# Asymmetric Scattering and Diffraction of Two-Dimensional Electrons at Quantized Tubes of Magnetic Flux

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(Received 1 June 1992)

The Hall conductivity of a high-mobility two-dimensional electron gas (2DEG) has been investigated in a distribution of quantized magnetic flux tubes (vortices) formed at a type-II superconducting "gate" layer. A pronounced suppression of the Hall effect was observed for long Fermi wavelengths (as compared to the submicron vortex size) indicating a situation where electrons are diffracted by the flux quanta. In contrast, for shorter Fermi wavelengths the Hall conductivity has been found to be insensitive to the extreme inhomogeneity of the magnetic field and determined by the average field.

PACS numbers: 72.10.Fk, 73.40.Qv, 73.50.Jt

There has been much interest in the last few years in a hybrid system in which an extremely inhomogeneous magnetic field created by a type-II superconductor is projected down onto a two-dimensional electron gas (2DEG) below [1-7]. This system can be fabricated by gating a GaAs/GaAlAs heterostructure 2DEG with a type-II superconducting film. An applied magnetic field is segregated within and near the superconductor into a distribution of magnetic flux tubes (vortices) with characteristic diameter $d \cong 2\lambda$ ($\lambda \cong 0.1$ $\mu$m and is the magnetic-field penetration depth). A point of fundamental interest here is the fact that the size of the magnetic flux tubes can be much smaller than transport relaxation lengths, in which case a vortex may be considered as a magnetic string, i.e., a flux tube of negligibly small cross-sectional area. Nonlocal weak localization [2,3] and electrodynamic coupling between a superconductor and the 2DEG [4] were recently reported.

In this Letter we report *ballistic* electron transport through a random distribution of vortices. When one considers such a ballistic transport it is not clear *a priori* how to obtain transport coefficients from the rather complicated electron motion. Fortunately, the picture is strikingly simplified when account is taken of the fact that electrons are only influenced by the vortices along a small part of their paths and, hence, vortices can be considered as additional scatterers introduced into the 2DEG [5,6,8]. We wish to emphasize two fundamental features of these scatterers: their asymmetric and essentially quantum character. The asymmetry results from the vector action of the magnetic field while the quantum character is clearly highlighted by the equivalence between the quasiclassical angle of deflection of the electrons due to the Lorentz force, $\beta \cong \lambda_F/d$ (see below), and the characteristic angle of diffraction of a wave with wavelength $\lambda_F$ at an obstacle of size $d$. In addition, scattering at the vector potential outside the magnetic-field region should be taken into account [6,7,9]. When all these features are considered a question which naturally arises is whether the system exhibits a Hall effect and, if so, what is its magnitude.

The scattering efficiency at the flux tubes is a function of $\lambda_F/d$ [6,9] and we have tried to vary this parameter over a maximum range in our experiment. To this end, a set of $GaAl_{0.3}As_{0.7}/GaAs$ heterostructures with decreasing electron concentrations, $n=(4.13,1.76,1.24,0.365)\times 10^{11}$ cm$^{-2}$ (in the dark), and lead superconducting gates with a very small value of $\lambda$ were employed. The Fermi wavelength varied from 40 to 130 nm to achieve $\lambda_F/d \cong 0.7$ which is probably currently the maximum feasible value of this ratio. The electron mobilities were in the range of 40–100 m$^2$/V s at 1.3 K and the mean free path of the electrons in all cases exceeded 2 $\mu$m, and for several samples exceeded 10 $\mu$m. Each sample used in the experiment contained two identical structures with Hall bar geometry fabricated on the same chip [see inset in Fig. 1(a)]. A superconducting film ($\cong 0.1$ $\mu$m in thickness) was deposited on one of the samples, while the other was used as a test sample to compare results with the case of a uniform magnetic field. To avoid ambiguity arising from an inevitable difference between 2D electron concentrations under the deposited metal gate and under the free surface, a 3-nm layer of aluminum was initially deposited over the whole structure, providing the same surface conditions for both Hall bars. It was also important to minimize the effect of spatial broadening of the vortices when they emerge from the superconductor. For the heterostructures used with a distance between the surface and the 2DEG of 70–85 nm, our calculations based on the Clem model of a vortex [10] yielded a vortex diameter at the 2DEG of approximately 200 nm (the penetration depth in the lead films was $\cong 60$ nm at 1.3 K with the upper critical field $\cong 1200$ G). To avoid macroscopic inhomogeneity of the magnetic field inside the samples because of vortex pinning, we performed measurements in the so-called "field-cooling" regime, i.e., every change of the applied magnetic field was followed by heating the sample to a temperature above $T_c=7.2$ K and then cooling it in the magnetic field down to 1.3 K. When the applied field was swept, pinning prevented vor-

