Localization of Langmuir Waves in a Fluctuating Plasma

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We have made preliminary observations of the spatial localization of an electron plasma wave in a fluctuating plasma. The effect is related to the general wave phenomenon known as Anderson localization which is applicable both quantum mechanically and classically. Our experiment consists of a multipole discharge plasma in which a launched ion-acoustic mode provides a density fluctuation. We have observed an increase in the damping of an additional launched electron plasma wave which follows the predictions of a Mathieu-type equation for the wave amplitude evolution.

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We have performed a simple plasma experiment that shows the relevance of the concept of localization [1] by disorder to the problem of wave propagation in a fluctuating plasma, which has been previously studied numerically [2] and experimentally [3] in the light of mode coupling theory. The experiment consists of a multipole discharge plasma in which a launched ion-acoustic mode provides the density fluctuation. We have observed an increase in the damping of an additional launched electron plasma wave. This damping, which does not involve any dissipative mechanism, is related to the general wave phenomenon known as Anderson localization.

Anderson localization (for a general review on localization, see Thouless [4]) can be described in quite simple terms by considering the propagation of a wave through a static disordered medium in one dimension. As the wave travels through the disordered medium, it undergoes many random scatterings. One might expect that these random scatterings would simply cause the envelope of the wave to randomly distort as the wave propagates through the system. In fact, something more interesting happens. The wave amplitude can exponentially increase as a function of distance along the system and then exponentially decrease, thus forming a spatially localized concentration of wave energy. The disorder of the medium has formed this localized concentration of wave energy in much the same way a resonant cavity enhances the wave energy at its resonant frequency. Similar to what happens in a resonant cavity, the wave energy has been enhanced by the constructive effects of repeated wave scatterings. However, unlike a resonant cavity, these wave scatterings have occurred at many different spatial locations. Anderson localization in one dimension is a robust effect. For almost any realization of the disorder in the one-dimensional medium the wave will exhibit localization somewhere; moreover, localization will occur no matter how weak the one-dimensional disorder is in magnitude.

Although localization was first discussed in the context of solid-state physics, it is in fact a rather general wave phenomenon. In the early 1960's Mott and Twose [5] and Borland [6] discussed the problem of the distortion of the electron Bloch function when it propagates through a randomly distorted one-dimensional potential field rather than through a purely periodic one. In a more classical setting, Nickel [7] studied theoretically and experimentally the transmission of microwaves through a waveguide filled with slabs of randomly varying dielectric constant. Localization effects have been experimentally observed in conductance measurements in thin metal films and wires and in semiconductor channels at low temperature [8], in the propagation of light through liquid suspensions of dielectric spheres [9], in the classical sound vibrations of a system of beads on a wire [10], and in surface water waves in an irregular water channel [11]. Localization has also been invoked to explain positron mobility observations through a gas of helium [12].

Another interpretation of Anderson localization is that, with a probability equal to 1, the wave meets large regions where the potential has a periodicity that implies gaps in the Floquet or Bloch analysis for the given energy. This is precisely the case of the experiment we are going to describe.

Longitudinal electron plasma waves in an unmagnetized plasma are described by an equation of the type

$$\frac{-1}{3v_T^2}\frac{\partial^2\psi}{\partial t^2}=[-\Delta+\mathcal{V}(x,t)]\psi\,, \qquad (1)$$

where ψ is the electric field, and $\mathcal{V}(x,t)=\omega_p^2(x,t)/3v_T^2$. The position x is taken along the direction of propagation of the plane longitudinal wave; $\omega_p^2(x,t)=n(x,t)e^2/\varepsilon_0 m$ is the square of the plasma frequency and is proportional to the plasma density; and $v_T=(\kappa T/m)^{1/2}$ is the electron thermal velocity. We consider the case where randomness is produced by a one-dimensional spectrum of nondispersive ion-acoustic waves: The random density is moving in the plasma with a constant velocity $c_s=(\kappa T/M)^{1/2}$ (M is the ion mass) much smaller than v_T. The wave equation can then be rewritten in a frame moving with the density profile and this leads to an Eq. (1) where the potential $\mathcal{V}(x)$ is static. The time Fourier transform of this new equation yields the time-independent Schrödinger equation $[-\Delta+\mathcal{V}(x)]\bar{\psi}=\mathcal{E}\bar{\psi}$, where the energy $\mathcal{E}=\omega^2/3v_T^2$; $\omega=2\pi f_e$, where f_e is the frequency of the electronic mode. All the known results about localization are hence applicable. In the special case where the ion-acoustic spectrum consists of a single wave with wavelength λ_s and amplitude φ_s, in the frame moving with the wave, $n(x)=[1+(e\varphi_s/\kappa T)\cos(2\pi x/\lambda_s)]\bar{n}$, where $\bar{n}$ is the average density. The Schrödinger equation then reduces to the Mathieu equation,

