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New Thermodynamic Bethe Ansatz Equations without Strings

C. Destri[(1),(a)] and H. J. de Vega[(2),(b)]

[(1)]*Dipartimento di Fisica, Università di Parma, and Istituto Nazionale di Fisica Nucleare, Gruppo Collegato di Parma, Parma, Italy*

[(2)]*Laboratoire de Physique Théorique et Hautes Energies, Université de Paris VI, Paris, France*

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We present a computational framework *not* based on the string hypothesis for the thermodynamics of statistical and quantum field theory models solvable by the Bethe ansatz. In the cases of the *XXZ* Heisenberg chain and the sine-Gordon quantum field theory we derive a *single* nonlinear integral equation which determines the free energy (or ground-state scaling function). Our approach is very effective at high temperature, and correctly reproduces the low-temperature central charge and the analytic structure of the corrections.

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The computation of thermodynamical functions for integrable models started with the seminal works of Refs. [1-3]. In Refs. [2,3] the free energy of the Heisenberg spin chain is written in terms of the solution of an infinite set of coupled nonlinear integral equations, derived on the basis of the so-called "string hypothesis." The same strategy was followed in Ref. [4] for the sine-Gordon quantum field theory (QFT). In this Letter we propose a simpler way to solve the thermodynamics by means of a single, rigorously derived, nonlinear integral equation. We restrict our attention here to the *XXZ* spin chain and to the sine-Gordon model, but the method can be applied to a wide class of models solvable by the Bethe ansatz.

The *XXZ* Hamiltonian for a periodic chain with $2L$ sites (generalization to other boundary conditions is possible) takes the form

$$H_{XXZ}=-J\sum_{n=1}^{2L}[\sigma_n^x\sigma_{n+1}^x+\sigma_n^y\sigma_{n+1}^y-\cos\gamma(\sigma_n^z\sigma_{n+1}^z+1)]\,, \tag{1}$$

where $0\le\gamma\le\pi$ (gapless regime). As is well known, H_{XXZ} is related to the diagonal-to-diagonal transfer matrix $T_L(\theta)$ of the six-vertex model by

$$T_L(\theta)\overset{\theta\to 0}{=}1-\frac{\theta}{2J\sin\gamma}H_{XXZ}+O(\theta^2)\,, \tag{2}$$

where

$$T_L(\theta)=R_{12}R_{34}\cdots R_{2L-12L}R_{23}R_{45}\cdots R_{2L1}\,, \tag{3}$$

$$R_{nm}=\frac{a+c}{2}+\frac{a-c}{2}\sigma_n^z\sigma_m^z+\frac{b}{2}(\sigma_n^x\sigma_m^x+\sigma_n^y\sigma_m^y)\,,$$
$$a=\frac{\sin(\gamma-\theta)}{\sin\gamma},\quad b=\frac{\sin\theta}{\sin\gamma},\quad c=1\,. \tag{4}$$

We now study the thermodynamics in the framework proposed in Ref. [5]. Thus, setting $\beta J\sin\gamma=\tilde\beta$, from Eq. (2) we read

$$e^{-\beta H_{XXZ}}=\lim_{N\to\infty}[T_L(2\tilde\beta/N)]^N\,,$$

so that the free energy per site can be written as

$$f(\beta)=-\frac{1}{\beta}\lim_{L\to\infty}\frac{1}{2L}\lim_{N\to\infty}\ln Z_{LN}(2\tilde\beta/N)\,, \tag{5}$$

where $Z_{LN}(\theta)\equiv\mathrm{Tr}[T_L(\theta)]^N$ is the six-vertex partition function on a periodic diagonal lattice with $L\times N$ sites. The two limits in Eq. (5) cannot be interchanged since the degeneracy of $T_L(0)=1$, that is 2^{2L}, is strongly L dependent. However, the crossing invariance of the six-vertex R matrix implies that under a rotation by $\pi/2$ of the entire lattice plus the substitution $\theta\to\gamma-\theta$, the numerical value of the partition function does not change (see, e.g., Ref. [6]). Therefore $Z_{LN}(\theta)=Z_{NL}(\gamma-\theta)$ and

$$f(\beta)=-\frac{1}{\beta}\lim_{L\to\infty}\frac{1}{2L}\lim_{N\to\infty}\ln Z_{NL}(\gamma-2\tilde\beta/N)\,. \tag{6}$$