 

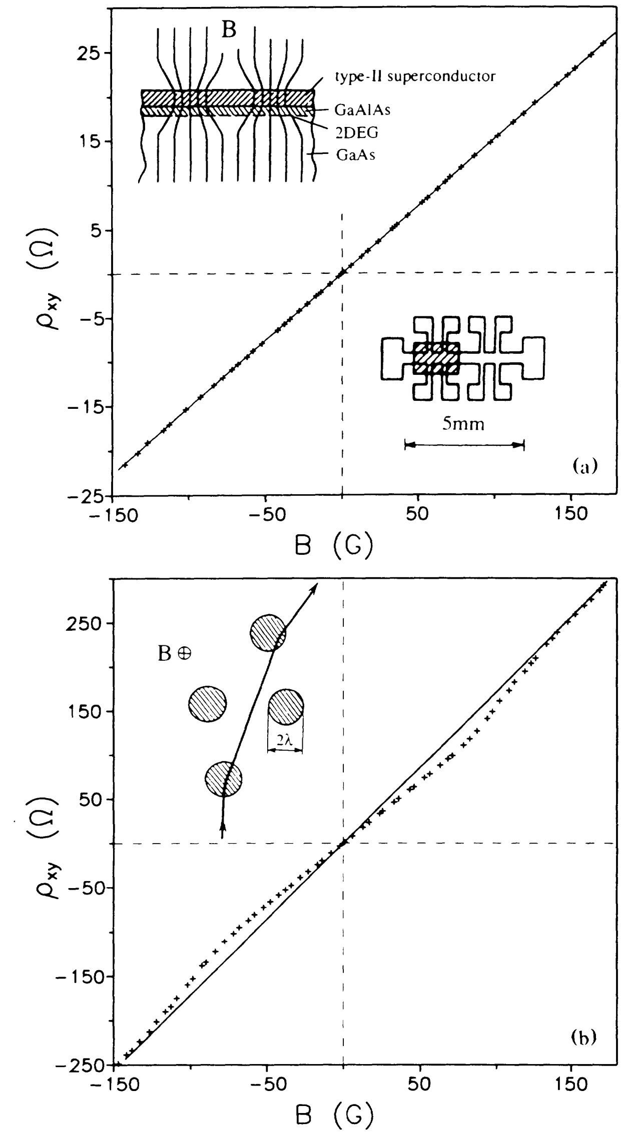


FIG. 1. The Hall resistivity of the 2DEG in the extremely inhomogeneous magnetic field of superconducting vortices for samples with electron concentrations of (a) $n=4.13\times10^{11}$ $cm^{-2}$ and (b) $n=3.65\times10^{10}$ $cm^{-2}$. The solid line represents $\rho_{xy}$ in the uniform magnetic field for the same 2DEGs. The insets to Fig. 1(a) show the experimental geometry. The inset to Fig. 1(b) shows schematically the electron motion through the vortices.