$$\frac{\partial^2\bar{\psi}}{\partial X^2}+(a-2q\cos 2X)\bar{\psi}=0\,, \qquad (2)$$

where $X=\pi x/\lambda_s$. This equation depends on two parameters

$$a=(2\lambda_s/\lambda_e)^2,\quad q=\frac{a}{2}\frac{f_p^2}{f_e^2-f_p^2}\frac{e\varphi_s}{\kappa T}\,, \qquad (3)$$

where $f_p=[\bar{n}e^2/\varepsilon_0 m]^{1/2}/2\pi$ is the average plasma frequency and

$$\lambda_e=[3v_T^2/(f_e^2-f_p^2)]^{1/2} \qquad (4)$$

is the electronic wavelength in the absence of density fluctuation. It is well known [13] that, in the (a,q) parameter plane, around each value of $a=i^2$ (i integer), there is a gap whose width increases with q. In these gaps, the solutions to the Mathieu equation, instead of being purely oscillatory, exhibit a spatially exponential behavior. Thus by varying the ion-acoustic wave frequency $f_s=k_s c_s/2\pi$ while keeping $\bar{n}$, f_e, and φ_s constant, we can experimentally explore the (a,q) parameter plane along a line that crosses these various gaps and observe purely oscillatory behavior alternating with spatially exponentially damped behavior for the electronic wave. This damping has nothing to do with any dissipative mechanism and, if strong enough, adds to underlying Landau damping of the electron wave in the absence of density fluctuations, as is observed in the experiment. A similar behavior has been previously reported for ripples on an electron beam [14]. A Mathieu equation has also been previously introduced to describe an experiment on electron plasma wave propagation in the presence of ion fluctuations but these fluctuations were purely temporal and the possibility of wave-number resonance and spatial localization was remote [15].

The experiment is performed in a double-plasma device [16] with multipolar magnetic confinement [17]. The vacuum chamber consists of a cylinder (52 cm long and 34 cm inner diameter) with twelve regularly spaced parallel rows of permanent magnets mounted on the outside. It is divided into two unequal parts by a system of three closely spaced parallel plane grids perpendicular to the cylinder axis at 10 cm from one end of the machine. The middle grid is biased positively and, by applying on it a fluctuating potential, is used to launch the plane electron plasma wave. The two outer grids, 7.6 mm apart, are grounded and provide an electromagnetic shielding for the wave. The base pressure is equal to 2×10^{-8} Torr; argon gas is used at 2×10^{-4} Torr pressure. The plasma is created by a discharge from a set of emissive filaments negatively biased at V_D. The electron plasma wave dispersion relation is very sensitive to the presence of primary electrons emitted by the filaments; the primary electrons tend to create an undesirable beamlike branch on the dispersion curve. In order to suppress this effect, we used two filaments located in the smaller source chamber with $V_D=-30$ V. The plasma diffuses through the grids into the target chamber where a Maxwell demon [18] controls the electron temperature T. We obtain a target plasma with an average plasma density $\bar{n}$ uniform to 10% on a distance of 15 cm from the outer grid and on a radius of 10 cm. In this plasma, we have launched an electron wave of frequency f_e and measured its wavelength λ_e by interferometry with a moving probe and have thus carefully checked that the electron Langmuir wave satisfies a Bohm-Gross–like dispersion relation given by Eq. (4). In the condition of the experiment, fitting the experimental results with Eq. (4) yields $f_p=141.5$ MHz and $T=6.6$ eV, in agreement with the average plasma density and temperature independently measured on a Langmuir probe.

