Galactic Dynamo and Nucleosynthesis Limit on the Dirac Neutrino Masses

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(Received 11 May 1992)

The dynamo mechanism for generating the magnetic fields of galaxies requires the existence of a primordial seed field, which induces oscillations between the left- and right-handed Dirac neutrinos. We consider the excitation of the right-handed neutrinos in the early Universe due to these oscillations and show that nucleosynthesis sets stringent upper limits on the size of the neutrino magnetic moments: $\mu_\nu \lesssim 6.5\times 10^{-34}\mu_B[1\ \mathrm{G}/B_{\rm seed}(T_{\rm now})]$. In the standard model these limits can be translated to a constraint on the neutrino masses, given by $\sum m_{\nu_i} \lesssim (2.1\times 10^{-15}\ \mathrm{eV})[1\ \mathrm{G}/B_{\rm seed}(T_{\rm now})]$. We also set upper limits on the transition magnetic moments.

PACS numbers: 98.80.Cq, 12.15.Ff, 14.60.Gh, 98.60.Jk

Galactic magnetic fields of the order 10^{-6} G have been observed in a number of galaxies. A possible way of understanding the existence of such fields is through a galactic dynamo mechanism, which amplifies a weak but coherent primordial field of order 10^{-21} G or larger on the comoving scale of 100 kpc [1]. Recent studies [2,3] indicate that although such a seed field is very weak, it must have originated, provided there is no large scale amplification mechanism, from a much stronger field in the very early Universe. For example, if the seed field is generated in the electroweak phase transition because of the cosmological variations in the gradients of the vacuum expectation value of the Higgs field, it has been estimated by Vachaspati [2] that at the electroweak phase transition, when $T=T_{\rm EW}\simeq M_W$, the random magnetic field would have the strength $B\simeq 10^{23}$-10^{24} G on the scale of the correlation length $L\simeq 1/M_W$. Because of the random walk performed by the fields involved, and due to the expansion of the Universe, such a huge magnetic field will, however, give rise to a background field on the scale 100 kpc of only $B\simeq 10^{-30}$ G. Thus, if no large scale amplification mechanism exists, this result implies that in the early Universe the seed field must have been very large indeed [3].

In the early Universe large magnetic fields may have played a very important role. In particular, a primordial magnetic field will cause left-handed Dirac neutrinos to oscillate into right-handed neutrinos [4], and hence there exists a possibility of thermalizing the right-handed neutrino ensemble before the freeze-out of the neutron-to-proton ratio at $T\simeq 0.7$ MeV. Right-handed neutrinos would then count as full additional degrees of freedom during the nucleosynthesis, with disastrous consequences for the helium abundance, which allows for only 0.3 extra neutrinos [5]. The purpose of the present paper is to study the nucleosynthesis constraints on primordial magnetic fields and the magnetic moments of the standard model neutrinos. We shall show that for realistic seed fields, one obtains substantial restrictions on the size of both the direct and transition magnetic moments of Dirac neutrinos, which in the standard model can be directly translated to bounds on the neutrino masses.

In order to obtain a primordial background field of a relevant size for the galactic dynamo effect to work, one must require that at the electroweak phase transition on the scale of one correlation length $1/M_W$

$$B\simeq cM_W^2/e\simeq c\times(10^{24}\ \mathrm{G})\,, \qquad (1)$$

where $c>1$ is a phenomenological parameter to be fitted in such a way that the present seed field emerges [3]. From (1) one may then find that in the early Universe, and over the scale of N correlation lengths, the random magnetic field reads as

$$B(T)\simeq\frac{cM_W^2}{eN}\left(\frac{T}{T_{\rm EW}}\right)^2, \qquad (2)$$

where the correlation length $L=N/T$. Note that in (2) the magnetic field depends on temperature also through N because of the randomness of the field.

Theoretically c should emerge from some cosmological particle physics considerations, but in the absence of any detailed mechanism, we shall here adopt the point of view that c is a parameter of unknown origin. In order to obtain a primordial magnetic field of the order 10^{-21} G on the scale 100 kpc at present, Eq. (2) implies that $c\simeq 3.7\times 10^9$. In actual model simulations of the dynamo effect [6], however, a seed field of about 10^{-17}-10^{-18} G is actually required to reproduce the observed galactic fields. Moreover, recent observations [7] of a spiral galaxy with $z=0.395$ suggest that the seed field must indeed be about 10^{-17} G, indicating the need for huge fields in the early Universe, with

$$c\simeq 3.7\times 10^{12}. \qquad (3)$$

In the present paper we shall not adopt any definite value for c. Hence Eq. (3) should be considered as a point of reference only, but it should be kept in mind that more than likely the magnetic field in the early Universe has been very large. Note that although within one correlation volume such a magnetic field is huge, when integrated over the volume of the whole Universe its contribution to the energy density remains small. This follows directly from (2) and is a consequence of the randomness of the field.