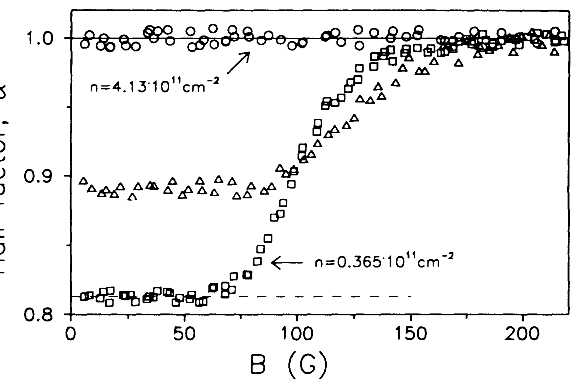


FIG. 2. The Hall factor in the distribution of vortices for different electron concentrations. Low-field parts of the curves correspond to the case of well-separated vortices. Vortices overlap in higher external fields providing uniform magnetic field at the 2DEG ($\alpha=1$). The central curve with $n=5.9\times10^{10}$ $cm^{-2}$ was obtained by illumination of the sample with $n=3.65\times10^{10}$ $cm^{-2}$.

tices from spreading easily into the film and caused large macroscopic field gradients. For more experimental details we refer to our recent papers [11].

Figures 1(a) and 1(b) show the Hall resistivity of samples with two strongly different electron concentrations. The small crosses represent the structure with the superconducting gate, and each cross corresponds to a new value of the external field $B$ in which the sample was cooled. The solid lines are a $\rho_{xy}$ sweep for the ungated sample. Perfect agreement between the cases of uniform and inhomogeneous fields is seen in Fig. 1(a) for $n=4.13\times10^{11}$ $cm^{-2}$ ($\lambda_F/d\cong0.2$). In contrast, a pronounced suppression of the Hall effect upon concentration of the magnetic flux into the flux tubes is observed in Fig. 1(b) for a sample with very low electron concentration ($n=3.65\times10^{10}$ $cm^{-2}$) when $\lambda_F/d$ is approximately 0.7. In applied magnetic fields $B$ less than about 70 G, where the vortices are spatially well separated, the slope of the curve is considerably shallower than in the uniform field. At higher fields, curves for the sample with the superconducting gate and for the ungated one gradually merge, leading to exactly the same Hall effect in fields above 150 G. This merging is a simple result of the fact that in large external fields the distance between individual vortices, $L(\mu m)\cong5[B(G)]^{-1/2}$, has decreased to a value comparable with the vortex diameter and magnetic fields due to adjacent vortices strongly overlap. Consequently the inhomogeneity rapidly smears out, yielding a somewhat uniform magnetic field at the 2DEG although superconductivity in the gate has not been destroyed. In addition to the difference in the Hall conductivity for the gated and the ungated samples, the change of the slope of $\rho_{xy}(B)$ is the clearest evidence of the suppression of the Hall effect in the microinhomogeneous field.

The high degree of precision and reproducibility of the data allows us to present the dependence of the Hall constant on the external magnetic field (Fig. 2) where the above features are seen more distinctly. By analogy with the case of the nondegenerate electron gas [12], it is helpful to define the Hall factor, $\alpha=\rho_{xy}/(B/ne)$, where $B/ne$ is the conventional Hall resistivity in a uniform field. Clearly, $\alpha$ may be viewed as the Hall constant normalized to its value in the homogeneous field. When the vortices are well separated, the value of $\alpha$ indicates the efficiency of asymmetric scattering for electrons at vortices. For short Fermi wavelengths, the Hall factor was equal to unity at all external fields within our experimental accuracy. For longer $\lambda_F$, $\alpha$ was substantially smaller and decreased to $\alpha \cong 0.8$ for the sample with the minimum electron concentration. At high fields $\alpha$ tended to its value in the uniform magnetic field for all samples. Strong evidence that the Hall factor is determined by the electron concentration and not by another uncontrolled variable was obtained by changing the concentration in the 2DEG. To achieve this, the GaAlAs/GaAs heterostructures were illuminated from the rear with an infrared LED. We note that the conventional method of applying a potential to the gate did not provide sufficient stability and reproducibility during the multiple thermal cycles. The middle curve in Fig. 2 with $\alpha \cong 0.9$ shows $\rho_{xy}$ behavior for the sample with the unilluminated concentration $n = 3.65 \times 10^{10}$ cm$^{-2}$, after a short excitation. The electron concentration in the illuminated sample increased to $n = 5.9 \times 10^{10}$ cm$^{-2}$, leading to an increase of the value of $\alpha$. The dependence of the Hall factor on the electron concentration is shown in Fig. 3 where data for all structures are collated, including data obtained with a partial or full illumination. It can be seen that ballistic electrons do not "feel" the inhomogeneity of the field above $n \cong 3 \times 10^{11}$ cm$^{-2}$ ($\lambda_F/d \cong 0.25$), but a rapid suppression of the Hall constant arises for smaller values of $n_{2D}$. Unfortunately, we did not manage to observe a further quenching since larger values of $\lambda_F/d$ are not currently available.