By adjusting the source plasma density and wall potential, an ion beam is produced with relative density of order 1 and variable velocity. By applying a purely sinusoidal signal of amplitude $V_{\rm exc}$ and frequency f_s on a plane plate at the end of the source plasma, we can thus launch an ion-acoustic wave that propagates almost without damping in the entire target plasma. The density fluctuations are measured with an axially and azimuthally movable spherical probe (2 mm diameter). Boxcar averaging the probe signal allows one to get a snapshot $\tilde{n}$ of the plasma density as a function of the distance x from the system of separating grids for a given phase of the ion-acoustic wave. The measured plasma density fluctuations for $V_{\rm exc}=1$ V (peak to peak) with $f_s=195$ and 67.5 kHz are shown in Figs. 1(a) and 1(b). Such measurements give both the wavelength λ_s and the mean relative density fluctuation $\delta n/\bar{n}=e\varphi_s/\kappa T$. It is experimentally easy to check that the ion-acoustic wave obeys the nondispersive relation $\lambda_s=v_p/f_s$, where $v_p=1.85$ km/s. Knowing that $c_s=3.98$ km/s, we deduce that the ion beam velocity $u=1.4c_s$, in agreement with the plasma potential difference $\Delta V_p=7$ V of the source and target plasma measured with two fixed Langmuir probes. We notice that, at the lower frequency [Fig. 1(b)], the signal contains second and third harmonics [frequency $(2\text{–}3)f_s$, wavelength $\lambda_s/(2\text{–}3)$] of the ion-acoustic wave. These harmonics are observed whenever $f_s\le 160$ kHz and cannot be easily suppressed for the driving voltage used in the experiment.

We now set the receiving probe at a given distance $x_p=5$ cm from the grid where an electron plasma wave is launched with a frequency f_e. The signal received on the

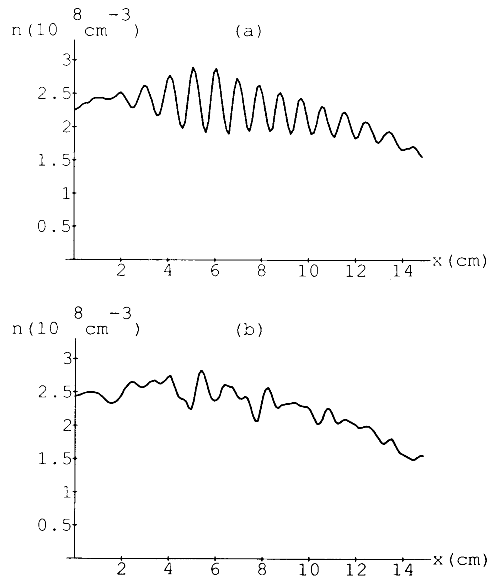

FIG. 1. Boxcar-averaged density modulation vs distance from the grid; $V_{\rm exc}=1$ V. (a) $f_s=195$ kHz, (b) $f_s=67.5$ kHz.

fixed probe is passed through a crystal detector, which gives the square of the amplitude of the wave at this position, and is then sent to the Y channel of an X-Y recorder. A plasma average density modulation is produced as described before. The frequency f_s of this modulation is continuously varied and a dc voltage proportional to f_s is applied to the X channel of the recorder. For a given value of f_e, if we assume that, when $V_{\rm exc}$ is fixed, the density modulation and hence φ_s do not vary with f_s, moving along the X axis corresponds to exploring the (a,q) plane of Eq. (3) along a straight line $2q/a=$const that crosses the various Mathieu gaps. The result is displayed by the continuous curves of Fig. 2 for $f_e=150$ MHz and various values of $V_{\rm exc}$, and of Fig. 3 for $V_{\rm exc}=0.4$ V and various values of f_e. Each curve of Fig. 3 is contained in a box of height 1 and labeled by its value of f_e. For each curve on these figures, the Langmuir wave energy is normalized to its value in the absence of ion-acoustic density modulation and we observe that the electronic wave amplitude decreases for certain values of f_s, or correspondingly certain values of λ_s as given by the nondispersive ion-acoustic dispersion relation.

