Two-Dimensional Fast Penetration of a Magnetic Field into a Homogeneous Plasma

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A fast penetration of a magnetic field into an initially homogeneous plasma is demonstrated. The penetration is induced by a density gradient along the current lines that is formed by the magnetic pressure in the two-dimensional flow. The penetration occurs for any magnitude of the plasma collisionality. If the collisionality is high enough the shock structure is determined by the resistivity, while in the low-collisionality case it is determined by the electron inertia.

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A magnetic field usually penetrates into a plasma to the (collisionless or collisional) skin depth. However, as has been recently shown [1,2], when the magnetic field is strong (the cyclotron frequency of the electron is larger than its collision frequency), nonlinear effects allow a fast, deeper penetration of the magnetic field into a plasma, provided the plasma is inhomogeneous. In that case the magnetic field penetrates into the plasma because the electrons move on time-varying orbits, along which B/n is constant (B is the magnetic field amplitude and n is the plasma density). The velocity of penetration u, perpendicular to the density gradient, is $(cB/8\pi en)|\nabla \ln n|$, where $-e$ is the electron charge and c the light velocity in vacuum. The shock structure in these studies [1,2] was determined by collisions. In a later study, the fast collisionless evolution of a magnetic field in the neighborhood of the electrodes was shown to result from the necessarily two-dimensional (2D) electron flow and the electron inertia [3]. Most recently one of us has shown that the shock structure in the fast penetration due to a density gradient can be determined in the collisionless case by the electron inertia, if the flow is two dimensional [4]. In the collisionless evolution no energy was dissipated in the bulk of the plasma, while in the collisional case the dissipation was shown to be large [5].

In all these previous studies the plasma was assumed to have a density gradient from the start. In this Letter we demonstrate magnetic field penetration into an initially homogeneous plasma, where the density gradient along the current lines arises due to magnetic pressure in the 2D flow. The velocity of penetration is larger than the mass velocity, and therefore the compression of the plasma is small. The penetration occurs for any magnitude of the plasma collisionality. If the collisionality is high enough the shock structure is determined by the resistivity, while in the low-collisionality case it is determined by the electron inertia. The initial configuration consists of an unmagnetized homogeneous plasma, adjacent to a vacuum region that is permeated by a magnetic field of only one nonzero component. In such a configuration the magnetic field cannot propagate into the plasma as a whistler wave or as any other plasma wave. The penetration that we describe here follows an initial small nonuniform penetration that could result, for example, from a faster penetration of the magnetic field along a cathode that emits the electrons. Our mechanism could therefore explain fast magnetic field penetration into plasmas that carry current between electrodes, such as in the plasma opening switch (POS) [6] or in the Z pinch. Such a penetration may occur also in various other plasmas, as a result of current channels of nonuniform widths.

The evolution of the plasma and of the magnetic field is described by the continuity equation

$$\frac{\partial n}{\partial t}+\frac{\partial V_z}{\partial z}=0\,, \tag{1}$$

the momentum balance equation

$$\frac{\partial V_z}{\partial t}+\frac{1}{2}\frac{\partial}{\partial z}b^2=0\,, \tag{2}$$

the equations of motion of the electron

$$\varepsilon \mathbf{v}\cdot\nabla v_x=-(E_x-v_z b)-\eta v_x\,, \tag{3a}$$

$$0=E_z+v_x b\,, \tag{3b}$$

Ampere's law

$$\mathbf{v}=-(1-n)\nabla\times\mathbf{b}\,, \tag{4}$$

and Faraday's law

$$-\frac{\partial b}{\partial t}=\frac{\partial E_x}{\partial z}-\frac{\partial E_z}{\partial x}\,. \tag{5}$$