According to the well-known Bethe ansatz (BA) solution, the largest eigenvalue $\Lambda_N^{\max}(\theta)$ of $T_N(\theta)$ is nondegenerate when $\theta\to\gamma$. Then the two limits in Eq. (6) commute

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and one finds [5]

$$-2\beta f(\beta)=\lim_{N\to\infty}\ln\Lambda_N^{\max}(\gamma-2\tilde{\beta}/N)\,. \tag{7}$$

Combining Eq. (7) with the results of Ref. [7], we can write

$$f(\beta)=\frac{1}{2\beta}\lim_{N\to\infty}\left[E_N(\beta)-2N\ln\frac{\sin(2\tilde{\beta}/N)}{\sin\gamma}\right], \tag{8}$$

where

$$E_N(\beta)=i\sum_{j=1}^{N}\phi(\lambda_j+i\gamma/2,\gamma/2-\tilde{\beta}/N)-\phi(\lambda_j-i\gamma/2,\gamma/2-\tilde{\beta}/N),\quad \phi(\lambda,x)\equiv i\ln\frac{\sinh(ix+\lambda)}{\sinh(ix-\lambda)}\quad [\phi(0,x)=0]\,. \tag{9}$$

The real numbers $\lambda_1,\ldots,\lambda_N$ are the roots of the ground-state BA equations,

$$z_N(\lambda_j)=j-(N+1)/2\,, \tag{10}$$

written in terms of the so-called *counting function* [8]

$$z_N(\lambda)=\frac{1}{2\pi}\left[\phi(\lambda,\tilde{\beta}/N)+\phi(\lambda,\gamma-\tilde{\beta}/N)-\frac{1}{N}\sum_{k=1}^{N}\phi(\lambda-\lambda_k,\gamma)\right]. \tag{11}$$

Our aim is to derive an integral equation for $z_N(\lambda)$.

Assuming that the root distribution becomes the continuous density $\sigma_c(\lambda)$ (normalized to 1) for large N would lead to the following linear integral equation:

$$2\pi\sigma_c(\lambda)=\phi'(\lambda,\tilde{\beta}/N)+\phi'(\lambda,\gamma-\tilde{\beta}/N)-\int_{-\infty}^{+\infty}d\mu\,\phi'(\lambda-\mu,\gamma)\sigma_c(\mu)\,, \tag{12}$$

with the explicit solution

$$\sigma_c(\lambda)=z_c'(\lambda),\quad z_c(\lambda)=\frac{1}{\pi}\arctan\left[\frac{\sinh(\pi\lambda/\gamma)}{\sin(\pi\tilde{\beta}/\gamma N)}\right]. \tag{13}$$

Roots $|\lambda_j|>O(\sqrt{\beta/N})$ have a spacing of order larger than $O(1/N)$ and cannot be described by densities. Therefore, contrary to the usual situation [8], we must go beyond the density description to obtain a bulk quantity like the free energy per site.