To explain the obtained results we shall view the vortices as scatterers introduced into the 2DEG [5,6]. Because of the strong pinning in our samples the vortices did not form a regular lattice and were randomly distributed [11]. Scattering asymmetry (i.e., preferred scattering of

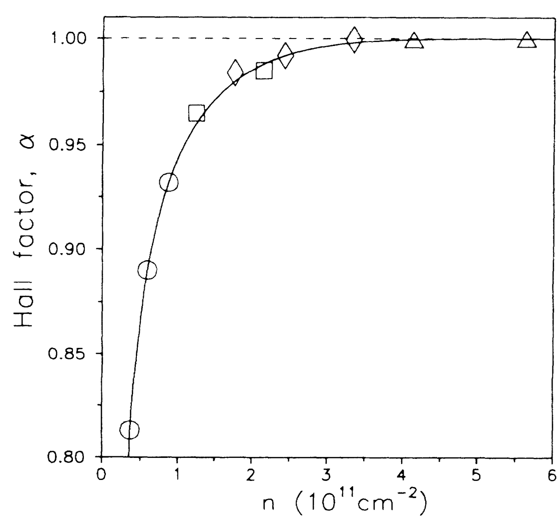


FIG. 3. Dependence of the Hall factor on the electron concentration. For a large concentration the Hall effect corresponds exactly to the case of uniform field. The solid line is a guide to the eye.

electrons in one particular direction) should lead to a Hall-like component in the 2DEG resistivity. A similar effect is known for systems with magnetic impurities which also exhibit asymmetric scattering [12]. The concentration of additional scatterers in our system, $p$, can be changed by changing the external magnetic field $B$. The conservation of magnetic flux yields $p = B/\phi_0$ where $\phi_0 = h/2e$ is the magnetic flux quantum. Since the resistivity is proportional to the concentration of scatterers, and hence is proportional to $B$, the Hall resistivity due to scattering at vortices may be written in the form $\rho_{xy} = \alpha B/ne$ where the Hall factor $\alpha$ depends on the details of the scattering. For arbitrary values of $\lambda_F/d$ and the enclosed flux $\phi$, there is no known solution for the problem of electron scattering at the flux tube. However, for $\phi = \phi_0$ an exact solution has been obtained in two opposing limits, $\lambda_F \ll d$ and $\lambda_F \gg d$, yielding $\alpha = 1$ [6] and $\alpha = 0$ [9], respectively. We explain these results qualitatively below.

In the case $\lambda_F \ll d$, a quasiclassical approach yields the correct result. The Lorentz force, $F = m\Delta v/\Delta t = ev_F B_v(r)$, acting on an electron during the time $\Delta t \cong d/v_F$ deflects it by the angle $\beta = \Delta v/v_F \cong 2\lambda_F/\pi d$ if account is taken of the fact that the magnetic field inside the vortex is $B_v \cong B_0 = \phi_0/\pi\lambda^2$. Each scattering event turns the electron in the same direction, resulting in its cyclotron-type motion. Along the cyclotron trajectory the electron passes through a large number of vortices ($\cong 2\pi/\beta \gg 1$) providing an effective averaging of the magnetic field "seen" by it. For a more quantitative result, the Hall force in the field term of the Boltzmann kinetic equation must be calculated: $F_H = p\int \Psi^* \hat{F}_y \Psi \, d^2r$, where $\Psi$ is the wave function of a scattering electron and $\hat{F}_y = e\hat{v}_x B_v$ is the operator for the Lorentz force. In the quasiclassical small-angle scattering limit the above integral is reduced to $pev_F \int B_v \, d^2r = ev_F B$ since for small-angle scattering $v_x \cong v_F$, and $\int B_v \, d^2r = \phi_0 = B/p$. Thus, no difference exists between the considered case and the case of uniform field, i.e., $\alpha = 1$. The validity of the quasiclassical approach for this essentially quantum problem relates to the quantized value of the magnetic flux [6] since quantum corrections to the quasiclassical scattering efficiency depend on $\phi$ as $\sin(\pi\phi/\phi_0)$ [13]. Note that for the case of arbitrary $\phi$, deviations of $\alpha$ from unity are to be expected [6,14].