The dimensionless quantities are the magnetic field b normalized to B_0, the density perturbation n normalized to the unperturbed density n_0, the time normalized to the ion cyclotron period ω_{ci}^{-1} ($\equiv Mc/eB_0$, where M is the ion mass), the coordinates normalized to the ion skin depth c/ω_{pi} [$\equiv(Mc^2/4\pi n_0 e^2)^{1/2}$], the electron velocity $\mathbf{v}$ and the ion velocity $\mathbf{V}$ normalized to V_A ($\equiv c\omega_{ci}/\omega_{pi}$), the electric field $\mathbf{E}$ normalized to B_0V_A/c, and the resistivity η normalized to B_0/n_0ec. Also, $\varepsilon\equiv m/M$ is the electron-to-ion mass ratio. The only nonzero component of the magnetic field is in the direction of the ignorable coordinate y. In writing the equations in the above form we assumed that the z derivatives are larger than the x derivatives, so that L_z/L_x, v_z/v_x, and V_x/V_z are all small. By L_x and L_z we denote the characteristic lengths in the two directions. We further assumed that the velocity of penetration u is much larger than V_z so that the compression is small and n is much smaller than unity. On the other hand, in order to neglect the partial time derivative in the electron equation of motion, we assume that u is much smaller than v_z, implying that the flow is indeed two dimensional. The various velocities therefore satisfy the relations

$$V_x \ll V_z \ll 1 \ll u \ll v_z \ll v_x \,. \tag{6}$$

The choice of this particular ordering corresponds to a magnetic field penetration in the z direction, where the velocity of penetration is x dependent. Such x dependence can result, for example, from a faster penetration of the magnetic field along the cathode. Looking for a solution where all quantities depend on $z-u(x)t$, we find from Eqs. (1) and (2) that

$$V_z=\frac{b^2}{2u},\quad n=\frac{b^2}{2u^2}\,. \tag{7}$$

Combining Eqs. (3)–(5), we obtain

$$\frac{\partial b}{\partial t}=-\frac{1}{2}\left(\frac{\partial n}{\partial z}\frac{\partial b^2}{\partial x}-\frac{\partial n}{\partial x}\frac{\partial b^2}{\partial z}\right)+\varepsilon\left(\frac{\partial b}{\partial z}\frac{\partial}{\partial x}-\frac{\partial b}{\partial x}\frac{\partial}{\partial z}\right)\frac{\partial^2 b}{\partial z^2}+\eta\frac{\partial^2 b}{\partial z^2}\,. \tag{8}$$

We will consider two cases. In the first case the collisionality is high enough so that the third term on the right-hand side (RHS) of Eq. (8) is much larger than the second term. In the second case, of low collisionality, the second term on the RHS of Eq. (8) is much larger than the third term.

Let us examine first the collisional case. We neglect the electron inertia, the second term on the RHS of Eq. (8). A self-similar solution is obtained for $b=b(\xi)$, where $\xi=[z-u(x)t]x^{-1/3}$, and $u(x)=u_0x^{-1/3}$. For a penetration into an unmagnetized plasma Eq. (8) becomes

$$\eta\frac{db}{d\xi}=-u_0 b-\frac{b^4}{6u_0^2}\,. \tag{9}$$

It has the shock solution

$$b=\left[\frac{-\exp(-3u_0\xi/\eta)}{1+\exp(-3u_0\xi/\eta)}\right]^{1/3}, \tag{10}$$

where $u_0=6^{-1/3}$. The thickness of the shock is $\theta(\eta)$ and thus $L_z=\theta(\eta x^{1/3})$. Defining $\delta_1\equiv z/x$, $\delta_2\equiv x^{1/3}$, and $\delta_3\equiv t/x^{4/3}$, we find that $v_x=\theta(\eta^{-1}\delta_2^{-1})$, $v_z=\theta[\eta^{-1}\times\delta_2^{-1}(\delta_1+\delta_3)]$, $u=\theta(\delta_2^{-1})$, $V_z=\theta(\delta_2)$, and $V_x=\theta[\delta_2\times(\delta_1+\delta_3)]$. In order that $v_z\gg u$, we require that $\delta_1+\delta_3\ll\eta$. In that case inequalities (6) are satisfied. It is easy to show that

$$u\ll\eta^{-1/2}\,. \tag{11}$$

Let us now turn to the case of low collisionality. We start by neglecting the resistive tem, the third term on the RHS of Eq. (8). A self-similar solution is obtained for $b=b(\xi)$ as well, but here $\xi=[z-u(x)t]x^{1/3}$ where $u(x)=u_0x^{-1/3}$. In this collisionless case Eq. (8) becomes

$$\frac{2}{3}\varepsilon\frac{d^2b}{d\xi^2}+\frac{2}{3}\frac{b^3}{u_0^2}+u_0=0\,. \tag{12}$$