Through simple manipulations we find from Eqs. (11) and (12) [9]

$$z_N'(\lambda)=z_c'(\lambda)-\int_{-\infty}^{+\infty}d\mu\,p(\lambda-\mu)\left[\frac{1}{N}\sum_{j=1}^{N}\delta(\mu-\lambda_j)-z_N'(\mu)\right], \tag{14}$$

where

$$p(\lambda)=\int_{-\infty}^{+\infty}\frac{dk}{2\pi}\tilde{p}(k)e^{ik\lambda},\quad \tilde{p}(k)=\frac{\sinh[(\pi/2-\gamma)k]}{2\sinh[(\pi-\gamma)k/2]\cosh(\gamma k/2)}\,. \tag{15}$$

The right-hand side of Eq. (14) can be expressed as a contour integral encircling the real axis, that is,

$$z_N'(\lambda)=z_c'(\lambda)+\int_{-\infty}^{+\infty}d\mu\,p(\lambda-\mu)z_N'(\mu)\left[\frac{1}{1+e^{2\pi iNz_N(\mu-i0)}}+\frac{1}{1+e^{-2\pi iNz_N(\mu+i0)}}\right], \tag{16}$$

where we used Eq. (10). Integrating Eq. (16) now yields

$$\ln\frac{a_N(\lambda)}{a_c(\lambda)}=\int_{-\infty}^{+\infty}d\mu\,p(\lambda-\mu)\ln\frac{1+a_N(\mu+i0)}{1+\bar{a}_N(\mu+i0)}\,, \tag{17}$$

which is the sought nonlinear integral equation written in terms of $a_N=\exp(2\pi iNz_N)$ [similarly $a_c=\exp(2\pi iNz_c)$]. Notice that $\bar{a}_N(\lambda)=1/a_N(\bar{\lambda})$. We evaluate the sum in Eq. (9) by a similar procedure, with the result

$$L_N(\beta)\equiv E_N(\beta)-E_c(\beta)=\frac{1}{\gamma}\int_{-\infty}^{+\infty}d\lambda\frac{\sinh(\pi\lambda/\gamma)\cos(\pi\tilde{\beta}/\gamma N)}{\cosh(2\pi\lambda/\gamma)-\cos(2\pi\tilde{\beta}/\gamma N)}\ln\frac{1+a_N(\lambda+i0)}{1+\bar{a}_N(\lambda+i0)}\,, \tag{18}$$

where $E_c(\beta)$ is that part of the energy that follows from the root density (13), that is,

$$E_c(\beta)=-2N\int_{-\infty}^{+\infty}\frac{dk}{k}\frac{\sinh[(\pi-\gamma)k]\sinh[(\gamma-2\tilde{\beta}/N)k]}{\sinh(\pi k)\cosh(\gamma k)}\,. \tag{19}$$

If Eqs. (17) and (18) are analytically continued to the axis $\mathrm{Im}\lambda=\pm\gamma/2$ they assume the same structure as those derived in Ref. [10] by different methods.

Inserting Eqs. (18) and (19) into Eq. (8) yields

$$f(\beta)=E_{XXZ}(\gamma)+\beta^{-1}L(\beta)\,, \tag{20}$$

where $E_{XXZ}(\gamma)$ is the ground-state energy per site of (1), namely,

$$E_{XXZ}(\gamma)=2J\left[\cos\gamma-\sin\gamma\int_{-\infty}^{+\infty}dk\frac{\sinh[(\pi-\gamma)k]}{\sinh(\pi k)\cosh(\gamma k)}\right], \tag{21}$$

while $L(\beta)$ is the $N\to\infty$ limit of (18), that is,

$$L(\beta)=\mathrm{Im}\int_{-\infty}^{+\infty}d\lambda\frac{\ln[1+a(\lambda+i0)]}{\gamma\sinh[\pi(\lambda+i0)/\gamma]}\,. \qquad (22)$$

Finally, $a(\lambda)\equiv a_\infty(\lambda)$ satisfies

$$-i\ln a(\lambda)=-\frac{2\pi\tilde\beta}{\gamma\sinh(\pi\lambda/\gamma)}+2\int_{-\infty}^{+\infty}d\mu\,p(\lambda-\mu)\,\mathrm{Im}\ln[1+a(\mu+i0)]\,. \qquad (23)$$

Thus the calculation of the free energy has been reduced to the problem of solving the single nonlinear integral equation (23) and then evaluating the integral (22).