For $\lambda_F/d \cong 1$ the small-angle scattering approximation breaks down and the quasiclassical approach is no longer valid (note that the quasiclassical angle $\beta$ diverges in the limit of long wavelengths and is meaningless). To demonstrate in which direction the deviation of the Hall factor from unity is expected for large $\beta$, we consider the case of $\lambda_F \gg d$. This limit is related to the well-known problem of electron diffraction at a magnetic string [9]. For large $\lambda_F$, the asymmetric deflection due to the Lorentz force is negligibly small because of the tiny magnetic-field-containing area. The dominant scattering in this case arises from the vector potential of the magnetic field (the Aharonov-Bohm effect) and is determined by the interference between electron partial waves passing on the left and on the right of the string. As usual the phase shift varies periodically with the enclosed flux and does not perturb the electron wave front for integer numbers of flux quanta, $Nh/e$. For superconducting flux quanta, $h/2e$, the scattering is still important but does not exhibit asymmetry with respect to the direction of the incident electron wave, leading to $\alpha=0$ [9,15]. Consequently, we attribute the observed decrease of $\alpha$ to the large-angle scattering at the vortices and a gradual transition from the quasiclassical situation to the case of a magnetic string.

Although there is no theory for finite $\lambda_F/d$, one might expect that diffraction corrections to the quasiclassical value of the Hall coefficient should vary as the square (or other well behaved) power of the small parameter $\lambda_F/d$. However, such a dependence seems inconsistent with the experimental results in Fig. 3 where a square law would correspond to a straight line since $n \sim \lambda_F^{-2}$. The more rapid quenching of $\alpha$ probably indicates that in the quasiclassical approximation the first-order quantum correction to order $(\lambda_F/d)^2$ contains the term $\sin(\pi\phi/\phi_0)$ [14] and therefore is zero in our case. The latter indicates that perturbation calculations cannot be used here and a transition region between the cases of $\lambda_F/d \gg 1$ and $\ll 1$ is not necessarily described by a power law of a small parameter.

In moderate magnetic fields, the mean free path of electrons due to scattering on vortices, $l=L^2/d$, can be less than the electron coherence length $L_\varphi$, which is several microns in our structures at 1.3 K (at 25 G, $l \cong 5$ $\mu$m). In higher fields an electron is scattered by vortices several times before losing phase coherence. Despite this the experimental data in Fig. 2 do not show any changes in $\alpha$ at the crossover between the cases of single and multiple vortex scattering. This is also confirmed by the fact that $\alpha$ did not change with increasing temperature when $L_\varphi$ was substantially reduced. We expect that the multiple scattering process is a second-order process since it does not appear to change $\alpha$ for both considered cases $\lambda_F/d \gg 1$ and $\ll 1$.

In conclusion, we have realized a system in which the Hall resistivity results from the scattering of 2D electrons at submicron tubes of magnetic flux. For short Fermi wavelengths the Hall effect was dictated by the average magnetic field in the system in accordance with quasiclassical considerations. In contrast a pronounced decrease of the Hall factor has been observed for low electron concentrations indicating a rapid quenching of the Hall effect in the limit of long Fermi wavelengths where the interference of electrons is important.

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