This is an equation for a nonlinear oscillator. We require that $db/d\xi=0$ for $b=0$ and for $b=-1$, and therefore Eq. (12) is integrated to

$$\varepsilon\left(\frac{db}{d\xi}\right)^2+\left(\frac{9}{2}\right)^{1/3}(b+b^4)=0\,. \tag{13}$$

The velocity is $u_0=6^{-1/3}$. The solution of Eq. (12) is periodic but the more physical solution is the shock solution since any nonzero dissipation will destroy the periodic solution. To show this we add the small collisional term

$$\eta\left\{\frac{d}{d\xi}\left[\ln\left(\frac{db}{d\xi}\right)\right]\right\}$$

to Eq. (12). Equation (13) is then

$$\varepsilon\left(\frac{db}{d\xi}\right)^2+\left(\frac{9}{2}\right)^{1/3}(b+b^4)+\eta\frac{db}{d\xi}=0\,. \tag{14}$$

Any nonzero η makes the points at which $db/d\xi=0$ fixed points. We therefore approximate the solution in the limit of small collisionality as follows: If half a period of the nonlinear oscillator is ξ_0, then for $0\le\xi\le\xi_0$, $b(\xi)$ is the solution of Eq. (12) while for $\xi<0$, $b(\xi)=-1$ and for $\xi>\xi_0$, $b(\xi)=0$.

In order to have a shock solution the resistivity has to be nonzero. It is easy to see, however, that if the resistivity is reduced the dissipation decreases as well. The dissipation vanishes in the limit of zero resistivity. This is different from the collisional case, in which the energy dissipation is independent of the magnitude of the resistivity. In this sense in the limit of zero resistivity the shock is collisionless. Also, in the low-collisionality case the shock structure is determined by the electron inertia and not by the resistivity.

The thickness of the shock ξ_0 is $\theta(\varepsilon^{1/2})$ and thus $L_z=\theta(\varepsilon^{1/2}\delta_2^{-1})$. It is possible that there will be kinetic instabilities associated with the narrow shock front. These instabilities could be important if they can grow significantly during the time that an electron spends inside the shock front, which is the electron-ion hybrid cyclotron period. In the low-collisionality case we find that

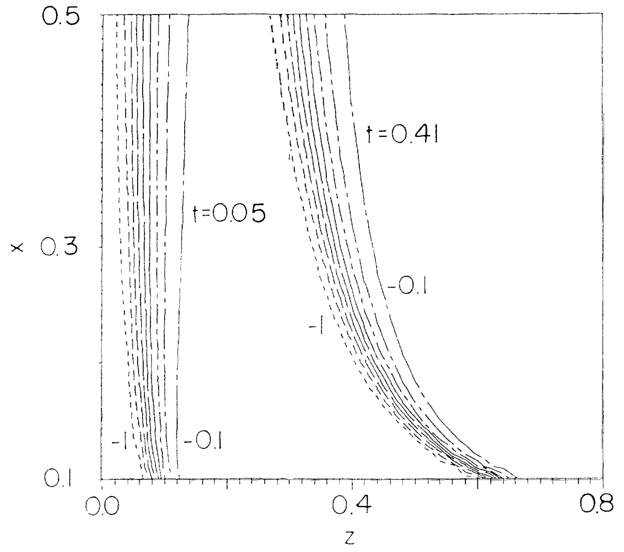

FIG. 1. The collisional shock. Plotted are the magnetic field contour lines, which correspond to values of b going from -1 to -0.1, for $t=0.05$ and $t=0.41$. $\eta=0.03$.