Consider now left-handed neutrinos propagating in the background of the field given in Eq. (2). In the absence of collisions, the probability for finding ν_R at time $t+\Delta t$ from a state initially prepared to be ν_L is given by

$$P_{L\to R}=[x^2/(x^2+V^2)]\sin^2[\tfrac{1}{2}(x^2+V^2)^{1/2}\Delta t]\,, \quad (4)$$

with

$$x=2\mu_\nu B_\perp,\quad V=\sqrt{2}G_F n_\gamma(T)\left[\Delta L-A\frac{T^2}{M_W^2}\right], \quad (5)$$

where $B_\perp$ is the magnetic field perpendicular to the neutrino momentum, which we may identify with the random seed field, μ_ν is the magnetic moment of the neutrino, n_γ is the photon density, $\Delta L\simeq 10^{-9}$ is the small lepton asymmetry (which can be neglected in the present case), and V accounts for the thermal background at $T\ll M_W$ [8,9], with A a constant depending on temperature and on the neutrino species; for ν_e, $A\simeq 55.0$. Here and in what follows, we shall neglect the masses of the neutrinos. In previous attempts [10] to estimate the spin flip in an external magnetic field, V was not taken correctly into account; as we shall see, V plays a crucial role in determining the flip rate.

As the left-handed neutrino propagates, it will scatter forward off the thermal background and the magnetic field, which will separate left-handed neutrinos from the right-handed ones; these will continue to propagate forward with a relative abundance determined by the probability (4). One may also say that a nonforward scattering will destroy the coherent evolution of the phase of the wave function, so that it constitutes a measurement of the spin contents of the left-handed neutrino ensemble. Hence the relevant distance scale Δt is the thermally averaged free path L_w of ν_L, which for ν_e reads $(T<m_\mu)$

$$L_w^{-1}=\Gamma_w=4.0G_F^2T^5\,. \quad (6)$$

(A similar expression can be found for $\nu_{\mu,\tau}$ [9].) When $T\gtrsim 1$ MeV, $\Gamma\gg H$, where $H=(4\pi^3 g_{\rm eff}/45)^{1/2}T^2/M_{\rm Pl}$ is the Hubble expansion rate. Hence we may neglect the expansion of the Universe and write simply $\Delta t=L_w$.

We may note that $VL_w=63.4\gg 1$. It follows that, over the length scale L_w, the probability (4) will average out to be

$$\langle P_{L\to R}\rangle=\frac{1}{2}\frac{x^2}{x^2+V^2}\,, \quad (7)$$

where the average magnetic field as seen by ν_L is given by Eq. (2) with $N=L_w/T$.

Right-handed neutrinos will be brought into equilibrium if their production rate $\Gamma_{L\to R}$ is larger than the expansion rate of the Universe, or

$$\frac{\Gamma_{L\to R}}{H}=\frac{\langle P_{L\to R}\rangle\Gamma_w}{H}\gtrsim 1\,. \quad (8)$$

It is easy to see that the ratio (8) grows as temperature increases. Thus, in order not to destroy the successful nucleosynthesis prediction of helium abundance, we must require that ν_R dropped out of equilibrium before the QCD phase transition, so that their number density was diluted by the subsequent entropy production. Hence the inequality (8) must have been violated. Setting $T\simeq 200$ MeV in (8) we then find the constraint

$$\mu_\nu\lesssim 2.4\times 10^{-16}\mu_B\left[\frac{10^{12}}{c}\right]=\frac{6.5\times 10^{-34}\mu_B}{B_{\rm seed}(T_{\rm now})/1\ {\rm G}}\,, \quad (9)$$

where μ_B is the Bohr magneton. With minor modifications due to the slightly different interaction rates, the limit (9) is valid for all neutrino species that are stable at nucleosynthesis. With large c, as required for the galactic dynamo to work, it is much more stringent than the present laboratory or astrophysical limits on the neutrino magnetic moments, which typically yield upper limits of the order $(10^{-10}\text{–}10^{-11})\mu_B$ [11].

