varepsilon=\varepsilon(\omega)$, in view of the symmetric excitation of the noise sidebands, but they differ in phase by $\phi=\phi(\omega)$. When $\varepsilon\ll E$ it is easily shown, to first order in ε/E, that

$$\mathrm{Re}E_T=\{E+2\varepsilon\cos(\phi/2)\cos(\omega t-\phi/2)\}\times\cos\{\omega_L t-(2\varepsilon/E)\sin(\phi/2)\cos(\omega t-\phi/2)\}\,. \tag{3}$$

The field has pure amplitude modulation (AM) for $\phi=0$, pure phase modulation (PM) for $\phi=\pi$, and combinations of AM and PM for intermediate values of ϕ.

The measured quantity for detection of the double-sideband intensity-fluctuation noise spectrum is

$$|E_L^*E_S+E_LE_I^*|^2=4E^2\varepsilon^2\cos^2(\phi/2)\,. \tag{4}$$

Only the AM component from the real field (3) contributes to the measured spectrum. By contrast, the measured quantities for the single-sideband spectra are

$$|E_L^*E_S|^2=|E_LE_I^*|^2=E^2\varepsilon^2[\cos^2(\phi/2)+\sin^2(\phi/2)]\,. \tag{5}$$

Both AM and PM now contribute, and the former has one-quarter of its magnitude observed in the double-sideband spectrum (4). Single-sideband detection is a well-known technique for detecting PM at radio frequencies.

In terms of the above model, the double-sideband measurements shown in Fig. 3 determine the AM component of the laser side-mode noise spectra, and this component also features in the single-sideband spectra with its strength reduced by a factor of 4. Subtraction of this AM part from the single-sideband spectra in Fig. 3 leaves the PM component of the sideband noise. These are Lorentzian spectra whose widths represent the degree of quenching of the subthreshold modes by the extra loss from the intracavity etalon. Their widths are accordingly close to the difference (~200 kHz [5]) between their own damping rate and that of the central lasing mode. In order to apply this simple classical model to the interpretation of our laser noise spectra, the amplitudes and phases of the two sidebands must be considered as stochastic random variables, whose fluctuations are determined by the laser dynamics. It is, however, clear from the measurements in Figs. 3 and 4, and confirmed by the detailed theory [8], that the signal and image fluctuations are strongly correlated.

The mechanism by which the amplitudes and phases of the two sidebands become locked is that of four-wave mixing, mediated by oscillations of the population inversion at the beat frequency. This is also an important feature of the theory in [4] and [5] which treats the injection of a detuned signal into a laser amplifier. It was shown that the interaction between the laser field at ω_L and the injected signal at $\omega_L+\omega$ induces a population-inversion oscillation at ω, provided that $\omega\lesssim\gamma_\parallel$, where $\gamma_\parallel$ is the population decay rate for the upper state of the lasing transition, of order $4\times10^8\ \mathrm{s}^{-1}$ [5]. This leads to excitation of a correlated image component at $\omega_L-\omega$. Our experiments establish that a similar process applies for the noise, where light spontaneously emitted within the lasing medium experiences the same four-wave mixing effect as a signal injected from outside. This common mechanism provides the physical basis underlying the intimate relationship between noise and gain [10]. Despite its origin in a random spontaneous process, the noise acquires correlated signal and image components similar to those that control the gain. The locking of three above-threshold modes by four-wave mixing [11] is a related phenomenon.

Our theory [8] shows that very close to threshold, where $C \rightarrow 1$, the AM and PM contributions are equal. The double-sideband spectrum (4) has exactly twice the strength of the single-sideband spectrum (5) in this case, consistent with the trend at the lowest powers shown in Figs. 3 and 4. The same doubling would occur for addition of uncorrelated signal and image components; however, such incoherent addition is obviously inconsistent with the measurements as C increases. At very high pumping rate, where $C \gg 1$, the relative phase $\phi(\omega) \rightarrow \pi$, independent of ω, corresponding to pure phase modulation. This leads to efficient cancellation of the signal and image amplitude-fluctuation contributions to the noise, removing all the strength from the double-sideband spectrum in the vicinity of Δ. This behavior is strikingly evident at the higher powers of Figs. 3 and 4.

In summary, the noise contributions from either side of the laser frequency are strongly correlated out to detunings comparable with $\gamma_{\parallel}$, the upper-state relaxation rate. For noise within the lasing mode, such correlations are consistent with the usual phase-diffusion model, where the amplitude fluctuations are progressively suppressed by gain saturation well above threshold, while the phase freely wanders with time. For the Ar^+ laser studied here, we show that strong correlations extend to the sub-threshold modes adjacent to the laser line. The amplitude and phase noise spectra have been identified by separate measurements of single- and double-sideband intensity-fluctuation spectra. Remarkably, with both sidebands present such that only the amplitude noise is observed, the cancellation is asymmetric with respect to the mode center, producing shifted symmetrical noise profiles of Lorentzian form. At higher laser powers, the beats of the two sidebands with the lasing mode are in almost exact antiphase (pure PM), resulting in nearly complete noise cancellation.

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