To each experimental continuous curve, we can associate a measured electronic wavelength as given by Eq. (4); it is easy to check that the observed wave amplitude depletion always occurs around a value of the ratio $2\lambda_s/\lambda_e$ equal to an integer, as predicted by Eq. (2). More precisely, we have superposed on Fig. 2 and Fig. 3 dashed curves which correspond to $\exp(-k_i x_p)$, where k_i is the spatial damping predicted by the Mathieu equation [13]. In Fig. 2, we recognize three groups of curves corresponding from right to left to the $a=1$, $a=4$, and $a=9$ gaps. The lower (upper) dashed curves correspond to $\delta n/\bar{n}$

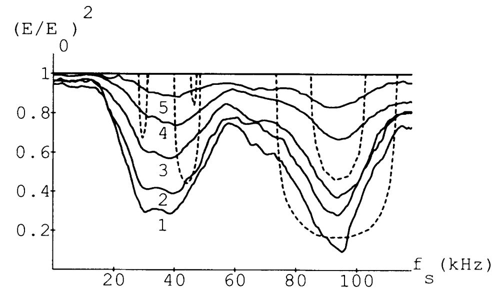

FIG. 2. Square of the normalized electron plasma wave amplitude at $x_p=5$ cm from the emitter vs f_s, for $f_e=150$ MHz and $V_{\rm exc}=1.4$, 1.2, 1.0, 0.8, and 0.6 V (continuous curves 1-5, respectively). $\text{Exp}(-k_i x_p)$ from the Mathieu equation with $2q/a=0.88$ (lower dashed curves) and 0.38 (upper dashed curves), for the $a=1$ (right), 4 (middle), and 9 (left) gaps.

$=0.11$ (0.05) or $2q/a=0.88$ (0.38) as given by Eq. (3) in the conditions of the experiment. We notice that the measured damping is bigger for the $a=4$ and $a=9$ gaps than the predicted damping. Such a behavior can be explained by taking into account the presence of stronger harmonics of the ion-acoustic density modulation at lower frequency, as we have already noticed in Fig. 1. The position of a gap on the abscissa is only determined by the wavelength ratio of the ion-acoustic density modulation to the electronic wave. But, as the ion-acoustic wave is nondispersive and the electronic wavelength is fixed for Fig. 2, the same position for a gap on the abscissa can correspond to different values of a; for example, an $a=4$ gap ($\lambda_s=\lambda_e$) for the ion-acoustic wave is superposed onto the $a=1$ gap for its second harmonic whose wavelength is $\lambda_s/2$. In each box of height 1 for the normalized electronic wave amplitude of Fig. 3, we have only superposed the theoretical prediction $\exp(-k_i x_p)$ for the $a=1$ gap with $\delta n/\bar{n}=0.05$, since the prediction for the $a=4$ and $a=9$ gaps shows the same behavior as in Fig. 2. We emphasize that no special fit has been made to obtain these curves and that they result only from the independently measured ion-acoustic and electron Langmuir wave

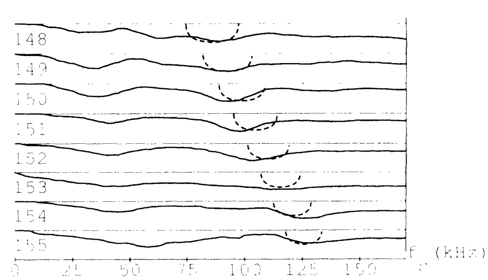

FIG. 3. Square of the normalized electron plasma wave amplitude at $x_p=5$ cm from the emitter vs f_s, for $V_{\rm exc}=0.4$ V and various f_e labeled in MHz on each curve contained in a box of height 1. $\text{Exp}(-k_i x_p)$ from the Mathieu equation with $\delta n/\bar{n}=0.05$ (dashed curve) for the $a=1$ gap.

dispersion relation. A strikingly good agreement is therefore obtained with the Mathieu theory.

We have reported experimental results that show the influence of density modulation on the propagation of an electron plasma wave. We have considered the case of a purely sinusoidal modulation where the wave amplitude is shown to obey a Mathieu-type equation. This is a crucial step for the observation of Anderson localization in a plasma. A further step would consist in replacing the purely sinusoidal modulation by a random one, as produced by an arbitrary wave-form generator, for example. The plasma seems like a promising medium for such studies since it allows both the randomness and the amplitude of the density fluctuations to be externally varied in a continuous way.

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