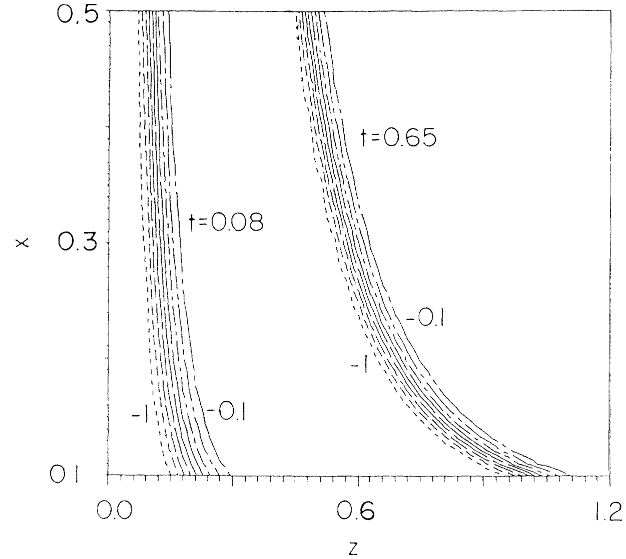

FIG. 2. The low-collisionality shock. Plotted are the magnetic field contour lines, which correspond to values of b going from -1 to -0.1, for $t=0.08$ and $t=0.65$. $\epsilon=0.01$ and $\eta/\epsilon=0.16$.

$$v_x=\theta(\delta_2\varepsilon^{-1/2}),\quad v_z=\theta(\delta_1\delta_2\varepsilon^{-1/2}),\quad u=\theta(\delta_2^{-1}),$$
$$V_z=\theta(\delta_2),\quad V_x=\theta(\delta_1\delta_2)\,. \tag{15}$$

In order that $v_z \gg u$, we require that $zx^{-1/3}>\varepsilon^{1/2}$. In that case inequalities (6) are satisfied. We also find that

$$u \ll \varepsilon^{-1/8}\,. \tag{16}$$

In dimensional units the shock velocity is $V_A(c/\omega_{pi}x)^{1/3}$, while the shock thickness is $(c/\omega_{pe})(c/\omega_{pi}x)^{1/3}$, where c/ω_{pe} is the electron skin depth.

In both collisional and low-collisionality cases the penetration of the magnetic field is larger at smaller values of x. This larger penetration could result from a faster penetration of the magnetic field along a cathode that emits the electrons. The mechanism could therefore explain fast magnetic field penetration into plasmas that carry current between electrodes, such as in the POS or in the Z pinch.

Figures 1 and 2 show the shock propagation of the magnetic field in the collisional case [Eq. (10)] and in the low-collisionality case [Eq. (14)], respectively. Plotted are the magnetic field contour lines (along which the current flows) for two different times. In Fig. 1, $\eta=0.03$. In Fig. 2, $\epsilon=0.01$ and $\eta/\epsilon=0.16$. For a plasma of density 10^{14} cm^{-3} and a magnetic field of 10 kG, $\eta=0.03$ corresponds to an electron temperature of 1 eV. Had a smaller, more realistic, mass ratio been used in the calculation, the current channel in Fig. 2 would have been narrower. The plasma compression is such that $n=0.56$ at $x=0.2$. At larger values of x the compression is larger and the approximation is less good.

In both cases, of high and low collisionality, the pushing of the plasma in the 2D flow causes density variation along the current lines. The density is not a function of ξ only, but also of x, $n=n(x,\xi)$. The density increases along the current lines, and as a result a fast penetration of the magnetic field ensues.

In a more complicated geometry, in which the magnetic field is of more than one nonzero component, there could possibly be additional mechanisms for field penetration. What we have shown here is that even in a geometry that could be approximated as a 2D geometry, a situation typical of various plasmas, the magnetic field can penetrate into the plasma.

As is usually the case with self-similar solutions, their relation to a physical solution with given initial boundary conditions has to be explored further. However, our self-similar solutions do show the possibility of fast penetration.

In summary, we demonstrated the possibility of a fast magnetic field penetration into an initially homogeneous plasma. The pushing of the plasma in the 2D flow generates a density gradient along the current lines. This in turn induces a penetration of the magnetic field that is faster than the plasma hydrodynamic motion. The penetration occurs also in the low-collisionality case in which there is no bulk energy dissipation. This mechanism could explain fast magnetic field penetration into the plasmas in the POS or in the Z pinch, and possibly also in other plasmas as a result of current channels of nonuniform widths.

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