It is possible to rewrite Eq. (23) in the alternative form

$$z(\lambda)=\tilde\beta q(\lambda)+\frac{1}{2\pi^2}\mathrm{Im}\int_{-\infty}^{+\infty}d\mu\,\phi'(\lambda-\mu,\gamma)\times\ln\cos[\pi z(\mu+i0)]\,, \qquad (24)$$

where

$$q(\lambda)=\frac{1}{\pi}\left[\frac{\sinh(2\lambda)}{\cosh(2\lambda)-\cos(2\gamma)}-\coth\lambda\right], \qquad (25)$$

and $z(\lambda)\equiv(1/2\pi i)\ln a(\lambda)$ is related to the original counting function,

$$z(\lambda)=\lim_{N\to\infty}N[z_N(\lambda)-\tfrac12\,\mathrm{sgn}\lambda]\quad(\lambda\text{ real})\,. \qquad (26)$$

By the residue theorem we then obtain

$$z(\lambda)=\tilde\beta q(\lambda)-\frac{1}{2\pi}\sum_{j\in\mathbb{Z}}[\phi(\lambda-\xi_j,\gamma)-\phi(\lambda,\gamma)]\,, \qquad (27)$$

where the real numbers ξ_j, defined by $z(\xi_j)=j-\frac12$, $j\in\mathbb{Z}$, can be identified with the $N\to\infty$ limit of the original BA roots $\lambda_1,\lambda_2,\ldots,\lambda_N$. The new algebraic BA equations $z(\xi_j)=j-\frac12$ embody all the information about the XXZ thermodynamics. Equation (27) shows that $z(\lambda)$ has periodicity $i\pi$ and has, as unique singularity on the real axis, a simple pole at the origin with residue $-\tilde\beta/\pi$.

Let us now study $f(\beta)$ for high temperatures. We set

$$z(\lambda)=\sum_{k=1}^{\infty}\tilde\beta^k b_k(\lambda)\,, \qquad (28)$$

which inserted in Eq. (24) yields, up to fourth order in $\tilde\beta$,

$$b_1(\lambda)=q(\lambda),\quad b_2(\lambda)=(1/4\pi)\phi''(\lambda,\gamma),\quad b_3(\lambda)=0\,, \qquad (29)$$

$$b_4(\lambda)=\frac{1}{6\pi}\left[\frac13-\frac{1}{\sin^2\gamma}\right]\phi''(\lambda,\gamma)+\frac{1}{144\pi}\phi''''(\lambda,\gamma)\,.$$

Then, from Eqs. (20) and (22),

$$f(\beta)=-\beta^{-1}\ln2+J\cos\gamma-\beta J^2(1+\tfrac12\cos^2\gamma)+\beta^2J^3\cos\gamma+O(\beta^3)\,, \qquad (30)$$

which indeed agrees with the high-T expansion [2]. We want to remark that Eq. (24) generates the functions $b_k(\lambda)$ *recursively*, with easy integrations which involve only delta functions and derivatives thereof. It is indeed a very efficient way to recover the high-temperature expansion from the BA solution [11].

We shall consider now the low-temperature regime. When $\beta\gg1$ Eqs. (23) and (24) indicate that $z(\lambda)\sim\tilde\beta q(\lambda)$, implying that $L(\beta)\sim O(\beta^{-1})$. The higher-order corrections come from values of λ larger than $(\gamma/\pi)\ln\beta$, where the assumption $\ln a\sim O(\beta)$ does not hold anymore. It is convenient to introduce the new function $A(x)=a[x+(\gamma/\pi)\ln(4\pi\tilde\beta/\gamma)]$, in terms of which we have, in the $\beta\to\infty$ limit,

$$L(\beta)=\frac{2}{\pi\tilde\beta}\int_{-\infty}^{+\infty}dx\,e^{-\pi x/\gamma}\,\mathrm{Im}\ln[1+A(x+i0)]\,, \qquad (31)$$

$$-i\ln A(x)=-e^{-\pi x/\gamma}+2\int_{-\infty}^{+\infty}dy\,p(x-y)\,\mathrm{Im}\ln[1+A(y+i0)]\,. \qquad (32)$$

To calculate the integral (31), we can use the lemma.