In the standard model (9) translates to

$$\sum m_{\nu_i}\lesssim 800\ {\rm eV}\left[\frac{10^{12}}{c}\right]=\frac{2.1\times 10^{-15}\ {\rm eV}}{B_{\rm seed}(T_{\rm now})/1\ {\rm G}}\,. \quad (10)$$

With c given in (3), Eq. (10) would represent a constraint on the ν_μ and ν_τ masses much more severe than any earthbound experimental limits. We should also emphasize that the mass limit (10), unlike the well-known cosmological upper limit of about 100 eV, holds also for unstable neutrinos. Of course, unstable neutrinos imply interactions beyond the standard model, which may modify the standard model relation between the magnetic moment and the neutrino masses.

It is also interesting to consider transition magnetic moments between different neutrino flavors. In this case we may encounter the additional feature of resonant conversion. For two flavors the probability analogous to (4) reads [12]

$$P_{\nu_{eL}\to\nu_{lR}}=\frac{x^2}{x^2+(V-\Delta m^2/2E)^2}\sin^2\tfrac{1}{2}[x^2+(V-\Delta m^2/2E)^2]^{1/2}\Delta t\,, \quad (11)$$

where $l=\mu,\tau$ and Δm^2 is the mass difference squared of the two mass eigenstates, $E\simeq\langle E\rangle=3.15T$, and now $x=2\mu_{12}B_\perp$ where μ_{12} is the transition magnetic moment.

According to (11), a resonance is obtained when

$$\Delta m^2=2.2\times 10^{-19}\frac{T^6}{1\ {\rm MeV}^4}\,, \quad (12)$$

or, as we are interested in the region 1 MeV $\lesssim T\lesssim 200$ MeV, for $2\times 10^{-7}\ {\rm eV}^2\lesssim\Delta m^2\lesssim 1.4\times 10^7\ {\rm eV}^2$.

However, as the neutrino propagates the distance L_w, the location of the resonance changes due to the slight cooling of the Universe. The resonance may then be very narrow in the sense that the condition $|V-\Delta m^2/2E|\ll x$

can be maintained only very briefly, during the time $\Delta t_{\rm res} \ll L_w$. In that case a resonant transition will not be visible since we may replace $P_{\nu_{eL}\to\nu_{lR}}$ by $\langle P_{\nu_{eL}\to\nu_{lR}}\rangle$, and the constraint on the transition magnetic moments can be read from Eq. (8) but with μ_ν replaced by μ_{12} in Eq. (9). [For $\Delta m^2 > 1.4\times10^7$ eV^2, V must be replaced by $\Delta m^2/2E$ in (8).]

For the more interesting case of a wide resonance, for which $\Delta t_{\rm res} \gg L_w$, the situation is different. At the resonance $P_{\nu_{eL}\to\nu_{lR}} \simeq \mu_{12}^2 c^2 T^2$ so that

$$\frac{\Gamma_{L\to R}}{H} \simeq 1.2\mu_{12}^2 c^2 \frac{T^5}{1\ \mathrm{MeV}^3}\,. \tag{13}$$

It is easy to find out that $|V-\Delta m^2/2E| \ll x$ implies $\mu_{12}cT^4/(1\ \mathrm{MeV}^3) \gg 40$, so that a wide resonance is obtained whenever $T \gg 12$ MeV, or $\Delta m^2 \gg 1$ eV^2. Equation (13) implies then that in this region

$$\mu_{12} \lesssim 5.1\times10^{-15}\mu_B \left(\frac{1\ \mathrm{eV}^2}{\Delta m^2}\right)^{5/12}\left(\frac{10^{12}}{c}\right)$$
$$= 1.4\times10^{-32}\mu_B \left(\frac{1\ \mathrm{G}}{B_{\rm seed}(T_{\rm now})}\right). \tag{14}$$

Again for c as large as in (3), this represents a very severe constraint on the transition magnetic moments, which for $\Delta m^2 \gtrsim 40^2$ eV^2 is more stringent than the nonresonant limit (9) with μ_ν replaced by μ_{12}.

There is thus an interesting connection between large scale astrophysics and the properties of Dirac neutrinos. In the standard model the present-day galactic magnetic fields provide a handle on the neutrino masses, assuming of course that the dynamo mechanism, with its requirement of a primordial seed field, is the correct explanation for the appearance of the galactic magnetic fields. We were able to limit in particular the masses of the muon and tau neutrino, whether stable or unstable, in a stringent way, as is evident from Eq. (10). In extended models galactic magnetic fields imply important constraints on the size of the magnetic moments, which is of topical interest because of the solar neutrino problem. Clearly, more accurate measurements of the galactic magnetic fields, as well as theoretical elaboration of the dynamo mechanism, would be of considerable interest for particle physics.

K.E. wishes to thank A. Brandenburg for explaining the galactic dynamo mechanism, and K. Kainulainen for many discussions on neutrino oscillations in the early Universe.

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