Lemma.— Assume that $F(x)$ satisfies

$$-i\ln F(x)=\varphi(x)+\int_{x_1}^{x_2}dy\,p(x-y)\,\mathrm{Im}\ln[1+F(y+i0)]\,, \qquad (33)$$

where $\varphi(x)$ is real for real x. Then

$$\mathrm{Im}\int_{x_1}^{x_2}dx\,\varphi'(x)\ln[1+F(x+i0)]=\tfrac12\,\mathrm{Im}[\varphi(x_2)\ln(1+F_2)-\varphi(x_1)\ln(1+F_1)]+\tfrac12\,\mathrm{Re}[l(F_1)-l(F_2)]\,, \qquad (34)$$

where $F_{1,2}=F(x_{1,2})$ and l is a dilogarithm function,

$$l(t)\equiv\int_o^t du\left[\frac{\ln(1+u)}{u}-\frac{\ln u}{1+u}\right]. \qquad (35)$$

To prove the lemma we consider the relation

$$l(F_1)-l(F_2)=\int_{x_1}^{x_2}dx\left\{\ln[1+F(x)]\frac{d}{dx}\ln F(x)-\ln F(x)\frac{d}{dx}\ln[1+F(x)]\right\},$$

and then use Eq. (33) and its derivative to substitute $\ln F(x)$ and $d\ln F(x)/dx$. After a little algebra this yields Eq. (34) (related identities were used in Ref. [10]).

The integral in Eq. (31) may now be exactly calculated by invoking the lemma with $F(x)=A(x)$, $\varphi=-\exp(-\pi x/\gamma)$, and $x_1=-\infty$, $x_2=+\infty$. We have $A(x_1)=0$, $A(x_2)=1$, and

$$2\,\mathrm{Im}\int_{-\infty}^{+\infty}dx\,e^{-\pi x/\gamma}\ln[1+A(x+i0)]=-(\gamma/\pi)l(1)=-\pi\gamma/6\,.$$

Then the free energy for low temperature reads

$$f(\beta)=E_{XXZ}(\gamma)-(\gamma/6J\sin\gamma)\beta^{-2}+o(\beta^{-2}), \quad (36)$$

in perfect agreement with Refs. [2,5,12]. The quantitative determination of the $o(\beta^{-2})$ terms is more involved. Qualitatively, however, it is rather easy to establish, from the asymptotic behavior of the kernel $p(\lambda-\mu)$ in Eq. (23) and from the $i\pi$ periodicity implied by Eq. (27), that these higher-order terms must be integer powers of $T^{\pi\gamma}$ and $T^{\gamma/(\pi-\gamma)}$.

Let us now consider the problem of calculating the (properly subtracted) ground-state energy $E(L)$ of 2D integrable massive models of QFT in a finite 1-volume L. In the context of perturbed conformal field theory, $E(L)$ is known as the ground-state scaling function. On the other hand, by the Euclidean symmetry of the functional integral, $L^{-1}E(L)$ is just the free energy density at temperature $T=L^{-1}$. We use the lattice regularization provided by the light-cone approach [7]. For the six-vertex model, the ground-state energy, on a ring of length L formed by N sites, takes a form similar to Eq. (18):

$$E_N(L)-E_c(L)=\frac{N}{\gamma L}\operatorname{Im}\int_{-\infty}^{+\infty}d\lambda\left[\operatorname{sech}\frac{\pi}{\gamma}(\lambda-\Theta)-\operatorname{sech}\frac{\pi}{\gamma}(\lambda+\Theta)\right]\ln[1+a_N(\lambda+i0)], \quad (37)$$

where

$$E_c(L)=\frac{N^2}{L}\left[-2\pi+\int_{-\infty}^{+\infty}d\lambda\frac{\phi(\lambda+2\Theta,\gamma/2)}{\gamma\cosh(\pi\lambda/\gamma)}\right], \quad (38)$$

and $a_N(\lambda+i0)$ obeys an equation like (17) with the new source term

$$-i\ln a_c(\lambda)=2\arctan\frac{\sinh(\pi\lambda/\gamma)}{\cosh(\pi\Theta/\gamma)}. \quad (39)$$

The L dependence in these expressions is hidden in the light-cone parameter Θ, which tends to infinity, in the continuum limit $N\to\infty$, as [7]

$$\Theta=(\gamma/\pi)\ln(4N/mL), \quad (40)$$

with m, the physical mass scale, held fixed. Define now

$$\epsilon(\lambda)=-\lim_{N\to\infty}\ln a_N(\gamma\lambda/\pi). \quad (41)$$

Then from Eqs. (17) and (42)-(45) we find

$$E(L)\equiv\lim_{N\to\infty}[E_N(L)-E_c(L)]$$
$$=-m\int_{-\infty}^{+\infty}\frac{d\lambda}{\pi}\sinh\lambda\operatorname{Im}\ln[1+e^{-\epsilon(\lambda+i0)}] \quad (42)$$

and

$$\epsilon(\lambda)=mL\sinh\lambda+2\int_{-\infty}^{+\infty}d\mu\,G(\lambda-\mu)\operatorname{Im}\ln[1+e^{-\epsilon(\mu+i0)}], \quad (43)$$

where $G(\lambda)=(\gamma/\pi)p(\gamma\lambda/\pi)$. Thanks to the correspondence between the light-cone six-vertex model and the sine-Gordon (or massive Thirring) model [7], Eqs. (42) and (43) give the ground-state scaling function $E(L)$ of the sine-Gordon field theory on a ring of length L (m denotes fermion mass). This is a rigorous and simpler alternative to the standard thermodynamic Bethe ansatz [4,13]. Notice that all UV-divergent terms are contained in $E_c(L)$ [cf. Eq. (38)]. $E_c(L)$ also contains the finite, scaling part $m^2L\cot\pi^2/2\gamma$ (also found in Ref. [14] by completely different means), which gives the ground-state energy density in the $L\to\infty$ limit.

For small values of mL the calculation closely parallels that for low T in the thermodynamics of the XXZ chain. Shifting $\lambda\to\lambda-\ln mL$ and applying the lemma leads to

$$E(L)\overset{ML\to 0}{=}-\pi/6L+O(1), \quad (44)$$

showing the expected value $c=1$ for the UV central charge. The large-mL regime easily follows from Eq. (43) by iteration. One finds the behavior typical of a massive quantum field theory,

$$E(L)\overset{L\to\infty}{\simeq}-\frac{2m}{\pi}K_1(mL)+O(e^{-2mL}), \quad (45)$$

with $K_1(z)$ the modified Bessel function of order 1. This nonanalytic behavior in $T=L^{-1}$ corresponds to the solitons contribution in the sine-Gordon language.

We would like to remark once more that, contrary to the traditional thermal Bethe ansatz [2-4], our approach does not rely on the string hypothesis for the structure of the roots of the BA equations. This makes our approach definitely simpler. Most notably, the whole construction of the thermodynamics no longer depends on whether γ/π is a rational or not, unlike the stringy approach. Furthermore, it should be stressed that Eqs. (42) and (43) are valid at the full quantum level. In this respect, it would be interesting, in further developments, to compare them to similar equations previously derived in the classical and semiclassical limits [15].

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(a)Electronic address: 37998::destri. Mail address: Dipartimento di Fisica, Viale delle Scienze, 43100 Parma, Italy.

(b)Mail address: LPTHE, Tour 16, 1er étage, Université Paris VI, 4 Place Jussieu, 75252, Paris CEDEX 05, France.

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